CHAPTER 3

PROPELLANT AND ROCKET PERFORMANCE

3.1. INTRODUCTION

This chapter is devoted to the analysis of rocket and propellant performance. Indeed, with the available data we have from the different rocket tests, especially from the pressure and thrust measurements, it is possible to derive a lot of interesting information that characterise the performance of the propellant and that of the rocket motor. These data compared with the thermodynamical calculations of the propellant will show the efficiency at which the propellant is burned in the rocket motor. Moreover, it will also show the weak points of our construction, and the ways to improve them. It will also be possible to calculate the rocket performance under other, completely different circumstances.

3.2. THE THRUST COEFFICIENT

When pressure and thrust are measured together, it is possible to derive the thrust coefficient:

$$C_f = \frac{F}{P_c A_t} \qquad (3.1)$$

In some rockets, we have done these measurements. The result of this analysis is given in figure 3.1.

The value of the Cf coefficient is also dependent upon the ratio of the exit area to the throat area (Ae/At). In our case we had the following values:

- NEBEL : 10,24
- CANDY : 9,75
- GX- : 19,80

These differences however do not appear in the calculation of Cf.

From the thermodynamical calculations we know the ratio of the heat capacities (Cp/Cv) to be equal to:

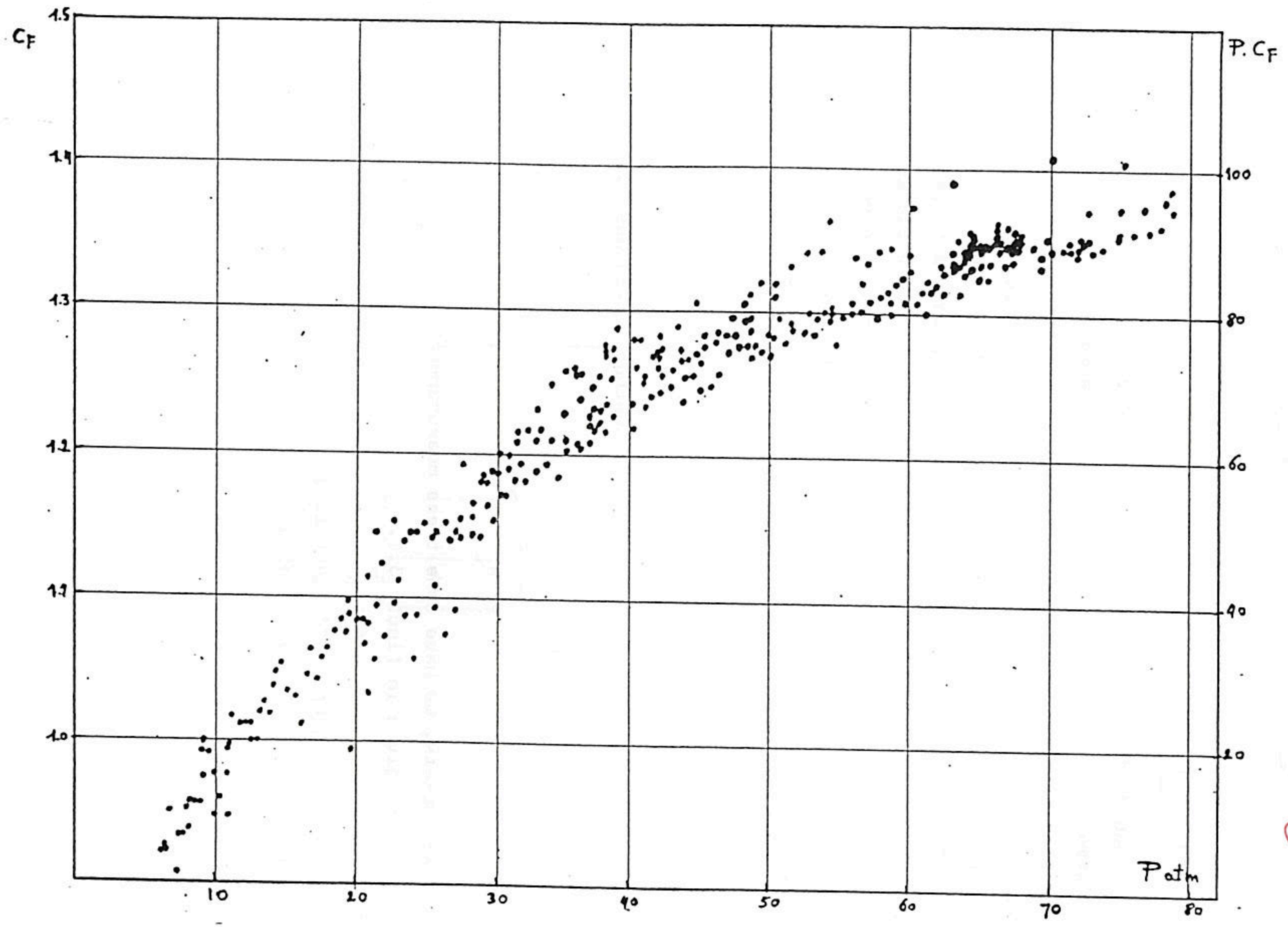

Fig.3.1. Values of the thrust coefficient as a function of chamber pressure,derived from different tests.

$$\gamma = 1{,}2447$$

Since the ratio of Ae to At and the chamber pressure is known, it is possible to calculate the outletpressure Pe as a function of the chamber pressure:

$$\frac{Ae}{At} = \frac{\left(\frac{\gamma - 1}{2}\right)^{1/2} \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma + 1}{2(\gamma - 1)}}}{\left(\frac{Pe}{Pc}\right)^{1/\gamma} \left(1 - \left(\frac{Pe}{Pc}\right)^{\frac{\gamma - 1}{\gamma}}\right)} \qquad (3.2)$$

For the Cf coefficient under ideal circumstances, we know the expression:

