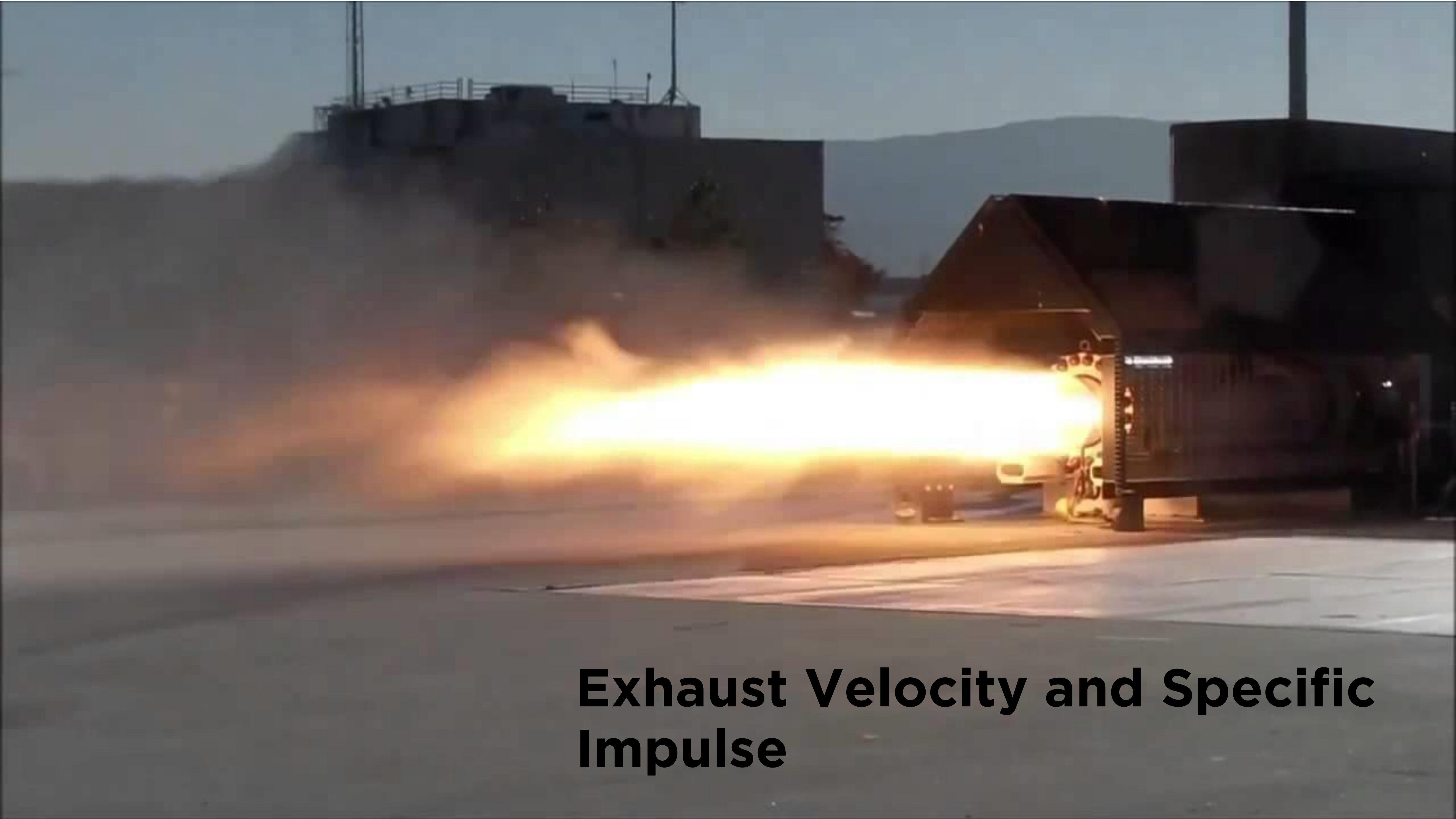

Exhaust Velocity and Specific Impulse

What is the specific impulse?

$$Impulse = Integral\ of\ thrust\ over\ time = Thrust * Time$$
$$I = Ft$$

Specific impulse:

$$I_{sp} = \frac{Thrust * Time}{propellant\ mass} = F * \frac{\Delta t}{\Delta \mathrm{m}} = \frac{F}{\frac{\Delta m}{\Delta t}} = \frac{F}{\dot{m}}$$

