	coefficient				
6	Pitch line height	H		–	27.5
7	Transverse pressure angle	α_t	$\tan^{-1}\left(\dfrac{\tan\alpha_n}{\cos\beta}\right)$	20.34160 deg	
8	Mounting distance	a	$\dfrac{zm_n}{2\cos\beta}+H+x_n m_n$	52.965	
9	Reference diameter	d	$\dfrac{zm_n}{\cos\beta}$	50.92956	–
10	Base diameter	d_b	$d\cos\alpha_t$	47.75343	
11	Addendum	h_a	$m_n(1+X_n)$	2.500	2.500
12	Tooth depth	h	$2.25m_n$	5.625	
13	Tip diameter	d_a	$d+2h_a$	55.929	–
14	Root diameter	d_f	d_a-2h	44.679	

The formulas of a standard helical rack are similar to those of Table 4.14 with only the normal profile shift coefficient xn = 0.
To mesh a helical gear to a helical rack, they must have the same helix angle but with opposite hands.

The displacement of the helical rack, l, for one rotation of the mating gear is the product of the transverse pitch and number of teeth.

$$l=\frac{\pi m_n}{\cos\beta}z \qquad (4.18)$$

According to the equations of Table 4.13, let transverse pitch pt = 8 mm and displacement l = 160 mm. The transverse pitch and the displacement could be resolved into integers, if the helix angle were chosen properly.

Table 4.14 The calculations for a helical rack in the transverse system

No.	Item	Symbol	Formula	Example	
				Pinion	Rack
1	Transverse module	m_t	Set Value	2.5	
2	Transverse pressure angle	α_t		20 deg	
3	Reference cylinder helix angle	β		10 deg 57’49”	
4	Number of teeth & helical hand	z		20 (R)	– (L)
5	Transverse profile shift coefficient	x_t		0	–
6	Pitch line height	H		–	27.5
7	Mounting distance	a	$\dfrac{zm_t}{2}+H+x_t m_t$	52.500	
8	Reference diameter	d	zm_t	50.000	–
9	Base diameter	d_b	$d\cos\alpha_t$	46.98463	
10	Addendum	h_a	$m_t(1+X_t)$	2.500	2.500
11	Tooth depth	h	$2.25m_t$	5.625	
12	Tip diameter	d_a	$d+2h_a$	55.000	–
13	Root diameter	d_f	d_a-2h	43.750	

In the meshing of transverse system helical rack and helical gear, the movement, l, for one turn of the helical gear is the transverse pitch multiplied by the number of teeth.

$$l=\pi m_t z \qquad (4.19)$$

4.4 Bevel Gears

Bevel gears, whose pitch surfaces are cones, are used to drive intersecting axes. Bevel gears are classified according to their type of the tooth forms into Straight Bevel Gear, Spiral Bevel Gear, Zerol Bevel Gear, Skew Bevel Gear etc. The meshing of bevel gears means the pitch cone of two gears contact and roll with each other. Let z1 and z2 be pinion and gear tooth numbers; shaft angle Σ ; and reference cone angles δ1 and δ2 ; then:

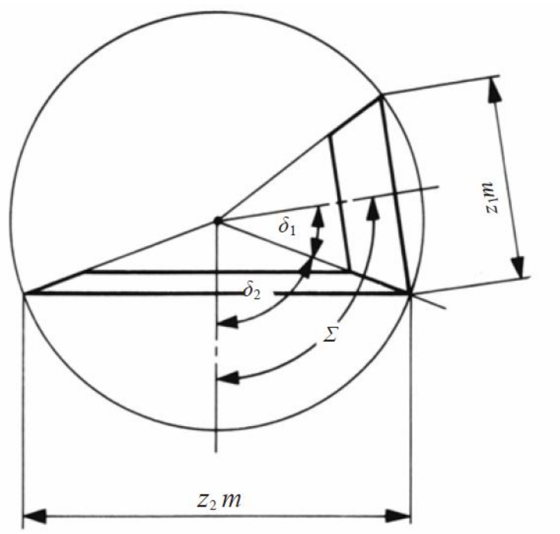


Fig. 4.8 The reference cone angle of bevel gear

$$\left.\begin{aligned} \tan\delta_1 &= \frac{\sin\Sigma}{\frac{z_2}{z_1}+\cos\Sigma} \\ \tan\delta_2 &= \frac{\sin\Sigma}{\frac{z_1}{z_2}+\cos\Sigma} \end{aligned}\right\} \qquad (4.20)$$