$$Cf_i = \sqrt{\frac{2\gamma^2}{\gamma - 1}\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma + 1}{\gamma - 1}} \left(1 - \left(\frac{Pe}{Pc}\right)^{\frac{\gamma - 1}{\gamma}}\right)} + \left(\frac{Pe - Pa}{Pc}\right)\frac{Ae}{At} \qquad (3.3)$$

Applying the known values of γ, Pc , Pe , Ae and At in this formula, we find Cf values which are higher than the values we found in our experiments! The difference is due to several reasons. First of all the velocity of the gases at the exit of the nozzle is not perpendicular to the axis of the nozzle, and since we measure thrust only in that direction, there is a loss. Secondly there are losses in the nozzle due to friction of the gases with the wall. One should also consider the fact that there are turbulances and that there is a heat loss.

The losses due to the fact that the nozzle is conical, can easily be calculated:

$$\lambda = \frac{1 + \cos\theta}{2} \qquad (3.4)$$

θ being the half angle of the divergent section of the nozzle. In all our cases this value was 15°. The correction factor λ than becomes:

$$\lambda = 0{,}983$$

The other losses are not easy to calculate from a theoretical point of view. From the comparison of the Cf data elaborated from the measurements, with the theoretical values, we found a correction coefficient of 0,868.

This means that for our rockets filled with a mixture of 60% of potassiumnitrate and 40% of sugar, the following expression for the thrust coefficient is valid:

$$Cf = \lambda \zeta_F \left(\sqrt{ \frac{2\gamma^2}{\gamma - 1} \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{\gamma-1}} \left(1 - \left(\frac{Pe}{Pc} \right)^{\frac{\gamma-1}{\gamma}} \right) } + \left(\frac{Pe - Pa}{Pc} \right) \frac{Ae}{At} \right) \qquad (3.5)$$

with: $\gamma = 1{,}2447$
$\lambda = 0{,}983$
$\zeta_F = 0{,}883$

It is easy to show that from a given pressure on the above given expression for Cf is no longer valid. This is due to the fact that the gases no longer expand till the end of the nozzle. At a given section of the nozzle there is a flow separation. (fig.3.2).

