Derivation of Equation (3-25)

$$\boxed{\frac{A_t}{A_x} = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}} \left(\frac{p_x}{p_1}\right)^{1/k} \sqrt{\frac{k+1}{k-1}\left[1-\left(\frac{p_x}{p_1}\right)^{\frac{k-1}{k}}\right]}} \quad (3\text{-}25)$$

using

$$\frac{V_y}{V_x} = \left(\frac{p_x}{p_y}\right)^{1/k} \quad (3\text{-}6) \qquad V_t = V_1 \left(\frac{k+1}{2}\right)^{1/k-1} \quad (3\text{-}21)$$

$$v_x = \sqrt{\frac{2gk}{k-1} RT_1\left[1-\left(\frac{p_x}{p_1}\right)^{\frac{k-1}{k}}\right]} \quad (3\text{-}15) \qquad v_t = \sqrt{\frac{2gk}{k+1} RT_1} \quad (3\text{-}23)$$

from continuity, $\frac{A_t v_t}{V_t} = \frac{A_x v_x}{V_x}$, or $\frac{A_t}{A_x} = \frac{V_t v_x}{V_x v_t}$

$$\Rightarrow \quad \frac{A_t}{A_x} = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}} \frac{V_1}{V_x} \frac{v_x}{v_t} = \left(\frac{k+1}{2}\right)^{1/k-1} \left(\frac{p_x}{p_1}\right)^{1/k} \frac{v_x}{v_t}$$

$$= \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}} \left(\frac{p_x}{p_1}\right)^{1/k} \sqrt{\frac{\frac{2gk}{k-1} RT_1\left[1-\left(\frac{p_x}{p_1}\right)^{\frac{k-1}{k}}\right]}{\frac{2gk}{k+1} RT_1}}$$

$$\Rightarrow \quad \boxed{\frac{A_t}{A_x} = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}} \left(\frac{p_x}{p_1}\right)^{1/k} \sqrt{\frac{k+1}{k-1}\left[1-\left(\frac{p_x}{p_1}\right)^{\frac{k-1}{k}}\right]}}$$