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Specific Impulse Units

From its definition:

$$I_{sp} = \frac{Thrust\ * Time}{propellant\ mass} = \frac{Ns}{kg}$$

But we know that $F = ma$ with m in kg and a in $\frac{m}{s^2}$

$$\text{So: } I_{sp} = \frac{kg*m}{s^2}\frac{s}{kg} = \frac{m}{s}$$

$$\frac{I_{sp}}{g} = s \text{ with } g = 9.81\frac{m}{s^2}$$

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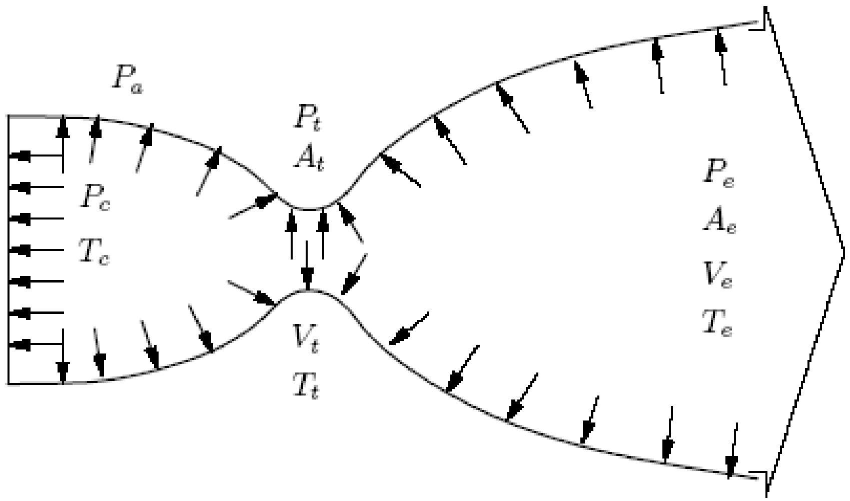
Within the Rocket Motor Chamber



External Atmospheric pressure Pa

Chamber Throat Exit

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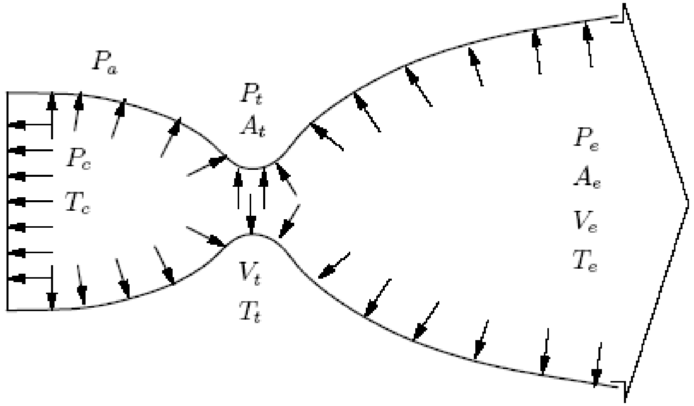
Chamber Forces Integration



External Atmospheric pressure Pa

Chamber Throat Exit

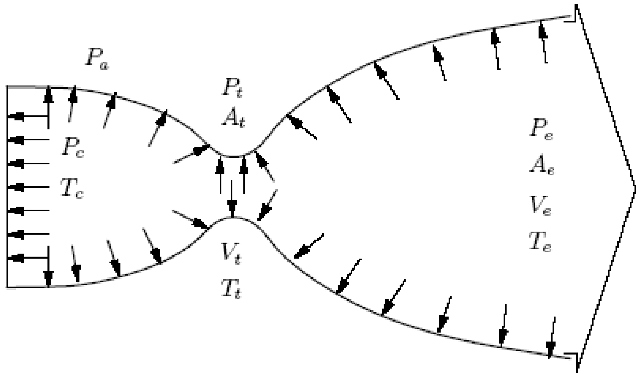
First, let's integrate the force due to the propellant ejection:

$$F_1 = \dot{m}V_e$$

Then, the force due to the difference in pressure between inside the chamber and the atmosphere:

$$F_2 = A_eP_e - A_eP_a = A_e(P_e - P_a)$$

Effective Exhaust Velocity



Summing up these forces we can write:

$$F = F_1 + F_{2_e} = \dot{m}V_e + A_e(P_e - P_a)$$

This allows us to estimate the effective exhaust velocity C of the rocket:

$$F = \dot{m}V_e + A_e(P_e - P_a) = \dot{m}C$$

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Specific Impulse and Effective Exhaust Velocity

We can now realise that:

$$I_{sp} = \frac{F}{\dot{m}}$$

However,

$$F = \dot{m}C$$

So,

$$I_{sp} = \frac{\dot{m}C}{\dot{m}} = C$$

The specific impulse of a rocket motor and its effective exhaust velocity are equivalent!

Eliott Wertheimer