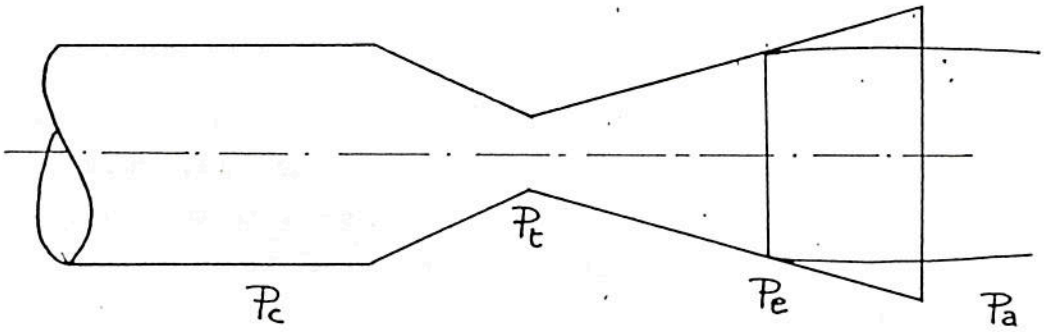

Fig.3.2. Flow separation due to overexpansion in the nozzle

This means that it is no longer possible to consider Ae to be equal to the real exit area of the nozzle. In reality Ae will be smaller. The point know is to find the value of the exit pressure (lower than the outside atmospheric pressure) at which the gasstream will separate from the nozzle wall. This is the socalled Summerfield criterion. In order to find this value of the pressure we will use the following method. We assume a value of Pe and we use this value in the expression of Ae/At. The value for Ae/At we get with this calculation will be used in the expression for Cf. If the calculated and the measured Cf values are not the same, Pe is adjusted and the calculation is repeated. For our experiments we came to a mean value of 0,45 bar. This value is higher than what is normally found (0,36). The difference is probably due to the fact that in our case the nozzle walls are less smouth than in the case of nozzles used by professionals.

To be strict the Summerfield criterion should be expressed as a function of the atmospheric pressure. This means that the results of our calculation can we written as:

$$\frac{Pe}{Pa} \geq 0{,}45$$

Comparing the ζ_F factor with lit erature one will find that our result is lower. This means that we have higher losses. It is clear that also this has to be attributed to the roughness of the nozzle wall, and to the sharp edges near the throat.

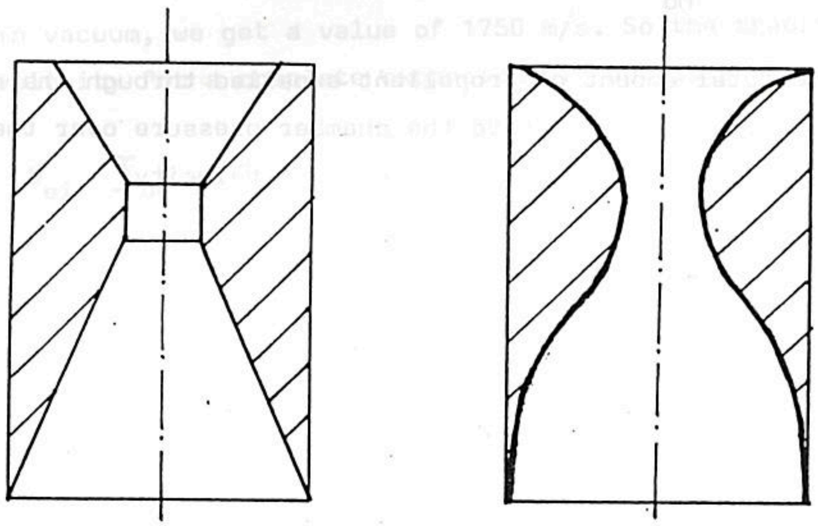

Fig.3.3. Differences between the nozzles used and the so called perfect nozzles.

