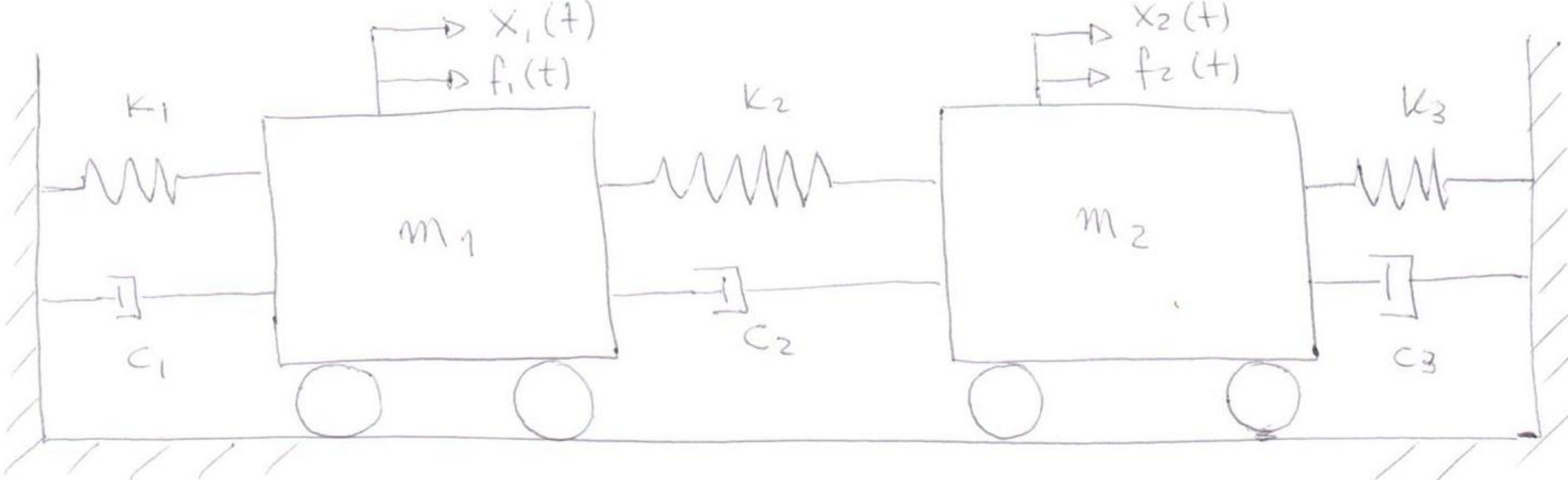
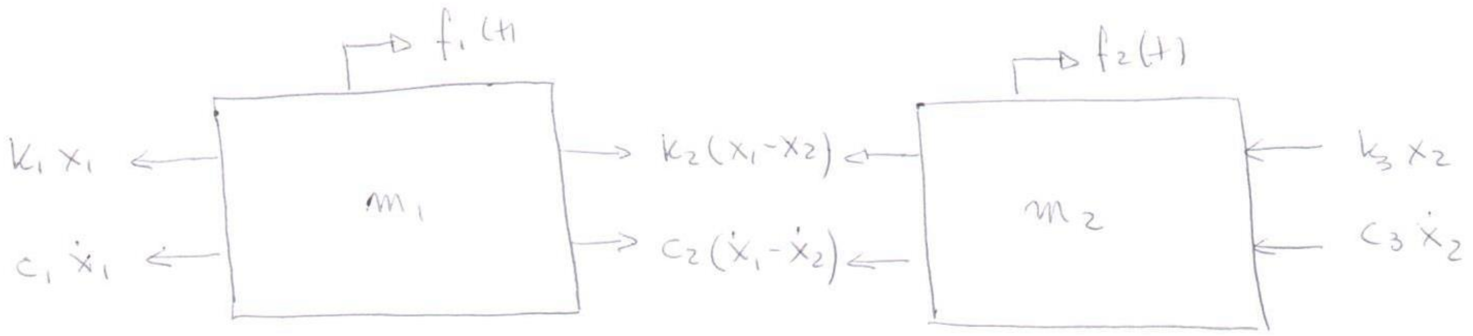
5.1 – Multi degree of freedom system

Given a two degree of freedom system,



5.1 – Multi degree of freedom system

Doing the free body diagram



and applying the Newton's second law

$$f_1(t) - k_1 x_1 - k_2(x_1 - x_2) - c_1\dot{x}_1 - c_2(\dot{x}_1 - \dot{x}_2) = m\,\ddot{x}_1(t)$$
$$f_2(t) - k_3 x_2 - c_3\dot{x}_2 + k_2(x_1 - x_2) + c_2(\dot{x}_1 - \dot{x}_2) = m\,\ddot{x}_2(t)$$

5.1 – Multi degree of freedom system

The system equations can be rewritten as

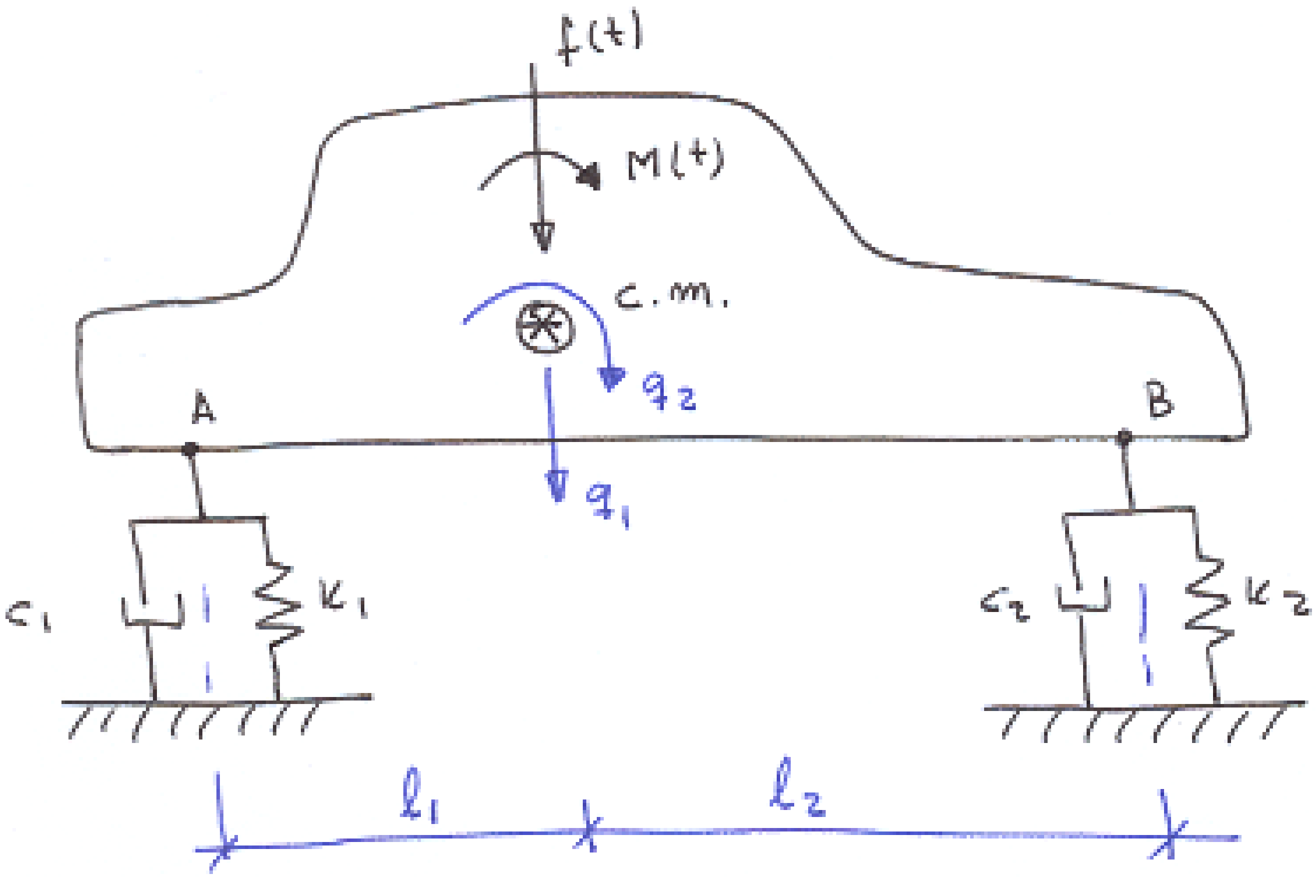
$$m\ddot{x}_1(t)+(k_1+k_2)x_1(t)+(c_1+c_2)\dot{x}_1(t)-k_2x_2(t)-c_2\dot{x}_2(t)=f_1(t)$$
$$m\ddot{x}_2(t)+(k_2+k_3)x_2(t)+(c_2+c_3)\dot{x}_2(t)-k_2x_1(t)-c_2\dot{x}_1(t)=f_2(t)$$

Or in matriz form

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\{\ddot{x}\}+\begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix}\{x\}+\begin{bmatrix} c_1+c_2 & -c_2 \\ -c_2 & c_2+c_3 \end{bmatrix}\{\dot{x}\}=\{f\}$$

