BENDING STRENGTH OF SPUR AND HELICAL GEARS

Generally, bending strength and durability specifications are applied to spur gears and helical gears (including double helical and internal gears) to beused in industrial machines in the following range:

Module m 1.5-25mm
Pitch diameter d0 25-3200mm
Tangential speed v 25m/s or slower
Rotational speed n 3600 rpm or slower

(1) Conversion Formulas

The equations that relate transmitted tangential force at the pitch circle, Ft(kgf), power P(kW), and torque, T(kgf・m) are basic to the calculations. The relations are as follows:

$$F_t = \frac{102P}{v} = \frac{1.95 \times 10^6 P}{d_b n} = \frac{2000T}{d_b} \quad (10.1)$$

$$P = \frac{F_t v}{102} = \frac{10^{-6}}{1.95} F_t d_b n \quad (10.2)$$

$$T = \frac{F_t d_b}{2000} = \frac{974P}{n} \quad (10.3)$$

Where v： Tangential speed of working pitch circle (m/s)
v = dbn / 19100

db : Working pitch diameter (mm)
n : Rotational speed (rpm)

(2) Bending Strength Equations

In order to satisfy the bending strength, the transmitted tangential force at the working pitch circle, Ft , is not to exceed the allowable tangential force at the working pitch circle, Ftlim, that is calculated taking into account the allowable bending stress at the root.

$$F_t \leqq F_{tlim} \qquad (10.4)$$

At the same time, the actual bending stress at the root, σF, that is calculated on the basis of the transmitted tangential force at the working pitch circle, Ft, must be less than the allowable bending stress at the root, σFlim.

$$\sigma_F \leqq \sigma_{Flim} \qquad (10.5)$$

Equation (10.6) presents the calculation of Ftlim (kgf).

$$F_{tlim} = \sigma_{Flim} \frac{m_n b}{Y_F Y_\varepsilon Y_\beta} \left(\frac{K_L K_{FX}}{K_V K_O} \right) \frac{1}{S_F} \qquad (10.6)$$

Equation (10.6) can be converted into stress by Equation (10.7) (kgf/mm2).

$$\sigma_F = F_t \frac{Y_F Y_\varepsilon Y_\beta}{m_n b} \left(\frac{K_V K_O}{K_L K_{FX}} \right) S_F \qquad (10.7)$$

(3) Determination of Factors

(3)-1 Facewidth b(mm)
If the gears in a pair have different facewidth, let the wider one be bw and the narrower one be bs.
And if:

$b_w - b_s \leqq m_n$ b_w and b_s can be put directly into Equation (10.6).

$b_w - b_s > m_n$ the wider one would be changed to $b_s + m_n$ and the narrower one, b_s, would be unchanged.

NOTE: Regarding the facewidth of round gear racks, see 10.2 (3) – 1.

BEVEL GEARBOX

OTHER PRODUCTS

(3)-2 Tooth Profile Factor YF

The tooth profile factor YF is obtainable from Figure 10.1 based on the equivalent number of teeth, zV, and profile shift coefficient, x, if the gear has a standard tooth profile with pressure angle αn = 20°, per JIS B 1701. Figure 10.1 also indicates (a) theoretical undercut limit, and (b) narrow tooth top limit. These will be helpful in determining gear specifications. For internal gears, obtain the factor by considering the equivalent racks.

(3)-3 Load Sharing Factor, Yε

Load sharing factor, Yε, is the reciprocal of transverse contact ratio, εa.