3.3. THE CHARACTERISTIC VELOCITY

The characteristic velocity is defined by:

$$C^* = \frac{Pc\ At}{\dot{m}} \qquad (3.6)$$

It is a constant for a given propellant since it only depends upon the thermodynamical parameters. This can easily be checked by the following expression valid under ideal circumstances:

$$C_i^* = \sqrt{\frac{R\ Tc}{M}\ \frac{1}{\gamma}\ \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{\gamma-1}}} \qquad (3.7)$$

This expression doesn't take into account the heat losses. In reallity one will not find the same value as this calculation will give.

In order to find the characteristic velocity of our propellant, we need to know the mass flow $\dot{m}$ for any given pressure in the chamber . This is possible with our knowledge of Cf. The calculation is however large and complicated. Therefor we will use an other approuch. Indeed when we integrate (3.6) we find:

$$C^* = \frac{At \int_0^{t_b} Pc\ dT}{Mb} \qquad (3.8)$$

Mb being the total amount of propellant expelled through the nozzle. So it is sufficient to integrate the chamber pressure over the burning time in order to get the characteristic velocity.

The following tabel gives the results for different tests

ROCKET	CHARACTERISTIC VELOCITY
CANDY 3	1001,7
GX-44	831,5
NEBEL 2	936,8
NEBEL 3	1134,8
NEBEL 4	1006,2
NEBEL 6	964,0

The mean value is 979 m/s which is very close to the theoretical value of 991 m/s. This gives us the possibility to calculate the loss coefficient for the characteristic velocity:

$$\xi_{C^*} = \frac{979\ m/s}{991.\ m/s} = 0{,}988$$

or:

$$C^* = 0{,}988\ C_i^*$$

3.4. THE EXHAUST VELOCITY

The exhaust velocity is a function of the chamber pressure for a given nozzle and can now be calculated for potassiumnitrate and sugar propellants:

$$U_e = Cf_i \, \zeta_F \, C_i^* \, \zeta_{C^*} \tag{3.9}$$

$$U_e = 0{,}872 \; Cf_i \, C_i^* = \zeta_{Ue} \; U_{ei} \tag{3.10}$$

$$U_e = \zeta_{Ue} \; U_{ei} \tag{3.11}$$

This is the real value of the exhaust velocity and it does not take into account the projection of the velocity along the nozzle axis. For flight calculations and in order to compare with test results we have to multiply Ue with λ.

For a complete expansion of the gases, which is of course only possible in vacuum, we get a value of 1750 m/s. So the theoretical exhaust velocity for a complete expansion without losses will become:

$$U_{ei} \, \zeta_{Ue} = 1750 \text{ m/s}$$

or: $U_{ei} = 2007$ m/s (compared with $U_{etheoretical} = 1975$ m/s)

3.5. THE THRUST

The thrust measured on the test bench is written as:

$$F = \lambda \, U_e \, \dot{m} \tag{3.12}$$

or: $$F = \lambda \, \zeta_{C^*} \; \zeta_{Cf} \; C_i \; Cf_i \; \dot{m} \tag{3.13}$$

For $\theta = 15°$:

$$F = 0{,}857 \; C_i^* \; C_{fi} \; \dot{m}$$

3.6. CONCLUSIONS

From this analysis we can see that there is a very good agreement between the values derived from the thermodynamical calculations and the values from the tests. This proves of course that both the calculations and the measurements were made properly.

We are confident that if we could use well rounded and polished nozzles , the exhaust velocity would significantly increase. With good nozzles the losses may change in the following way:

λ = 0,983 to 0,99 with Bell shape
ζ_F = 0,883 to 0,92 with proper roundings and smouth surfaces.

The overall loss would than change from:

0,983 x 0,883 x 0,988 = 0,857

to: 0,99 x 0,92 x 0,978 = 0,891

This would give an increase of:

$$\frac{0,891 - 0,857}{0,857} \times 100 = 4\ \%$$

So the mean specific impulse from our tests of 108,8 s would increase to 113 s, or NEBEL 4 with its specific impulse of 137 s would have given 142,5 s.