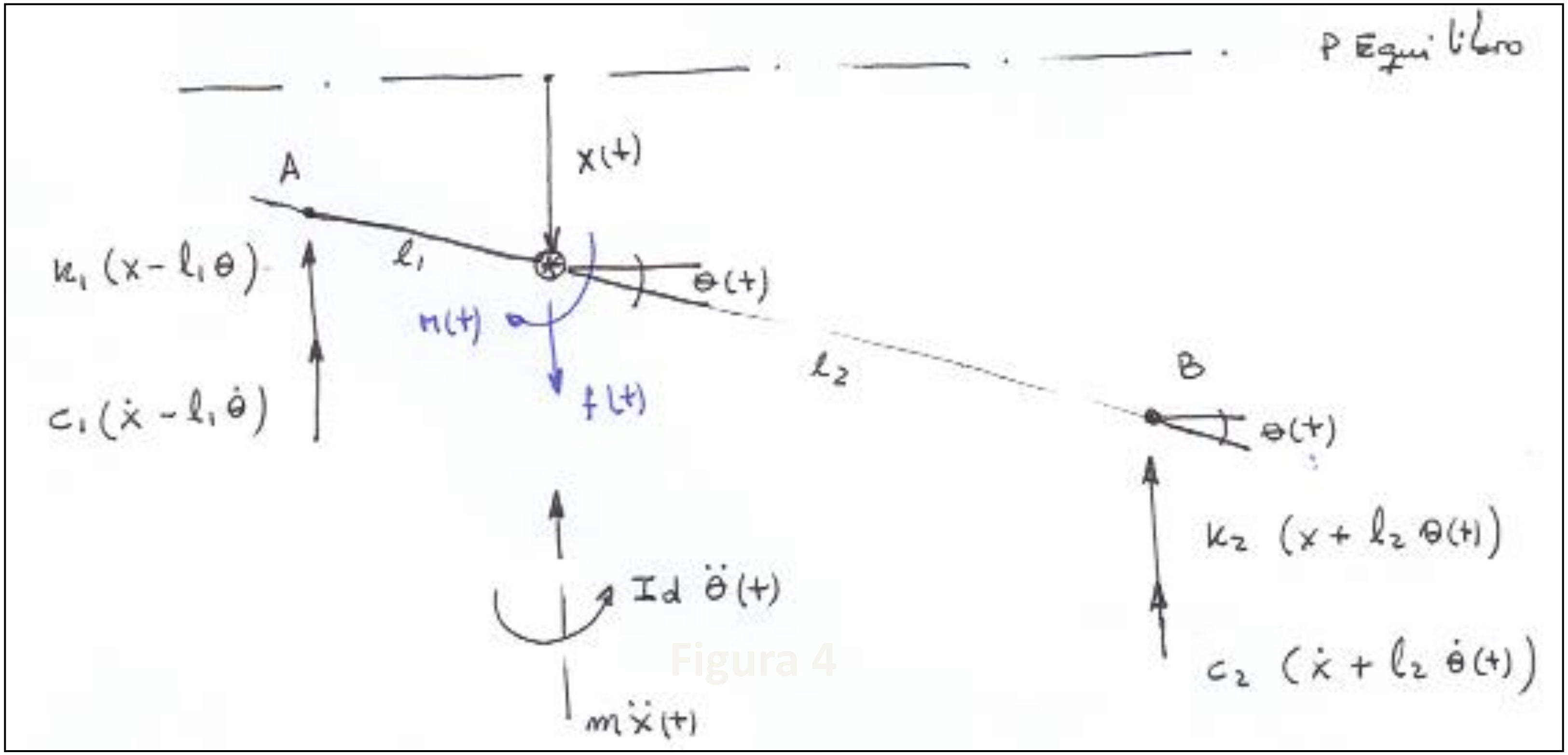
5.1 – Multi degree of freedom system: An Application

The two degree of freedom model can represent a simple model of the suspension car, the rotor and isotropic support and the isolation system, among others.



5.1 – Multi degree of freedom system: An Application

The free body diagram is



◈ 5.1 – Multi degree of freedom system: An Application

and applying the Newton's second law

$$f(t) - k_1(x - l_1\theta) - c_1(\dot{x} - l_1\dot{\theta}) - k_2(x + l_2\theta) - c_2(\dot{x} + l_2\dot{\theta}) = m\ddot{x}(t)$$

$$k_1(x - l_1\theta)l_1 + c_1(\dot{x} - l_1\dot{\theta})l_1 + M(t) - k_2(x + l_2\theta)l_2 - c_2(\dot{x} + l_2\dot{\theta})l_2 = I_d\ddot{\theta}(t)$$

5.1 – Multi degree of freedom system: An Application

and in matrix form

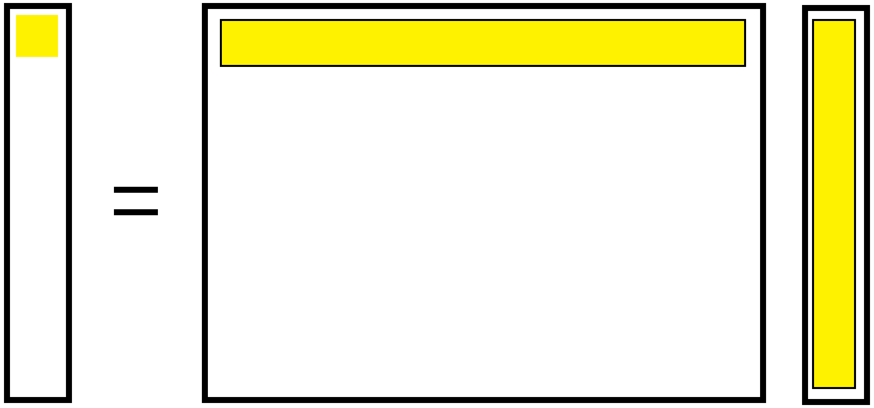
$$\begin{bmatrix} m & 0 \\ 0 & I_d \end{bmatrix}\{\ddot{q}\}+\begin{bmatrix} c_1+c_2 & c_2l_2-c_1l_1 \\ c_2l_2-c_1l_1 & c_2l_2^2+c_1l_1^2 \end{bmatrix}\{\dot{q}\}+\begin{bmatrix} k_1+k_2 & k_2l_2-k_1l_1 \\ k_2l_2-k_1l_1 & k_2l_2^2+k_1l_1^2 \end{bmatrix}\{q\}=\{f(t)\}$$

$$\{q\}=\begin{Bmatrix} x(t) \\ \theta(t) \end{Bmatrix} \qquad \{f(t)\}=\begin{Bmatrix} f(t) \\ M(t) \end{Bmatrix}$$

5.1 – Influence Coefficient Method

Stiffness: The reaction force introducing by the elastic properties is, as saw, given by

$$Q_i = \sum_{j=1}^{n} k_{ij} q_j$$



5.1 – Influence Coefficient Method

If we supposed that the “s” coordinates is 1 and another ones, with j≠s, are 0, the resultant force to produce such situation it will, numerically, equals to the column of stiffness matrix

$$q_s = 1, \; and \;\; q_{j \neq s} = 0 \quad \Rightarrow \quad \boxed{Q_i = k_{is}}$$

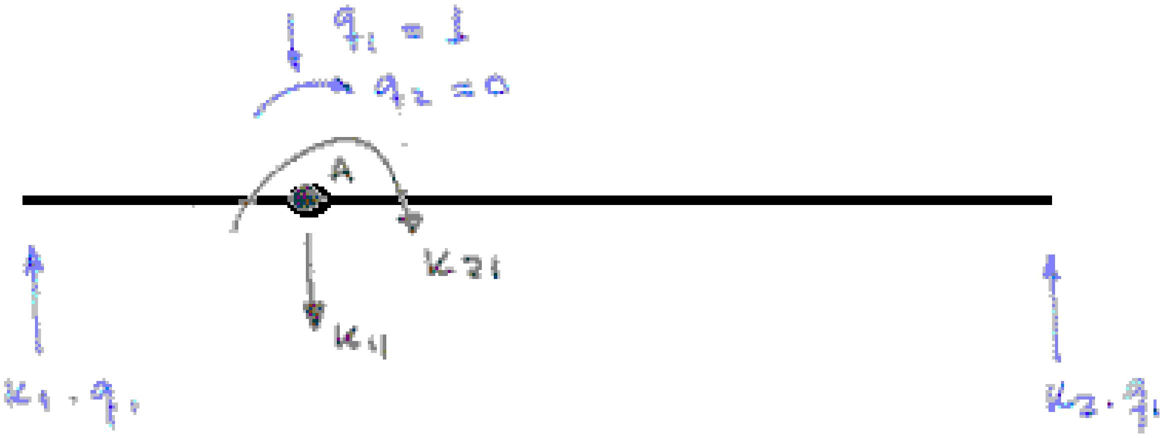
This procedure allows us to find the K matrix.

The same concept can be applied to the damping and inertial matrix. In theses cases we use the velocity and acceleration coordinates, instead the displacement coordinates.

5.1 – Influence Coefficient Method: Example 1

Use this method for the example above, considering

$$a)\ \ q_1 = 1\ \ and\ \ q_2 = 0;$$



5.1 – Influence Coefficient Method: Example 1

The resultant force is given by

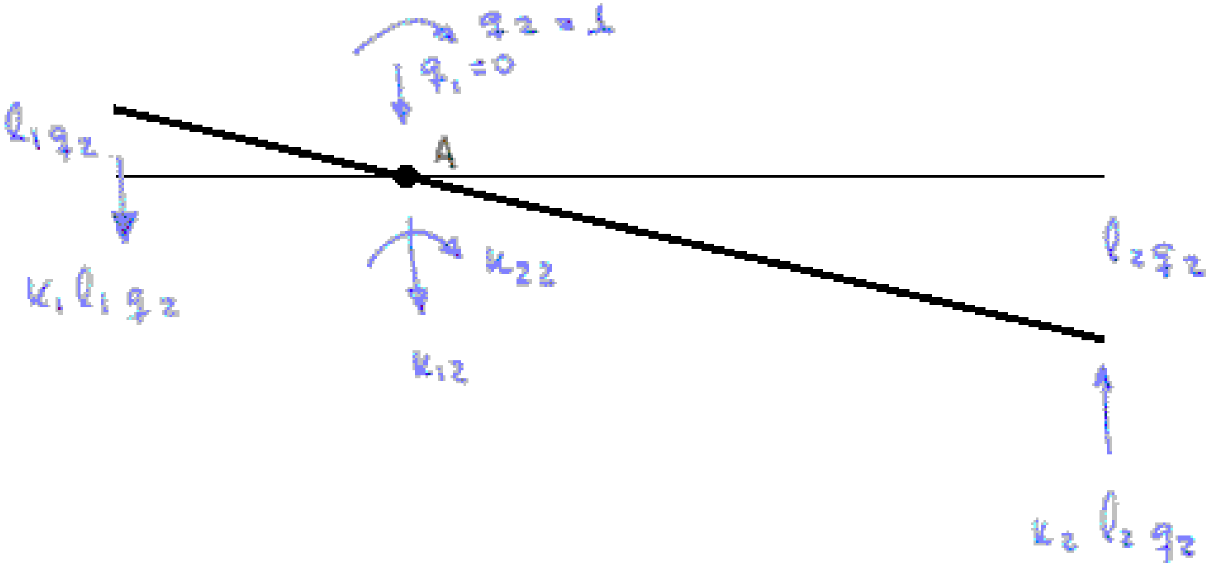
$$\sum F_{vertical} = 0 \Rightarrow -k_1 - k_2 + k_{11} = 0 \Rightarrow \quad k_{11} = k_1 + k_2$$

$$\sum M_A = 0 \quad \Rightarrow \; k_1 l_1 - k_2 l_2 + k_{21} = 0 \Rightarrow \; k_{21} = k_2 l_2 - k_1 l_1$$