$$Y_\varepsilon = \frac{1}{\varepsilon_\alpha} \qquad (10.8)$$

Fig.10.1 Chart showing tooth profile factor

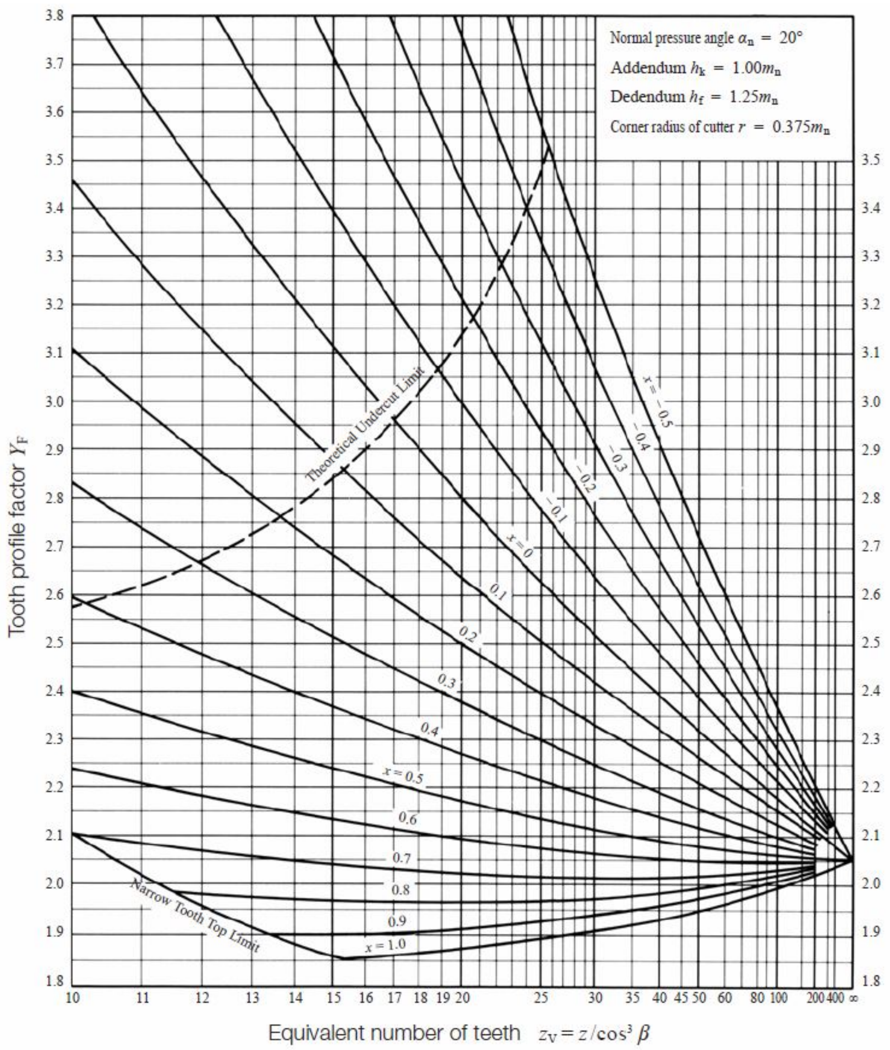

Table 10.1 Transverse contact ratio of standard spur gears, εα

($\alpha_0 = 20°$)

No. of teeth	17	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	110	120
17	1.514																			
20	1.535	1.557																		
25	1.563	1.584	1.612																	
30	1.584	1.605	1.633	1.654																
35	1.603	1.622	1.649	1.670	1.687															
40	1.614	1.635	1.663	1.684	1.700	1.714														
45	1.625	1.646	1.674	1.695	1.711	1.725	1.736													
50	1.634	1.656	1.683	1.704	1.721	1.734	1.745	1.755												
55	1.642	1.664	1.691	1.712	1.729	1.742	1.753	1.763	1.771											
60	1.649	1.671	1.698	1.719	1.736	1.749	1.760	1.770	1.778	1.785										
65	1.655	1.677	1.704	1.725	1.742	1.755	1.766	1.776	1.784	1.791	1.797									
70	1.661	1.682	1.710	1.731	1.747	1.761	1.772	1.781	1.789	1.796	1.802	1.808								
75	1.666	1.687	1.714	1.735	1.752	1.765	1.777	1.786	1.794	1.801	1.807	1.812	1.817							
80	1.670	1.691	1.719	1.740	1.756	1.770	1.781	1.790	1.798	1.805	1.811	1.817	1.821	1.826						
85	1.674	1.695	1.723	1.743	1.760	1.773	1.785	1.794	1.802	1.809	1.815	1.821	1.825	1.830	1.833					
90	1.677	1.699	1.726	1.747	1.764	1.777	1.788	1.798	1.806	1.813	1.819	1.824	1.829	1.833	1.837	1.840				
95	1.681	1.702	1.729	1.750	1.767	1.780	1.791	1.801	1.809	1.816	1.822	1.827	1.832	1.836	1.840	1.844	1.847			
100	1.683	1.705	1.732	1.753	1.770	1.783	1.794	1.804	1.812	1.819	1.825	1.830	1.835	1.839	1.843	1.846	1.850	1.853		
110	1.688	1.710	1.737	1.758	1.775	1.788	1.799	1.809	1.817	1.824	1.830	1.835	1.840	1.844	1.848	1.852	1.855	1.858	1.863	
120	1.693	1.714	1.742	1.762	1.779	1.792	1.804	1.813	1.821	1.828	1.834	1.840	1.844	1.849	1.852	1.856	1.859	1.862	1.867	1.871
RACK	1.748	1.769	1.797	1.817	1.834	1.847	1.859	1.868	1.876	1.883	1.889	1.894	1.899	1.903	1.907	1.911	1.914	1.917	1.922	1.926

$$\varepsilon_\alpha = \frac{\sqrt{r_{k1}^2 - r_{g1}^2} + \sqrt{r_{k2}^2 - r_{g2}^2} - a \sin \alpha_b}{\pi m \cos \alpha_o}$$

Transverse Contact Ratio is calculated as follows.