Generally, a shaft angle Σ = 90° is most used. Other angles (Figure 4.8) are sometimes used. Then, it is called “bevel gear in nonright angle drive”. The 90° case is called “bevel gear in right angle drive”. When Σ = 90°, Equation (4.20) becomes :

$$\left.\begin{aligned} \delta_1 &= \tan^{-1}\left(\frac{z_1}{z_2}\right) \\ \delta_2 &= \tan^{-1}\left(\frac{z_2}{z_1}\right) \end{aligned}\right\} \qquad (4.21)$$

Miter gears are bevel gears with Σ = 90° and z1 = z2. Their transmission ratio z2 / z1 = 1.

Figure 4.9 depicts the meshing of bevel gears. The meshing must be considered in pairs. It is because the reference cone angles δ1 and δ2 are restricted by the gear ratio z2 / z1. In the facial view, which is normal to the contact line of pitch cones, the meshing of bevel gears appears to be similar to the meshing of spur gears.

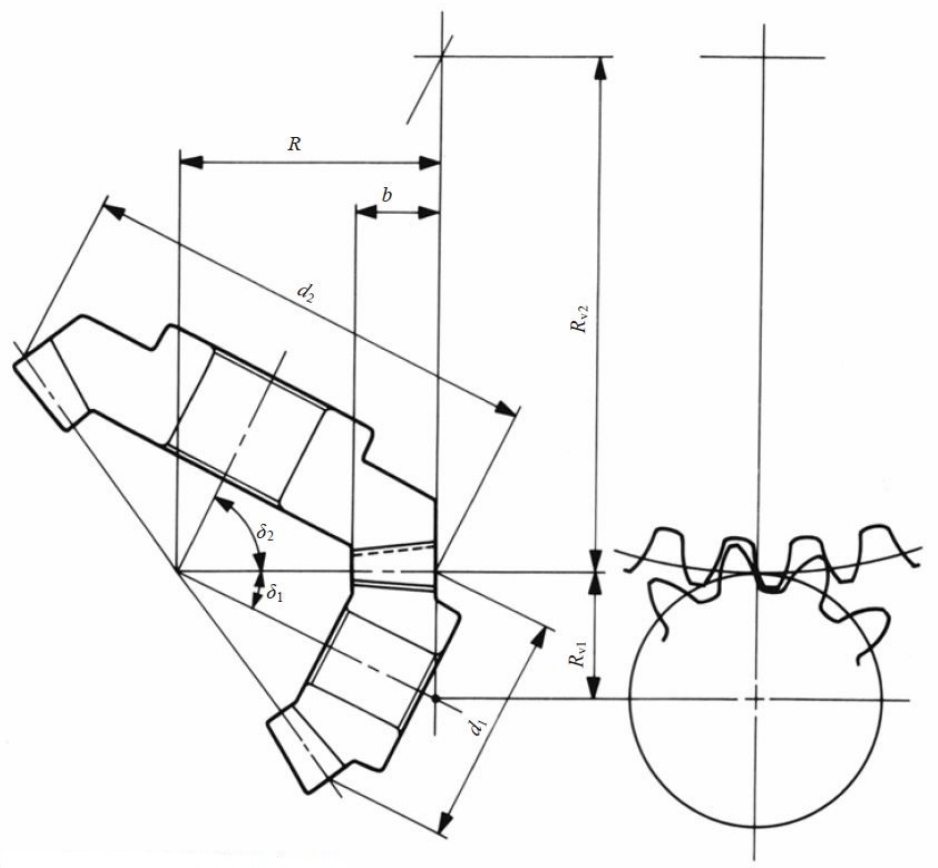


Fig. 4.9 The meshing of bevel gears

(1) Gleason Straight Bevel Gears
A straight bevel gear is a simple form of bevel gear having straight teeth which, if extended inward, would come together at the intersection of the shaft axes. Straight bevel gears can be grouped into the Gleason type and the standard type.
In this section, we discuss the Gleason straight bevel gear. The Gleason Company defines the tooth profile as: tooth depth h = 2.188m; tip and root clearance c = 0.188m; and working depth hw = 2.000m.