Then we find the first column of the stiffness matrix

◈ 5.1 – Influence Coefficient Method: Example 1

$$b)\ \ q_2 = 1\ \ and\ \ q_1 = 0;$$



5.1 – Influence Coefficient Method: Example 1

The resultant force is given by

$$\sum F_{vertical} = 0 \Rightarrow -k_{12} + k_1 l_1.1 - k_2 l_2.1 = 0 \Rightarrow \quad k_{12} = k_2 l_2 - k_1 l_1$$

$$\sum M_A = 0 \quad \Rightarrow k_1\, l_1 l_1 + k_2\, l_2 l_2 + k_{22} = 0 \Rightarrow \quad k_{22} = k_1\, l_1^2 - k_2\, l_2^2$$

$$K = \begin{bmatrix} k_1 + k_2 & k_2 l_2 - k_1 l_1 \\ k_2 l_2 - k_1 l_1 & k_2 l_2^2 + k_1 l_1^2 \end{bmatrix}$$

5.1 – Influence Coefficient Method: Example 1

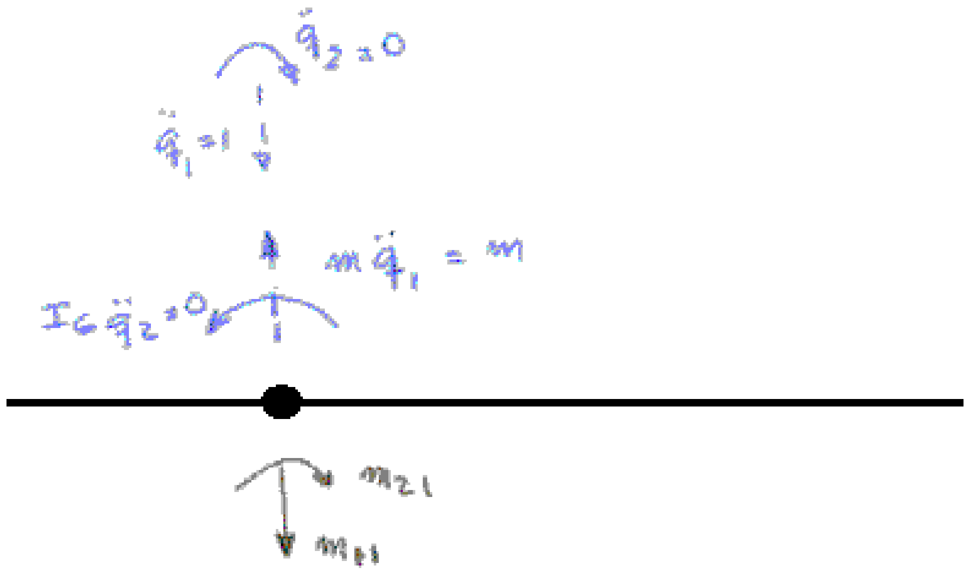
In a similar way, doing

$$a)\ \dot{q}_1 = 1\ \ and\ \ \dot{q}_2 = 0;\ and$$
$$b)\ \dot{q}_2 = 1\ \ and\ \ \dot{q}_1 = 0,$$

$$C = \begin{bmatrix} c_1 + c_2 & c_2 l_2 - c_1 l_1 \\ c_2 l_2 - c_1 l_1 & c_2 l_2^2 + c_1 l_1^2 \end{bmatrix}$$

Lastly, for the inertial matrix

$$a)\ \ \ddot{q}_1 = 1 \ \ and \ \ \ddot{q}_2 = 0;$$



5.1 – Influence Coefficient Method: Example 1

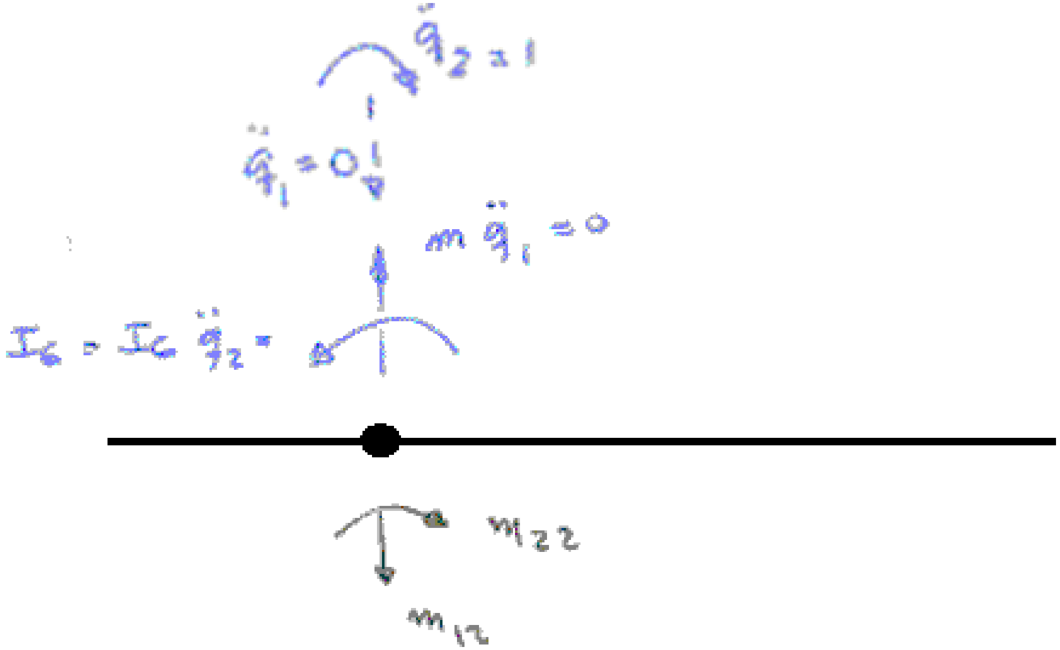
Then, the first column of the inertial matrix

$$\sum F_v = m_{11} - m\ddot{q}_1 = 0 \;\Rightarrow\; m_{11} = m$$

$$\sum M_A = m_{21} = 0 \qquad \Rightarrow\; m_{21} = 0$$

5.1 – Influence Coefficient Method: Example 1

$$b)\ \ \ddot{q}_2 = 1\ \ and\ \ \ddot{q}_1 = 0;$$



5.1 – Influence Coefficient Method: Example 1

$$\sum F_v = m_{12} = 0 \;\Rightarrow\; m_{12} = 0$$

$$\sum M_A = 0 \;= m_{22} - I_G\,\ddot{q}_2 = m_{22} - I_G = 0 \quad \Rightarrow\; m_{22} = I_G$$

$$\boxed{M = \begin{bmatrix} m & 0 \\ 0 & I_G \end{bmatrix}}$$

Then, the equation of motion is, in matrix simplified notation

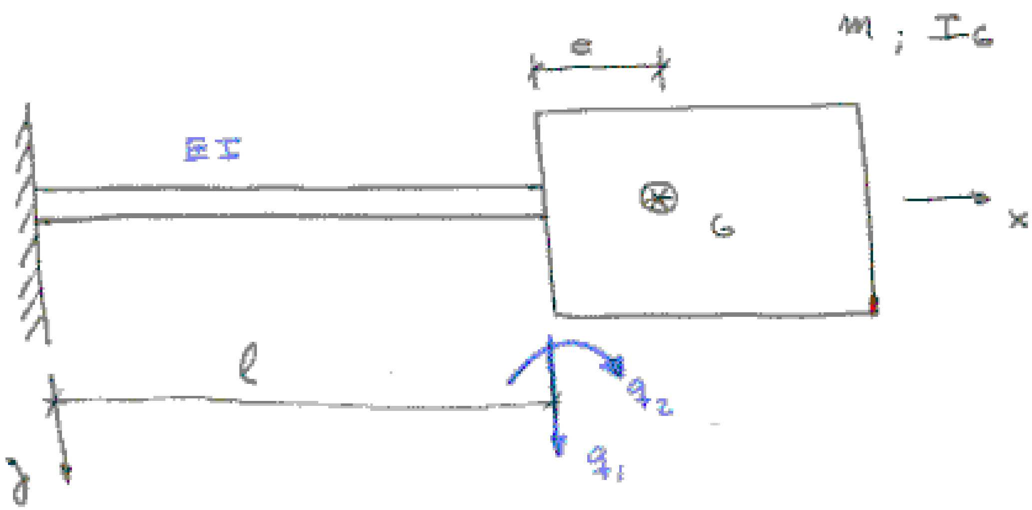
$$M\{\ddot{q}(t)\}+C\{\dot{q}(t)\}+K\{q(t)\}=f(t)$$

where

$$\begin{bmatrix} m & 0 \\ 0 & I_G \end{bmatrix}\begin{Bmatrix} \ddot{q}_1(t) \\ \ddot{q}_2(t) \end{Bmatrix}+\begin{bmatrix} c_1+c_2 & c_2l_2-c_1l_1 \\ c_2l_2-c_1l_1 & c_2l_2^2+c_1l_1^2 \end{bmatrix}\begin{Bmatrix} \dot{q}_1(t) \\ \dot{q}_2(t) \end{Bmatrix}+\begin{bmatrix} k_1+k_2 & k_2l_2-k_1l_1 \\ k_2l_2-k_1l_1 & k_2l_2^2+k_1l_1^2 \end{bmatrix}\begin{Bmatrix} q_1(t) \\ q_2(t) \end{Bmatrix}=\begin{Bmatrix} f(t) \\ M(t) \end{Bmatrix}$$