For Spur Gears :

$$\varepsilon_\alpha = \frac{\sqrt{r_{k1}^2 - r_{g1}^2} + \sqrt{r_{k2}^2 - r_{g2}^2} - a \sin \alpha_b}{\pi m \cos \alpha_0}$$

For Helical Gears :

$$\varepsilon_\alpha = \frac{\sqrt{r_{k1}^2 - r_{g1}^2} + \sqrt{r_{k2}^2 - r_{g2}^2} - a \sin \alpha_{bs}}{\pi m_s \cos \alpha_s} \qquad (10.9)$$

Where：
rk : Tip diameter (mm)
αb : Working pressure angle (degree)
rg : Reference radius (mm)
αbs : Transverse working pressure angle (degree)
a : Center distance (mm)
α0 : Reference pressure angle (degree)
αs : Reference transverse pressure angle (degree)

Table 10.1 shows the transverse contact ratio εa of a standard spur gear (α0 = 20°)

(3)-4 Helix Angle Factor, Yβ
Helix angle factor, Yβ, can be obtained from Equation

$$\left.\begin{array}{ll} 0 \leqq \beta \leqq 30° \text{ then} & Y_\beta = 1 - \dfrac{\beta}{120} \\ \beta \geqq 30° \quad \text{then} & Y_\beta = 0.75 \end{array}\right\} (10.10)$$

(3)-5 Life Factor, KL
We can choose the proper life factor, KL, from Table 10.2. The number of cyclic repetitions means the total loaded meshing during its lifetime.

Table 10.2 Life factor

No. of cyclic repetitions	Hardness (1) H_B120 ~ 220	Hardness (2) H_B221 or over	Gears w. carburizing/nitriding
10000 or fewer	1.4	1.5	1.5
Approx. 100000	1.2	1.4	1.5
Approx. 10^6	1.1	1.1	1.1
10^7 or greater	1.0	1.0	1.0

NOTES
(1) Cast steel gears apply to this column.
(2) For induction hardened gears, use the core hardness.

(3)-6 Size Factor of Root Stress, KFX
Generally, this factor, KFX, is unity.

$$K_{FX} = 1.00 \tag{10.11}$$

(3)-7 Dynamic Load Factor, KV
Dynamic load factor, KV, can be obtained from Table 10.3 based on the precision of the gear and the tangential speed at working pitch circle.

Table 10.3 Dynamic load factor, KV

Precision grade of gears from JIS B 1702		Tangential speed at working pitch circle (m/s)						
Tooth profile								
Unmodified	Modified	1 or under	Over 1 to 3 incl.	Over 3 to 5 incl.	Over 5 to 8 incl.	Over 8 to 12 incl.	Over 12 to 18 incl.	Over 18 to 25 incl.
	1	—	—	1.0	1.0	1.1	1.2	1.3
1	2	—	1.0	1.05	1.1	1.2	1.3	1.5
2	3	1.0	1.1	1.15	1.2	1.3	1.5	
3	4	1.0	1.2	1.3	1.4	1.5		
4	—	1.0	1.3	1.4	1.5			
5	—	1.1	1.4	1.5				
6	—	1.2	1.5					

(3)-8 Overload Factor KO
Overload factor, KO, can be obtained from Equation

$$K_O = \frac{\text{Actual tangential force}}{\text{Nominal tangential force, } F_t} \tag{10.12}$$

If tangential force is unknown, Table 10.4 provides guiding values. Load grades on affected machinery are introduced on page 572, as reference.

Table 10.4 Overload Factor, KO

Impact from Prime Mover	Impact from Load Side of Machine		
	Uniform Load	Medium Impact Load	Heavy Impact Load
Uniform Load (Motor, Turbine, Hydraulic Motor)	1.0	1.25	1.75
Light Impact Load (Multicylinder Engine)	1.25	1.5	2.0
Medium Impact Load (Single Cylinder Engine)	1.5	1.75	2.25

(3)-9 Safety Factor for Bending Failure, SF
Safety factor, SF, is too complicated to be determined precisely. Usually, it is set to at least 1.2.