The characteristics are :

** Design specified profile shifted gears
In the Gleason system, the pinion is positive shifted and the gear is negative shifted. The reason is to distribute the proper strength between the two gears. Miter gears, thus, do not need any shift.
** The tip and root clearance is designed to be parallel
The face cone of the blank is turned parallel to the root cone of the mate in order to eliminate possible fillet interference at the small end of the teeth.

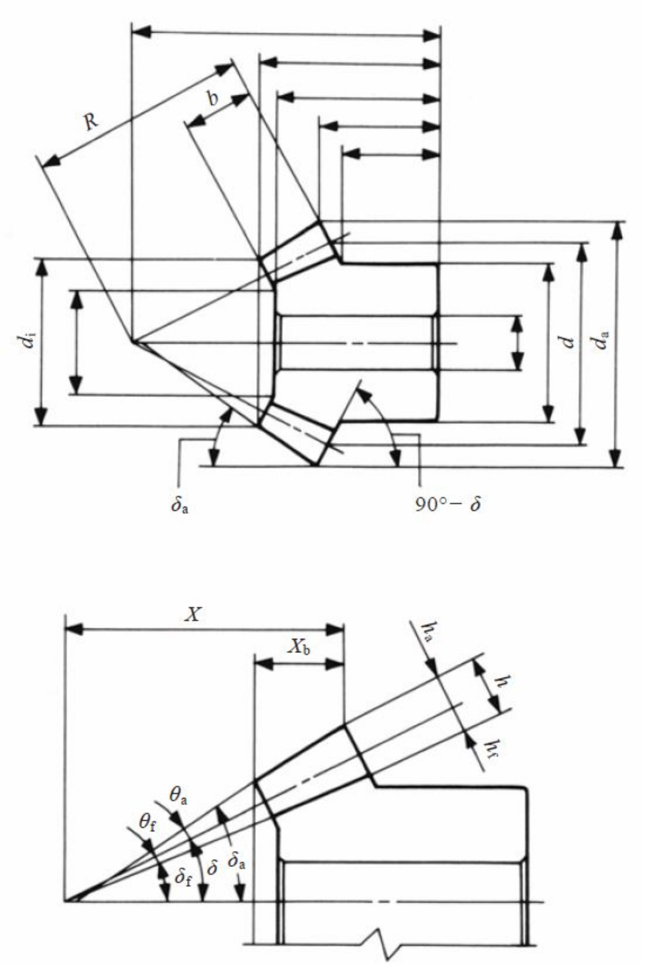


Fig. 4.10 Dimensions and angles of bevel gears

Table 4.15 shows the minimum number of the teeth to prevent undercut in the Gleason system at the shaft angle Σ = 90.°

Table 4.15 The minimum numbers of teeth to prevent undercut

Pressure angle	Combination of number of teeth z_1/z_2					
(14.5°)	29/29 and higher	28/29 and higher	27/31 and higher	26/35 and higher	25/40 and higher	24/57 and higher
20°	16/16 and higher	15/17 and higher	14/20 and higher	13/30 and higher		
(25°)	13/13 and higher					

Table 4.16 presents equations for designing straight bevel gears in the Gleason system. The meanings of the dimensions and angles are shown in Figure 4.10 above. All the equations in Table 4.16 can also be applied to bevel gears with any shaft angle.
The straight bevel gear with crowning in the Gleason system is called a Coniflex gear. It is manufactured by a special Gleason “Coniflex” machine. It can successfully eliminate poor tooth contact due to improper mounting and assembly.