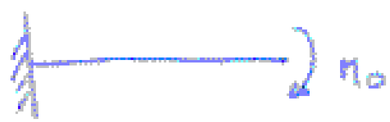
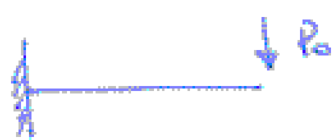
◆ 5.1 – Influence Coefficient Method: Example 2

The second example represents a water tank in the cities, the middle of the part of the Stockbridge, among others real systems.



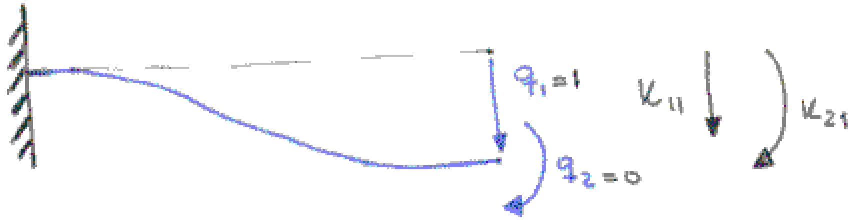
5.1 – Influence Coefficient Method: Example 2

It is known, for elasticity theory, that the external excitation applied in the free end of a beam elements produce the follow deformation:

	$\begin{cases} y_o = \dfrac{M_o l^2}{2EI} \\ \theta_o = \dfrac{M_o l}{EI} \end{cases}$
	$\begin{cases} y_o = \dfrac{P_o l^3}{3EI} \\ \theta_o = \dfrac{P_o l^2}{2EI} \end{cases}$

5.1 – Influence Coefficient Method: Example 2

$$a)\ \ q_1 = 1 \ \ and \ \ q_2 = 0;$$



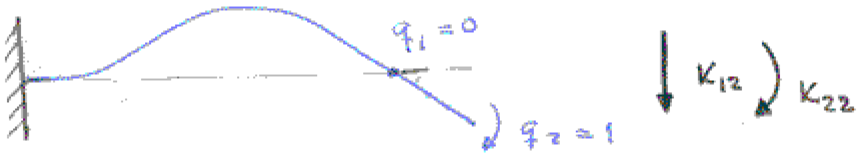
$$\left.\begin{aligned} \frac{k_{11} l^3}{3EI} + \frac{k_{21} l^2}{2EI} = 1 \\ \frac{k_{11} l^2}{2EI} + \frac{k_{21} l}{EI} = 0 \end{aligned}\right\}$$

$$k_{11} = \frac{12EI}{l^3}$$

$$k_{21} = -\frac{6EI}{l^2}$$

◈ 5.1 – Influence Coefficient Method: Example 2

$$b)\ \ q_2 = 1\ \ and\ \ q_1 = 0;$$



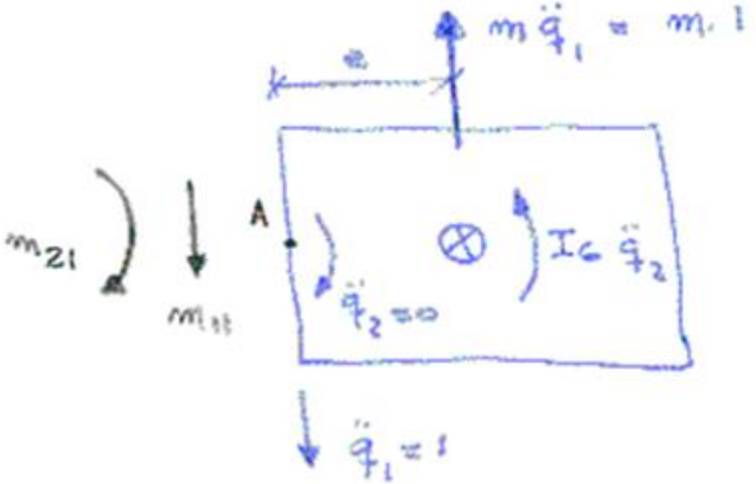
$$\left.\begin{array}{l} \dfrac{k_{12} l^2}{2EI} + \dfrac{k_{22} l}{EI} = 1 \\ \dfrac{k_{12} l^3}{3EI} + \dfrac{k_{22} l^2}{2EI} = 0 \end{array}\right\} \qquad k_{12} = -\dfrac{6EI}{l^2} \qquad k_{22} = \dfrac{4EI}{l}$$

$$\Rightarrow \quad K = \begin{bmatrix} 12EI/l^3 & -6EI/l^2 \\ -6EI/l^2 & 4EI/l \end{bmatrix}$$

5.1 – Influence Coefficient Method: Example 2

For the inertial matrix

$$a)\;\; \ddot{q}_1 = 1 \;\; and \;\; \ddot{q}_2 = 0;$$

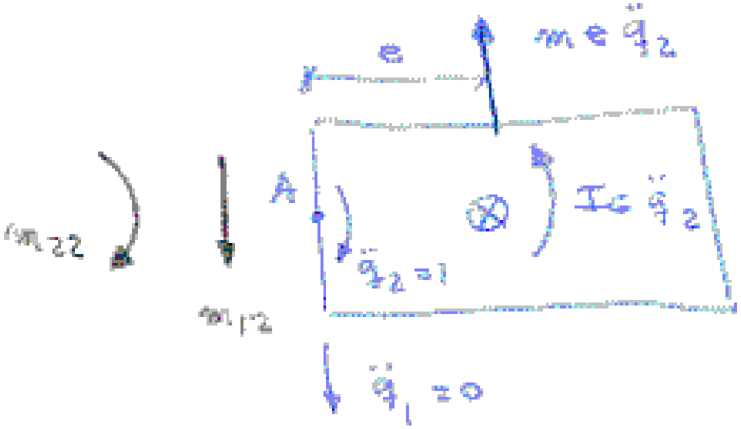


$$\sum F_v = m_{11} - m\ddot{q}_1 = 0 \;\Rightarrow\; m_{11} = m$$

$$\sum M_A = m_{21} - m\ddot{q}_1 . e = 0 \qquad \Rightarrow\; m_{21} = me$$

- 5.1 – Influence Coefficient Method: Example 2

$$b)\ \ \ddot{q}_2 = 1 \ \ and \ \ \ddot{q}_1 = 0;$$



$$\sum F_v = m_{12} - me\ddot{q}_2 = 0 \ \Rightarrow \ m_{12} = me$$

$$\sum M_A = 0 \quad = m_{22} - I_G\,\ddot{q}_2 - me^2\,\ddot{q}_2 = 0 \qquad \Rightarrow \ m_{22} = I_G + me^2$$

Then, the equation of motion is

$$\begin{bmatrix} m & me \\ me & I_G + me^2 \end{bmatrix} \begin{Bmatrix} \ddot{q}_1(t) \\ \ddot{q}_2(t) \end{Bmatrix} + \begin{bmatrix} 12EI/l^3 & -6EI/l^2 \\ -6EI/l^2 & 4EI/l \end{bmatrix} \begin{Bmatrix} q_1(t) \\ q_2(t) \end{Bmatrix} = \begin{Bmatrix} f(t) \\ m(t) \end{Bmatrix}$$

or

$$M\{\ddot{q}(t)\} + C\{\dot{q}(t)\} + K\{q(t)\} = f(t)$$

5.1 – Influence Coefficient Method of Flexibility

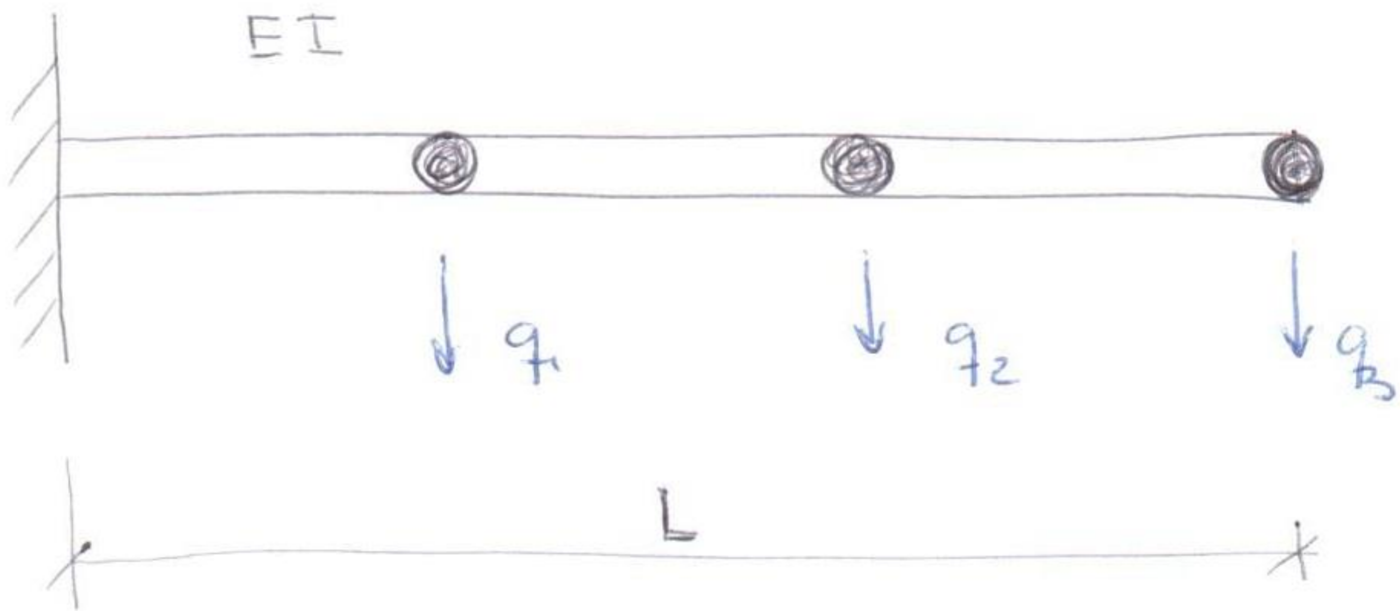
For mechanical systems, the calculation of the stiffness matrix, through the influence coefficients of stiffness, requires the resolution of an equation system.
This leads, in general, to an excessive computational cost.
For another hands, K can be calculated by the inverse of the flexible matrix.