(3)-10 Allowable Bending Stress at Root, σFlim
For a unidirectionally loaded gear, the allowable bending stresses at the root, σFlim, are shown in Tables 10.5 to 10.9. In these tables, the value of σFlim is the quotient of the fatigue limit under pulsating tension divided by the stress concentration factor 1.4. If the load is bidirectional, and both sides of the tooth are equally loaded, the value of allowable bending stress, σFlim, should be taken as 2/3 of the given value in the table. The core hardness means the hardness at the center region of the root.

Table 10.5 Gears without surface hardening

	Material (Arrows indicate the ranges)	Hardness		Tensile strength lower limit kgf/mm^2 (Reference)	σ_{Flim} kgf/mm^2
		H_B	H_V		
Cast steel gear	SC37			37	10.4
	SC42			42	12
	SC46			46	13.2
	SC49			49	14.2
	SCC3			55	15.8
				60	17.2
Normalizing carbon steel gear		120	126	39	13.8
		130	136	42	14.8
		140	147	45	15.8
	S25C	150	157	48	16.8
		160	167	51	17.6
		170	178	55	18.4
	S35C	180	189	58	19
		190	200	61	19.5
	S43C	200	210	64	20
	S48C	210	221	68	20.5
	S53C	220	231	71	21
	S58C	230	242	74	21.5
		240	252	77	22
		250	263	81	22.5

Quenched and tempered carbon steel gear		160	167	51	18.2
		170	178	55	19.4
		180	189	58	20.2
		190	200	61	21
	S35C	200	210	64	22
		210	221	68	23
		220	231	71	23.5
		230	242	74	24
	S43C, S48C	240	252	77	24.5
		250	263	81	25
	S53C	260	273	84	25.5
	S58C	270	284	87	26
		280	295	90	26
		290	305	93	26.5
Quenched and tempered alloy steel gear		220	231	71	25
		230	242	74	26
		240	252	77	27.5
		250	263	81	28.5
	SMn443	260	273	84	29.5
		270	284	87	31
		280	295	90	32
	SNC836	290	305	93	33
	SCM435	300	316	97	34
	SCM440	310	327	100	35
	SNCM439	320	337	103	36.5
		330	347	106	37.5
		340	358	110	39
		350	369	113	40
		360	380	117	41

Table 10.6 Induction hardened gears

		Material (Arrows indicate the ranges)	Heat treatment before induction hardening	Core hardness H_B	Core hardness H_V	Surface Hardness (1) H_V	σ_{Flim} kgf/mm²
Hardened throughout	Structural carbon steel		Normalized	160	167	More than 550	21
		S43C		180	189	〃	21
		S48C		220	231	〃	21.5
				240	252	〃	22
			Quenched and tempered	200	210	More than 550	23
				210	221	〃	23.5
		S43C		220	231	〃	24
		S48C		230	242	〃	24.5
				240	252	〃	25
				250	263	〃	25
	Structural alloy steel		Quenched and tempered	230	242	More than 550	27
				240	252	〃	28
		SCM440		250	263	〃	29
				260	273	〃	30
		SMn443		270	284	〃	31
		SNCM439		280	295	〃	32
		SCM435		290	305	〃	33
		SNC836		300	316	〃	34
				310	327	〃	35
				320	337	〃	36.5
Hardened except root area							75% of the above

Remarks： If a gear is not quenched completely, or not evenly, or has quenching cracks, the σFlim will drop dramatically.
NOTE (1) : If the hardness after quenching is relatively low, the value of σFlim should be that given in Table 10.5.

Table 10.7 Carburized and quenched gears

	Material (Arrows indicate the ranges)	Core hardness H_B	Core hardness H_V	σ_{Flim} kgf/mm²
Structural carbon steel	S15C S15CK	140	147	18.2
		150	157	19.6
		160	167	21
		170	178	22
		180	189	23
		190	200	24
Structural alloy steel	SCM415, SCM420, SNCM420, SNC415, SNC815	220	231	34
		230	242	36
		240	252	38
		250	263	39
		260	273	41
		270	284	42.5
		280	295	44
		290	305	45
		300	316	46
		310	327	47
		320	337	48
		330	347	49
		340	358	50
		350	369	51
		360	380	51.5
		370	390	52

NOTE (2)
The table on the left only applies to those gears which have adequate carburized depth and surface hardness.
If the carburized depth is relatively thin, the value of σFlim should be stated for quenched/tempered gears, having no surface hardened.

