Derivation of Equation (3-24)

$$\boxed{\dot{w} = A_t P_1 g \frac{k\sqrt{[2/(k+1)]^{\frac{k+1}{k-1}}}}{\sqrt{gkRT_1}}} \quad (3\text{-}24)$$

using equations (3-2), (3-21), (3-23) and $p = \rho RT$ eqn of state

$$\left\{ \begin{array}{ll} \dot{w} = \dfrac{A_t v_t}{V_t} \quad (3\text{-}2) & V_t = V_1 \left(\dfrac{k+1}{2}\right)^{1/k-1} \quad (3\text{-}21) \\ v_t = \sqrt{\dfrac{2gk}{k+1} RT_1} \quad (3\text{-}23) & \end{array} \right.$$

$$\Rightarrow \quad \dot{w} = \frac{A_t}{V_t}\sqrt{\frac{2gk}{k+1}RT_1}$$

since $V_t = V_1\left(\frac{k+1}{2}\right)^{1/k-1}$,
$$\dot{w} = \frac{A_t}{V_1\left(\frac{k+1}{2}\right)^{1/k-1}}\sqrt{\frac{2gk}{k+1}RT_1}$$

and $V_1 = \frac{RT_1}{P_1}$
$$\dot{w} = \frac{A_t P_1}{RT_1\left(\frac{k+1}{2}\right)^{1/k-1}}\sqrt{\frac{2gk}{k+1}RT_1}$$

$$= A_t P_1 \sqrt{\frac{2gkRT_1}{R^2 T_1^2 \left(\frac{k+1}{2}\right)^{2/k-1}(k+1)}}$$

where $\left(\frac{k+1}{2}\right)^{2/k-1}(k+1) = \left(\frac{k+1}{2}\right)^{2/k-1} 2\left(\frac{k+1}{2}\right) = 2\left(\frac{k+1}{2}\right)^{\frac{k+1}{k-1}}$

$$\therefore \quad \dot{w} = A_t P_1 \sqrt{\frac{gk}{RT_1\left(\frac{k+1}{2}\right)^{\frac{k+1}{k-1}}}}$$

$$\boxed{\Rightarrow \quad \dot{w} = \frac{A_t P_1 g k\sqrt{\left[\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\right]}}{\sqrt{gkRT_1}}}$$