$$AF = A\,K\,q \qquad\qquad AF = q$$

$$q_j = \sum_{i=1}^{n} \alpha_{ji}\, f_i = \sum_{i=1}^{n} a_{ji}\, f_i$$

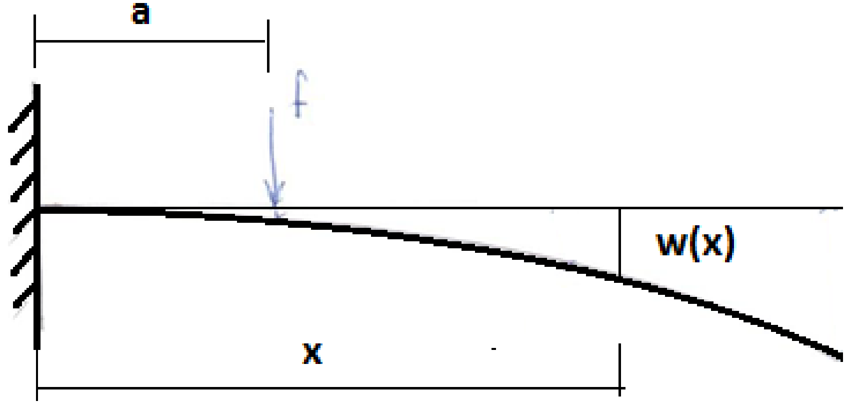
5.1 – Influence Coefficient Method of Flexibility

Considering the case below



5.1 – Influence Coefficient Method of Flexibility

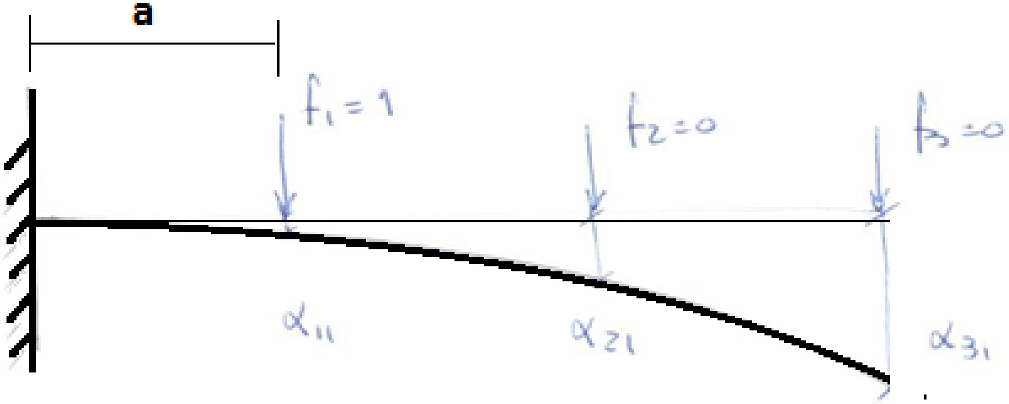
As known



$$y(x)=\frac{1}{E\,I}\left[\frac{1}{6}(x-a)^3\;\;\mu(x-a)-\frac{x^3}{6}+a\,\frac{x^2}{2}\right]$$

5.1 – Influence Coefficient Method of Flexibility

$$a)\ F_1 = 1\ and\ F_{j \neq s} = 0$$



The first column of the flexibility matrix will be

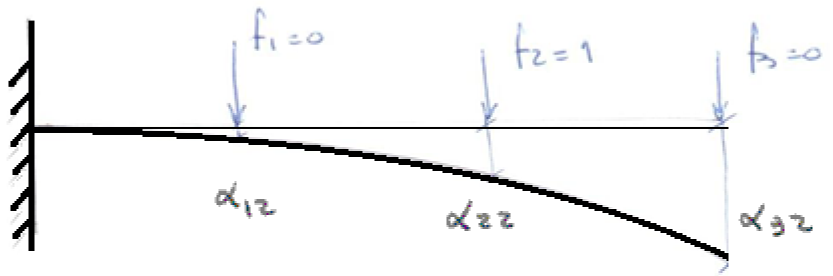
$$\alpha_{11}\left(x=\frac{L}{3}\right)=\frac{1}{EI}\left[-\frac{\left(L/3\right)^3}{6}+\frac{L}{3}\frac{\left(L/3\right)^2}{2}\right]=\frac{1}{EI}\left(\frac{L^3}{81}\right)$$

$$\alpha_{21}\left(x=\frac{2}{3}L\right)=\frac{1}{EI}\left[\frac{1}{6}\left(\frac{L}{3}\right)^3-\left(\frac{2L}{3}\right)^3\frac{1}{6}+\frac{L}{3}\left(\frac{\left(2L/3\right)^2}{2}\right)\right]=\frac{L^3}{EI}\frac{5}{162}$$

$$\alpha_{31}\left(x=L\right)=\frac{1}{EI}\left[\frac{1}{6}\left(\frac{2L}{3}\right)^3-\frac{L^3}{6}+\frac{L}{3}\frac{L^2}{2}\right]=\frac{L^3}{EI}\frac{4}{81}$$

5.1 – Influence Coefficient Method of Flexibility

$$b)\ \ F_2 = 1 \ \ and \ \ F_{j \neq s} = 0$$



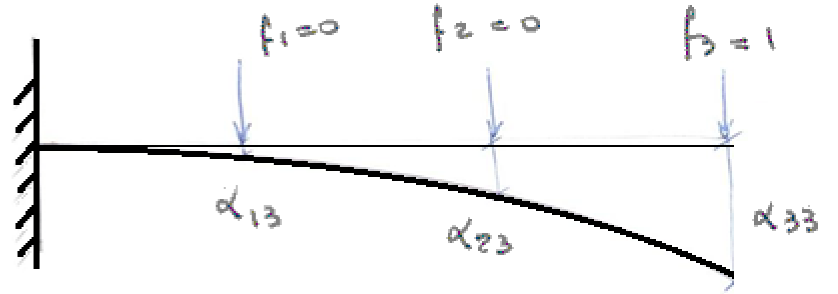
$$\alpha_{12}\left(x = \frac{L}{3}\right) = \frac{1}{EI}\left[-\frac{\left(L/3\right)^3}{6} + \frac{2L}{3}\,\frac{\left(L/3\right)^2}{2}\right] = \frac{1}{EI}L^3\left(\frac{5}{162}\right)$$

$$\alpha_{22}\left(x = \frac{2L}{3}\right) = \frac{1}{EI}\left[-\frac{\left(2L/3\right)^3}{6} + \frac{2L}{3}\,\frac{\left(2L/3\right)^2}{2}\right] = \frac{1}{EI}L^3\left(\frac{8}{81}\right)$$

$$\alpha_{32}\left(x = L\right) = \frac{1}{EI}\left[\frac{1}{6}\left(\frac{L}{3}\right)^3 - \left(\frac{L}{3}\right)^3 + \frac{2L}{3}\,\frac{L^2}{2}\right] = \frac{1}{EI}L^3\left(\frac{14}{81}\right)$$

5.1 – Influence Coefficient Method of Flexibility

$$b)\ \ F_3 = 1 \ \ and \ \ F_{j \neq s} = 0$$



$$\alpha_{13}\left(x = \frac{L}{3}\right) = \frac{1}{EI}\left[-\frac{\left(L/3\right)^3}{6} + L\,\frac{\left(L/3\right)^2}{2}\right] = \frac{1}{EI}L^3\left(\frac{4}{81}\right)$$

$$\alpha_{23}\left(x = \frac{2L}{3}\right) = \frac{1}{EI}\left[-\frac{\left(2L/3\right)^3}{6} + L\,\frac{\left(2L/3\right)^2}{2}\right] = \frac{1}{EI}L^3\left(\frac{14}{81}\right)$$

$$\alpha_{33}(x = L) = \frac{1}{EI}\left[-\frac{L^3}{6} + L\frac{L^2}{2}\right] = \frac{1}{EI}L^3\left(\frac{1}{3}\right)$$