Table 10.8 Nitrided Gears Excerpted from JGMA403-01(1976)

Material	Surface Hardness (Reference)	Core Hardness H_B	Core Hardness H_V	σ_{Flim} kgf/mm²
Structural alloy steel except nitriding steel	H_V 650 or more	220	231	30
		240	252	33
		260	273	36
		280	295	38
		300	316	40
		320	337	42
		340	358	44
		360	380	46
Nitriding steel SACM645	H_V 650 or more	220	231	32
		240	252	35
		260	273	38
		280	295	41
		300	316	44

NOTE (1)

The table on the left only applies to those gears which have adequate nitrided depth.
If the nitrided depth is relatively thin, the value of σFlim should be stated for gears which have no surface hardened.

Table 10.9 Stainless Steel and Free-Cutting Brass Gears Excerpted from JGMA6101-02 (2007)

Material	Hardness	Yield Point MPa	Tensile Strength MPa	σ_{Flim} MPa
Stainless Steel SUS304	Less than 187HB	More than 206 (Durability)	More than 520	103
Free-Cutting Brass C3604	More than 80HV	—	More than 333	39.3

(Reference) Load Grades on Affected Machinery Quoted from JGMA402-01 (1975)

Affected Machinery	Load Grade	Affected Machinery	Load Grade
Stirring machine	M	Food machinery	M
Blower	U	Hammermill	H
Brewing and distilling machine	U	Hoist	M
Automotive machinery	M	Machine tool	H
Clarifier	U	Metal working machinery	H
Sorting machine	M	Tumbling mill	M
Porcelain machine (Medium load)	M	Tumbler	H
Porcelain machine (Heavy load)	H	Blender	M
Compression Machine	M	Petroleum Refinery	M
Conveyer (Uniform load)	U	Papermaking machine	M
Conveyer (Non-uniform / heavy load)	M	Peeling machine	H
Crane	U	Pump	M
Crushing machine	H	Rubber machinery (Medium load)	M
Dredging boat (Medium load)	M	Rubber machinery (Heavy load)	H
Dredging boat (Heavy load)	H	Water treatment machine (Light load)	U
Elevator	U	Water treatment machine (Medium load)	M
Extruding machine	U	Screen (Sifter)	U
Fan (Household use)	U	Screen (Sand strainer)	M
Fan (Industrial use)	M	Sugar refinery machinery	M
Supplying machine	M	Textile machinery	M
Supplying machine (reciprocated)	H		

NOTE
1.
This sheet was created in reference to AGMA 151.02
2.
In this sheet, symbols are used to classify what load grades are:
U: Uniform load, M: Medium load and
H: Heavy load
3.
This sheet indicates general tendency of load grades.
For use in heavy load, one-higher-grade should be adopted.

For details, please refer to the AGMA standard mentioned in NOTE 1.

(4) Example of Calculation

Spur gear design details

No.	Item	Symbol	Unit	Pinion	Gear
1	Normal module	m_n	mm	2	
2	Normal pressure angle	α_n	Degree	20°	
3	Reference cylinder helix angle	β		0°	
4	Number of teeth	z		20	40
5	Center distance	a	mm	60	
6	Profile shift coefficient	x		+ 0.15	− 0.15
7	Pitch diameter	d_0	mm	40.000	80.000
8	Working pitch diameter	d_b		40.000	80.000
9	Facewidth	b		20	20
10	Precision grade			JIS 5 (Without tooth modification)	JIS 5 (Without tooth modification)
11	Manufacturing method			Hobbing	
12	Surface roughness			12.5S	
13	Rotational speed	n	rpm	1500	750
14	Tangential speed	v	m/s	3.142	
15	Direction of load			Unidirectional	
16	Duty cycle		Cycles	10^7 cycles or over	
17	Material			SCM415	
18	Heat treatment			Carburizing and quenching	
19	Surface hardness			H_V600 – 640	
20	Core hardness			H_B260 – 280	
21	Effective case depth		mm	0.3 – 0.5	

Bending Strength Factors of Spur Gear

No.	Item	Symbol	Unit	Pinion	Gear
1	Allowable bending stress at root	σ_{Flim}	kgf/mm²	42.5	
2	Normal module	m_n	mm	2	
3	Facewidth	b		20	
4	Tooth profile factor	Y_F		2.568	2.535
5	Load sharing factor	Y_ε		0.619	
6	Helix angle factor	Y_β		1.0	
7	Life factor	K_L		1.0	
8	Size factor of root stress	K_{FX}		1.0	
9	Dynamic load factor	K_V		1.5	
10	Overload factor	K_O		1.0	
11	Safety factor	S_F		1.2	
12	Allowable tangential force on working pitch circle	F_{tlim}	kgf	594.1	601.9

Related links:
Strength and Durability of Gears – A page of The ABC's of Gears / Basic Guide – B

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