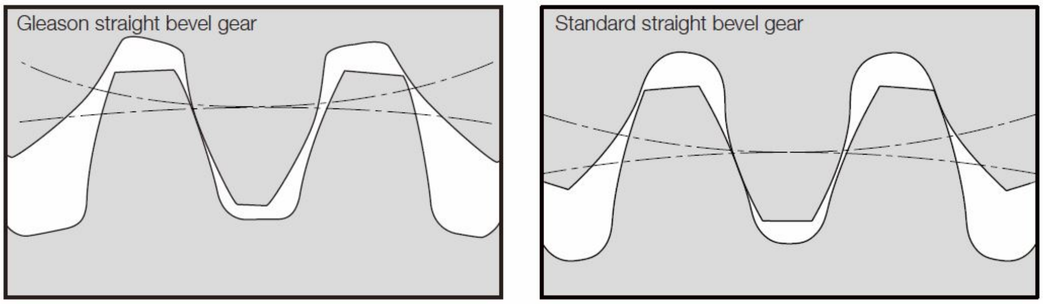
Tale 4.16 The calculations of straight bevel gears of the Gleason system

No.	Item	Symbol	Formula	Example	
				Pinion(1)	Gear(2)
1	Shaft angle	Σ	Set Value	90 deg	
2	Module	m		3	
3	Reference pressure angle	α		20 deg	
4	Number of teeth	z		20	40
5	Reference diameter	d	zm	60	120
6	Reference cone angle	δ_1 δ_2		26.56505 deg	63.43495 deg

			$\tan^{-1}\left(\dfrac{\sin \Sigma}{\dfrac{z_2}{z_1}+\cos \Sigma}\right)$ $\Sigma - \delta_1$		
7	Cone distance	R	$\dfrac{d_2}{2 \sin \delta_2}$	67.08204	
8	Facewidth	b	It should not exceed R / 3	22	
9	Addendum	h_{a1} h_{a2}	$2.000m - h_{a2}$ $0.540m + \dfrac{0.460m}{\left(\dfrac{z_2 \cos \delta_1}{z_1 \cos \delta_2}\right)}$	4.035	1.965
10	Dedendum	h_f	$2.188m - h_a$	2.529	4.599
11	Dedendum angle	θ_f	$\tan^{-1}(h_f / R)$	2.15903 deg	3.92194 deg
12	Addendum angle	θ_{a1} θ_{a2}	θ_{f2} θ_{f1}	3.92194 deg	2.15903 deg
13	Tip angle	δ_a	$\sigma + \theta_a$	30.48699 deg	65.59398 deg
14	Root angle	δ_f	$\sigma - \theta_f$	24.40602 deg	59.51301 deg
15	Tip diameter	d_a	$d + 2h_a \cos \sigma$	67.2180	121.7575
16	Pitch apex to crown	X	$R \cos \sigma - h_a \sin \sigma$	58.1955	28.2425
17	Axial facewidth	X_b	$\dfrac{b \cos \delta_a}{\cos \theta_a}$	19.0029	9.0969
18	Inner tip diameter	d_i	$d_a - \dfrac{2b \sin \delta_a}{\cos \theta_a}$	44.8425	81.6609

The first characteristic of a Gleason Straight Bevel Gear that it is a profile shifted tooth. From Figure 4.11, we can see the tooth profile of Gleason Straight Bevel Gear and the same of Standard Straight Bevel Gear.

Fig. 4.11 The tooth profile of straight bevel gears



(2) Standard Straight Bevel Gears
A bevel gear with no profile shifted tooth is a standard straight bevel gear. The are also referred to as Klingelnberg bevel gears. The applicable equations are in Table 4.17.

Table 4.17 The calculations for a standard straight bevel gears

No.	Item	Symbol	Formula	Example	
				Pinion(1)	Gear(2)
1	Shaft angle	Σ	Set Value	90 deg	
2	Module	m		3	
3	Reference pressure angle	α		20 deg	
4	Number of teeth	z		20	40
5	Reference diameter	d	zm	60	120
6	Reference cone angle	δ_1		26.56505 deg	63.43495 deg

		δ_2			
7	Cone distance	R		67.08204	
8	Facewidth	b	It should not exceed R / 3	22	
9	Addendum	h_{a1} h_{a2}		4.035	1.965
10	Dedendum	h_f	$2.188m - h_a$	2.529	4.599
11	Dedendum angle	θ_f	$\tan^{-1}(h_f / R)$	2.15903 deg	3.92194 deg
12	Addendum angle	θ_{a1} θ_{a2}	θ_{f2} θ_{f1}	3.92194 deg	2.15903 deg
13	Tip angle	δ_a	$\sigma + \theta_a$	30.48699 deg	65.59398 deg
14	Root angle	δ_f	$\sigma - \theta_f$	24.40602 deg	59.51301 deg
15	Tip diameter	d_a	$d + 2h_a \cos \sigma$	67.2180	121.7575
16	Pitch apex to crown	X	$R \cos \sigma - h_a \sin \sigma$	58.1955	28.2425
17	Axial facewidth	X_b		19.0029	9.0969
18	Inner tip diameter	d_i		44.8425	81.6609

These equations can also be applied to bevel gear sets with other than 90° shaft angles.