5.1 – Influence Coefficient Method of Flexibility

And the matrix K and M are

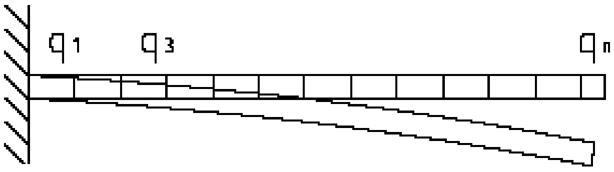
$$M = \begin{bmatrix} m_e & 0 & 0 \\ 0 & m_e & 0 \\ 0 & 0 & m_e/2 \end{bmatrix}$$

$$K = A^{-1}$$

$$A = \frac{L^3}{EI} \begin{bmatrix} 1/81 & 5/162 & 4/81 \\ 5/162 & 8/81 & 14/81 \\ 4/81 & 14/81 & 1/3 \end{bmatrix}$$

$$M_{3x3}\ \ddot{q}(t)_{3x1} + A^{-1}_{3x3} q(t)_{3x1} = f(t)_{3x1}$$

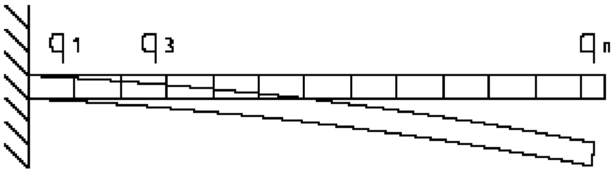
The equation of motion of the multi-degree systems is given by



$$M\,\ddot{q}(t)+C\,\dot{q}(t)+K\,q(t)=f(t)$$

$q(t)$	generalized coordinates, nx1
$f(t)$	generalized forces, nx1
M	generalized mass matrix, nxn
K	generalized stiffness matrix, nxn
C	generalized damping matrix, nxn

- Mathematical Model of Non Rotating Systems



In the frequency domain

$$\left[-\Omega^2 M + i\Omega C + K\right] Q(\Omega) = F(\Omega)$$

Using the below transformation matrix, and taking only a few eigenvectors who are into the frequency range of interest

$$Q(\Omega)_{nx1} = \hat{\Phi}_{nx\hat{n}} \hat{P}(\Omega)_{\hat{n}x1}$$

and pre-multiplying by $\hat{\Phi}_{\hat{n}x1}{}^{T}$

Mathematical Model of Non Rotating Systems

Considering a orthogonal properties of the eigenvectors and the orthonormalized characteristic, it is possible to obtain

$$\Theta^T M \Theta = I \qquad \Theta^T K \Theta = \Lambda$$

$$\Theta^T C \Theta = \begin{bmatrix} \backslash & & \\ & 2\xi_j \Omega_j & \\ & & \backslash \end{bmatrix}$$

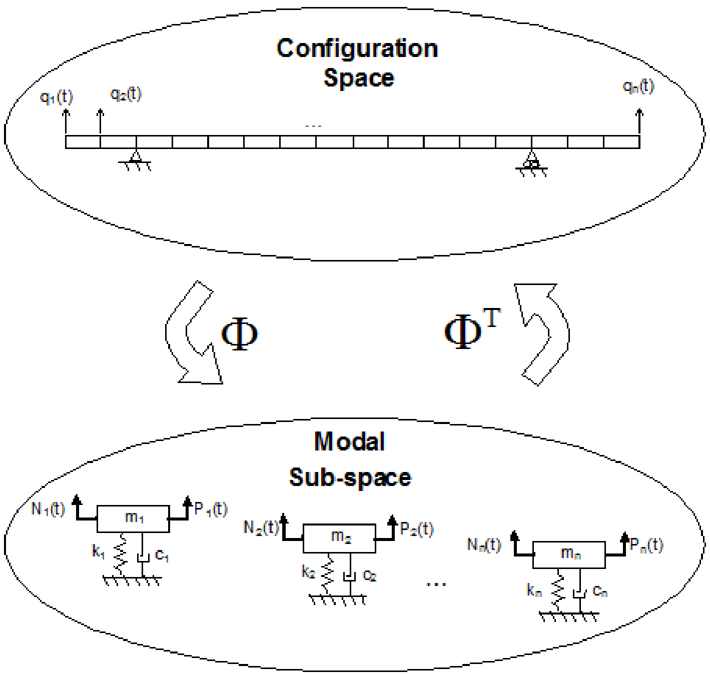
The equation of motion in the modal sub-space is given by

$$\left\{ -\Omega^2 \left[I_{\hat{n}x\hat{n}} \right] + i\Omega \left[\Gamma_{\hat{n}x\hat{n}} \right] + \Lambda_{\hat{n}x\hat{n}} \right\} \hat{P}(\Omega) = \hat{N}(\Omega) = \hat{\Theta}^T F(\Omega)$$

◈ Mathematical Model of Non Rotating Systems

So, it is possible to obtain the response $\hat{P}(\Omega) \Rightarrow Q(\Omega)$

$$\hat{P}(\Omega) = \left[\hat{D}(\Omega)\right]^{-1} \hat{\Theta}^{T} F(\Omega)$$



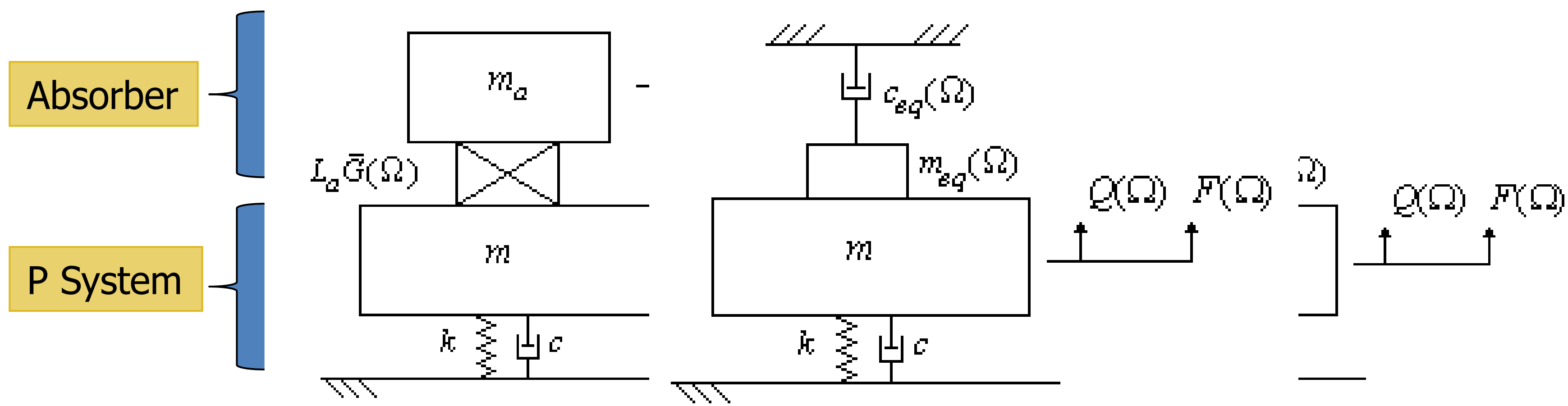
Then, the solution in the configuration space, q(t), is:

$$Q(\Omega) = \hat{\Theta}\left[\hat{D}(\Omega)\right]^{-1} \hat{\Theta}^{T} F(\Omega)$$

In the modal sub-space of the primary system the equations of motion are uncoupled.
Each line can be considered as one degree of freedom system

Equivalent Generalized Parameters

How to model the dynamic absorbers, such that, when attached to the structure to be controlled, the model of the compound system will be simple and inexpensive of the computational point of view

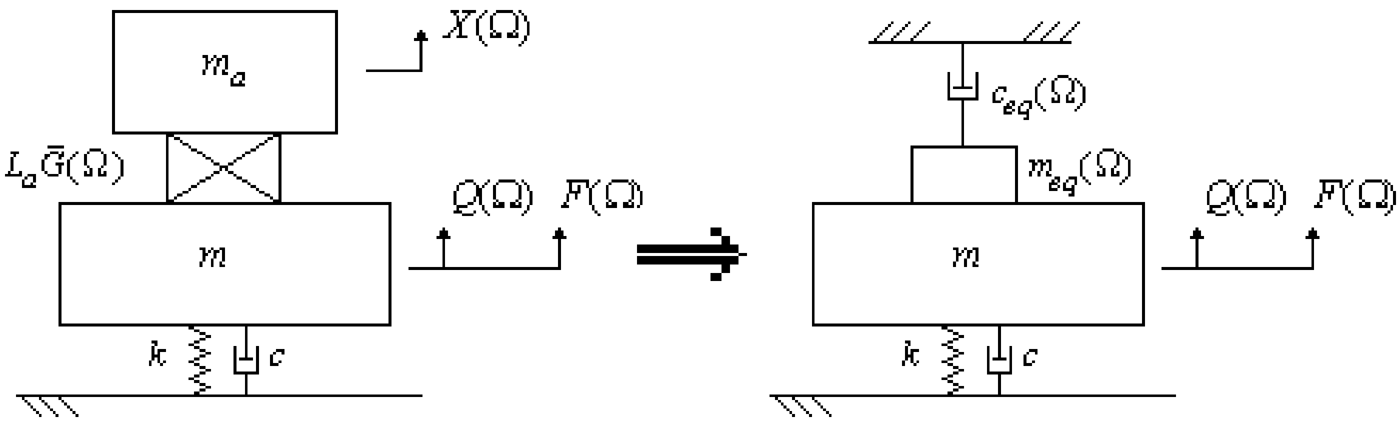


$$Z(\Omega) = \frac{F(\Omega)}{i\Omega\ X_b(\Omega)}$$

$$Z(\Omega) = c_{eq}(\Omega) + i\Omega m_{eq}(\Omega)$$

(ESPÍNDOLA e SILVA,1992)

Equivalent Generalized Parameters



$$c_{eq}(\Omega) = m\,\Omega_a \frac{r(\Omega)\,\eta(\Omega)\varepsilon^3}{\left(\varepsilon^2 - r(\Omega)\right)^2 + \left(r(\Omega)\eta(\Omega)\right)^2}$$

$$m_{eq}(\Omega) = -m\frac{r(\Omega)\left\{\varepsilon^2 - r(\Omega)\left[1+\eta(\Omega)^2\right]\right\}}{\left(\varepsilon^2 - r(\Omega)\right)^2 + \left(r(\Omega)\,\eta(\Omega)\right)^2}$$

- where

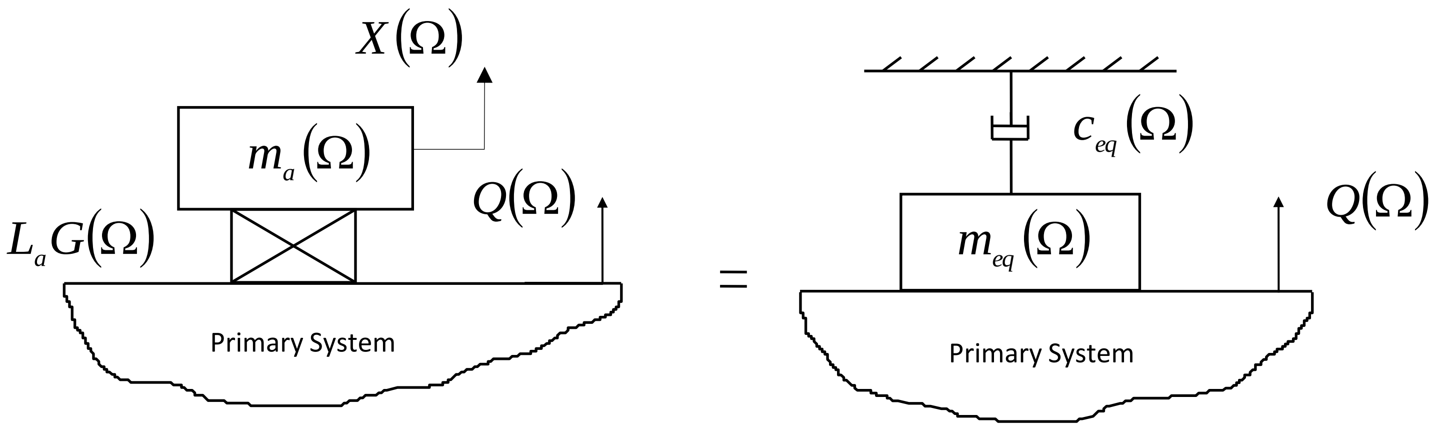
$$r(\Omega) = \frac{G(\Omega)}{G(\Omega_a)} \qquad \varepsilon = \frac{\Omega}{\Omega_a} \qquad \eta(\Omega) = \frac{\mathrm{Im}(G(\Omega))}{\mathrm{Re}(G(\Omega))}$$

Equivalent Generalized Parameters

Different types of model to different applications

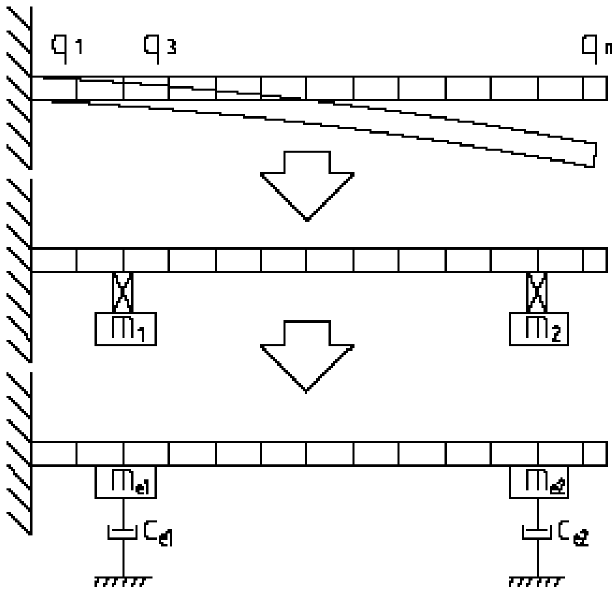
Type of Model	$m_e(\Omega)$	$c_e(\Omega)$	Ω_a
Viscous Model	$-m\frac{\{\varepsilon^2-[1+(2\zeta\varepsilon)^2]\}}{(\varepsilon^2-1)^2+(2\zeta\varepsilon)^2}$	$m\Omega_a\frac{2\zeta\varepsilon^4}{(\varepsilon^2-1)^2+(2\zeta\varepsilon)^2}$	$\sqrt{\frac{k}{m}}$
Viscoelastic	$-m\frac{r(\Omega)\{\varepsilon^2-r(\Omega)[1+\eta(\Omega)^2]\}}{(\varepsilon^2-r(\Omega))^2+(r(\Omega)\eta(\Omega))^2}$	$m\Omega_a\frac{r(\Omega)\eta(\Omega)\varepsilon^3}{(\varepsilon^2-r(\Omega))^2+(r(\Omega)\eta(\Omega))^2}$	$\sqrt{\frac{LG_r(\Omega_a)}{m}}$
Hydraulic	$-m\frac{\{\varepsilon^2-[1+(2\zeta\varepsilon)^2]\}}{(\varepsilon^2-1)^2+(2\zeta\varepsilon)^2}$	$m\Omega_a\frac{2\zeta\varepsilon^4}{(\varepsilon^2-1)^2+(2\zeta\varepsilon)^2}$	$\sqrt{\frac{\gamma g S_1\left(1+\frac{S_1}{S_2}\right)}{m_l}}$
Electromechanical	$T^2C\frac{\{1-\varepsilon)^2}{(1-\varepsilon^2)^2+(2\zeta_e\varepsilon)^2}$	$T^2C\frac{2\zeta_e\varepsilon^2}{(1-\varepsilon^2)^2+(2\zeta_e\varepsilon)^2}$	$\sqrt{\frac{1}{LC}}$

Equivalent Generalized Parameters



The primary system "feels" the absorber as being a equivalent mass $m_{eq}(\Omega)$ attached to the generalized coordinate $q_j(t)$ and a equivalent viscous damper with constant $c_{eq}(\Omega)$, connected to the ground.
Therefore, the dynamics of the resultant system (primary + absorbers) can be formulated in terms of the generalized coordinates of the primary system, where $Q(\Omega)$ is representative, despite the new system now having added degrees of freedom. This is the main advantage of the generalized equivalent quantities concept.

The Mathematical Model of Compound System



In the stationary system, the equation of motion for the compound system is

$$\left[-\Omega^2\tilde{M}+i\Omega\tilde{C}+K\right]Q(\Omega)=F(\Omega)$$

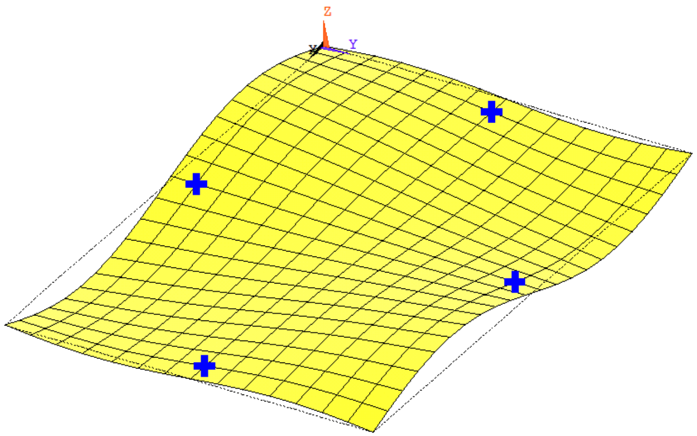
$$\tilde{M}=M+\begin{bmatrix}0 & & & & \\ & m_{eq}^{(1)}(\Omega) & & 0 & \\ & & \ddots & & \\ & 0 & & m_{eq}^{(p)}(\Omega) & \\ & & & & 0\end{bmatrix}=M+M_{eq}(\Omega)$$

$$\tilde{C}=C+\begin{bmatrix}0 & & & & \\ & c_{eq}^{(1)}(\Omega) & & 0 & \\ & & \ddots & & \\ & 0 & & c_{eq}^{(p)}(\Omega) & \\ & & & & 0\end{bmatrix}=C+C_{eq}(\Omega)$$

Then, the solution in the modal sub-space for the primary system

$$Q(\Omega)=\Phi\ \tilde{P}(\Omega)$$

The Mathematical Model of Compound System



Pre-multipliyng the equation of motion by the transpose eigenvectors, considering the orthonormalized characteristic, it is possible to obtain

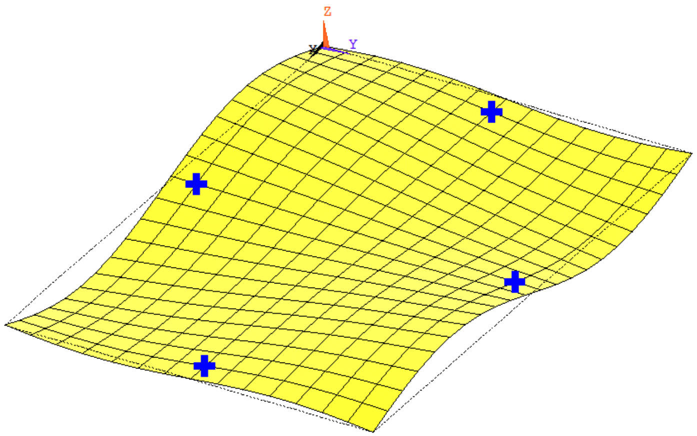
$$\Theta^T \tilde{M} \Theta = I + M_A(\Omega) \qquad \Theta^T K \Theta = \Lambda$$

$$\Theta^T \tilde{C} \Theta = \begin{bmatrix} \diagdown & & \\ & 2\xi_j \Omega_j & \\ & & \diagdown \end{bmatrix} + C_A(\Omega)$$