(3) Gleason Spiral Bevel Gears
A spiral bevel gear is one with a spiral tooth flank as in Figure 4.12. The spiral is generally consistent with the curve of a cutter with the diameter dc. The spiral angle β is the angle between a generatrix element of the pitch cone and the tooth flank. The spiral angle just at the tooth flank center is called the mean spiral angle βm. In practice, the term spiral angle refers to the mean spiral angle.

Fig.4.12 Spiral Bevel Gear (Left-hand)

All equations in Table 4.20 are specific to the manufacturing method of Spread Blade or of Single Side from Gleason. If a gear is not cut per the Gleason system, the equations will be different from these.

The tooth profile of a Gleason spiral bevel gear shown here has the tooth depth h = 1.888m; tip and root clearance c = 0.188m; and working depth hw = 1.700m. These Gleason spiral bevel gears belong to a stub gear system. This is applicable to gears with modules m > 2.1.

Table 4.18 shows the minimum number of teeth to avoid undercut in the Gleason system with shaft angle Σ = 90° and pressure angle αn = 20°.

Table 4.18 The minimum numbers of teeth to prevent undercut β=35°

If the number of teeth is less than 12, Table 4.19 is used to determine the gear sizes.

Table 4.19 Dimentions for pinions with number of teeth less than 12

Table 4.20 shows the calculations for spiral bevel gears in the Gleason system

Table 4.20 The calculations for spiral bevel gears in the Gleason system

No.	Item	Symbol	Formula	Example	
				Pinion (1)	Gesr (2)
1	Shaft angle	Σ	Set Value	90 deg	
2	Module	m		3	
3	Normal pressure angle	α_n		20 deg	
4	Mean spiral angle	β_m		35 deg	
5	Number of teeth and spiral hand	z		20 (L)	40 (R)
6	Transverse pressure angle	α_t		23.95680	
7	Reference diameter	d	zm	60	120
8	Reference cone angle	σ_1 σ_2		26.56505 deg	63.43495 deg
9	Cone distance	R		67.08204	

10	Facewidth	b	It should be less than 0.3R or 10m	20	
11	Addendum	h_{a1} h_{a2}		3.4275	1.6725
12	Dedendum	h_f	$1.888m - h_a$	2.2365	3.9915
13	Dedendum angle	θ_f	$\tan^{-1}(h_f / R)$	1.90952 deg	3.40519 deg
14	Addendum angle	θ_{a1} θ_{a2}	θ_{f2} θ_{f1}	29.97024 deg	1.90952 deg
15	Tip angle	σ_a	$\sigma + \theta_a$	29.97024 deg	65.34447 deg
16	Root angle	σ_f	$\sigma - \theta_f$	24.65553 deg	60.02976 deg
17	Tip diameter	d_a	$d + 2h_a \cos \sigma$	66.1313	121.4959
18	Pitch apex to crown	X	$R \cos \sigma - h_a \sin \sigma$	58.4672	28.5041
19	Axial facewidth	X_b		17.3565	8.3479
20	Inner tip diameter	d_i		46.1140	85.1224

All equations in Table 4.20 are also applicable to Gleason bevel gears with any shaft angle. A spiral bevel gear set requires matching of hands; left-hand and right-hand as a pair.

(4) Gleason Zerol Bevel Gears
When the spiral angle bm = 0, the bevel gear is called a Zerol bevel gear. The calculation equations of Table 4.16 for Gleason straight bevel gears are applicable. They also should take care again of the rule of hands; left and right of a pair must be matched. Figure 4.13 is a left-hand Zerol bevel gear.

Fig. 4.13 Left-hand zerol bevel gear

4.5 Screw Gears