Then, the solution in the modal sub-space for the primary system

$$\left\{ -\Omega^2 \left[I_{\tilde{n}x\tilde{n}} + \tilde{M}_A(\Omega) \right] + i\Omega \left[\Gamma_{\tilde{n}x\tilde{n}} + \tilde{C}_A(\Omega) \right] + \Lambda_{\tilde{n}x\tilde{n}} \right\} \tilde{P}(\Omega) = \tilde{N}(\Omega)$$

◈ The Mathematical Model of Compound System



$$\tilde{P}(\Omega) = [D(\Omega)]^{-1} \Theta^{T} F(\Omega)$$

Then, the solution in the configuration space, q(t), is:

$$Q(\Omega) = \Theta [D(\Omega)]^{-1} \Theta^{T} F(\Omega)$$

The FRF of the compound systems is:

$$H(\Omega) = \Theta [D(\Omega)]^{-1} \Theta^{T}$$

The optimization problem is defined by:

$$\min f(x) \quad where \quad f : R^{nd} \to R$$

$$with \; x \in R^{nd}$$

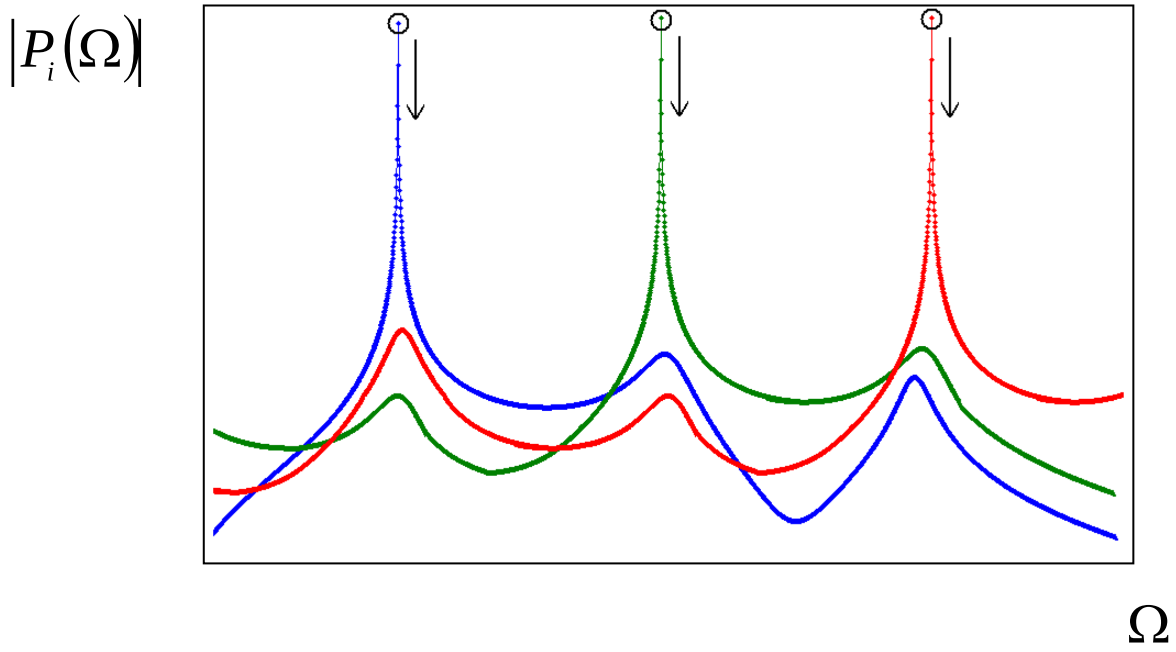
$$subject\ to:$$

$$h_i(x) = 0 \qquad i = 1,2,\ldots,m_i$$

$$g_j(x) \geq 0 \qquad j = m_i + 1, m_i + 2, \ldots, l$$

Nonlinear Optimization Techniques

Graphically:



$$\|P(\Omega)\|_2 = \sqrt{(MÁX)^2 + (MÁX)^2 + (MÁX)^2 + \dots}$$

◆ Nonlinear Optimization Techniques

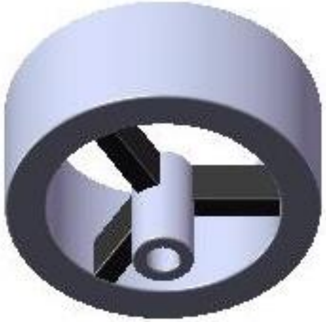
The objective function is defined by

$$f_{cost}(x) = \left\| \max_{\Omega_1<\Omega<\Omega_2} \left| \hat{P}(\Omega, x) \right\| \right\|$$

where

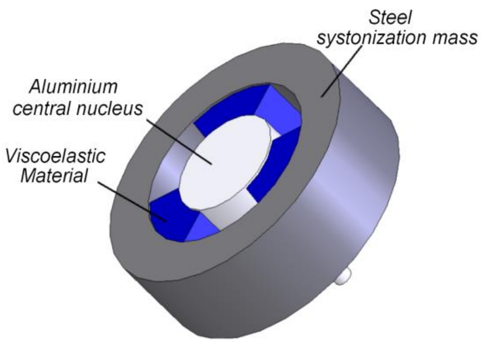
$$x = \left(\Omega_{a1}, \Omega_{a2}, \ldots, \Omega_{ap}\right)$$

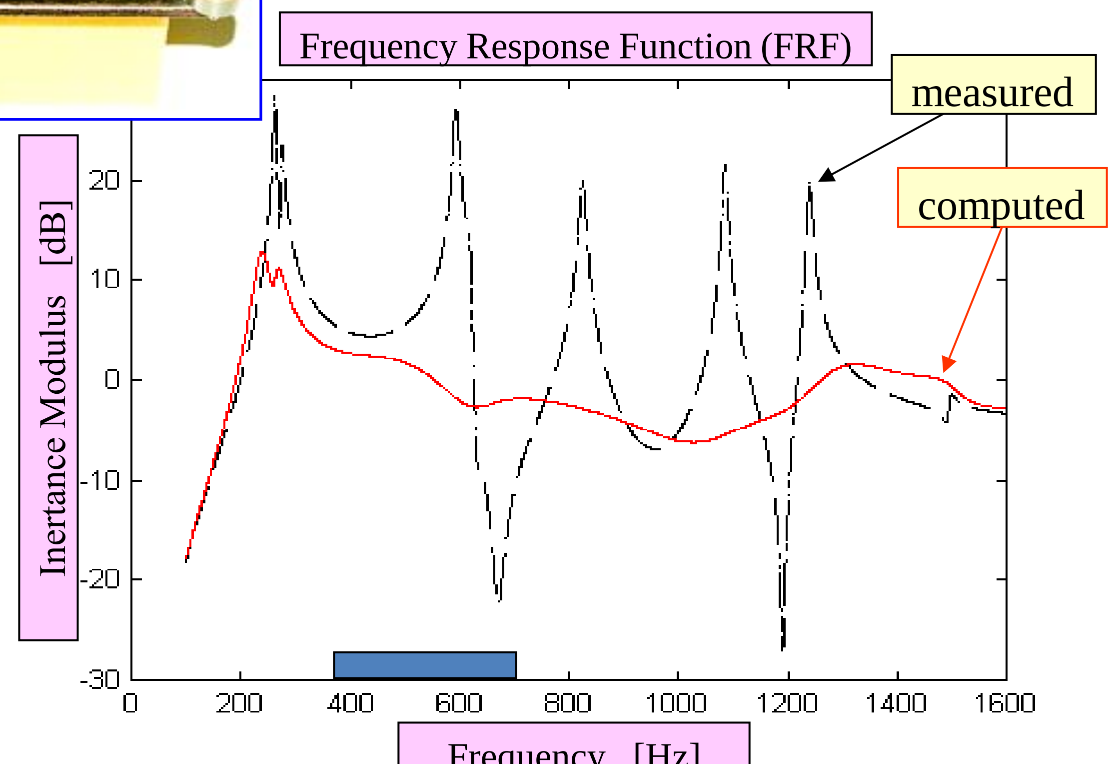
After optimization procedure, the DVA's natural frequencies Ω_n are known. Then, it is possible to do a physical realization.



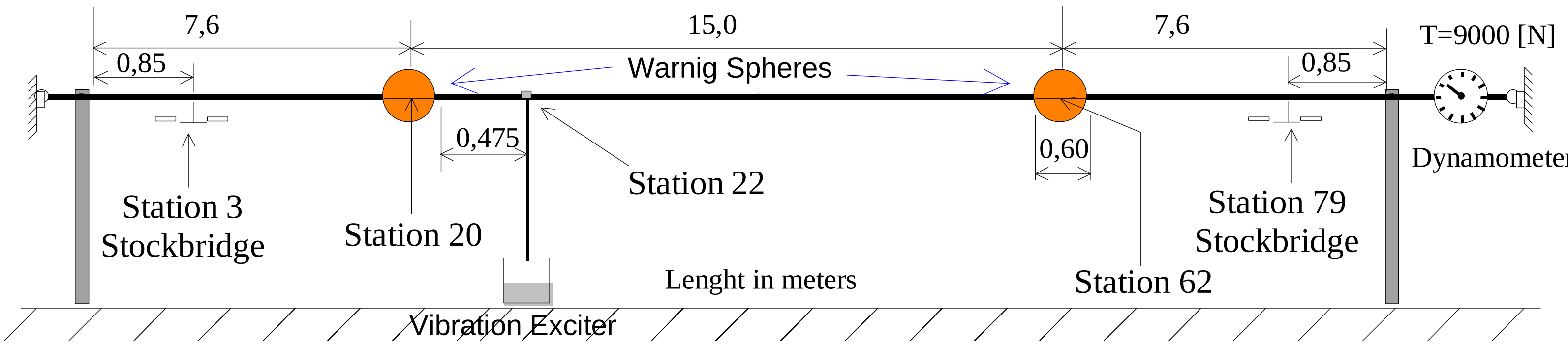
A thin steel plate – doctoral work

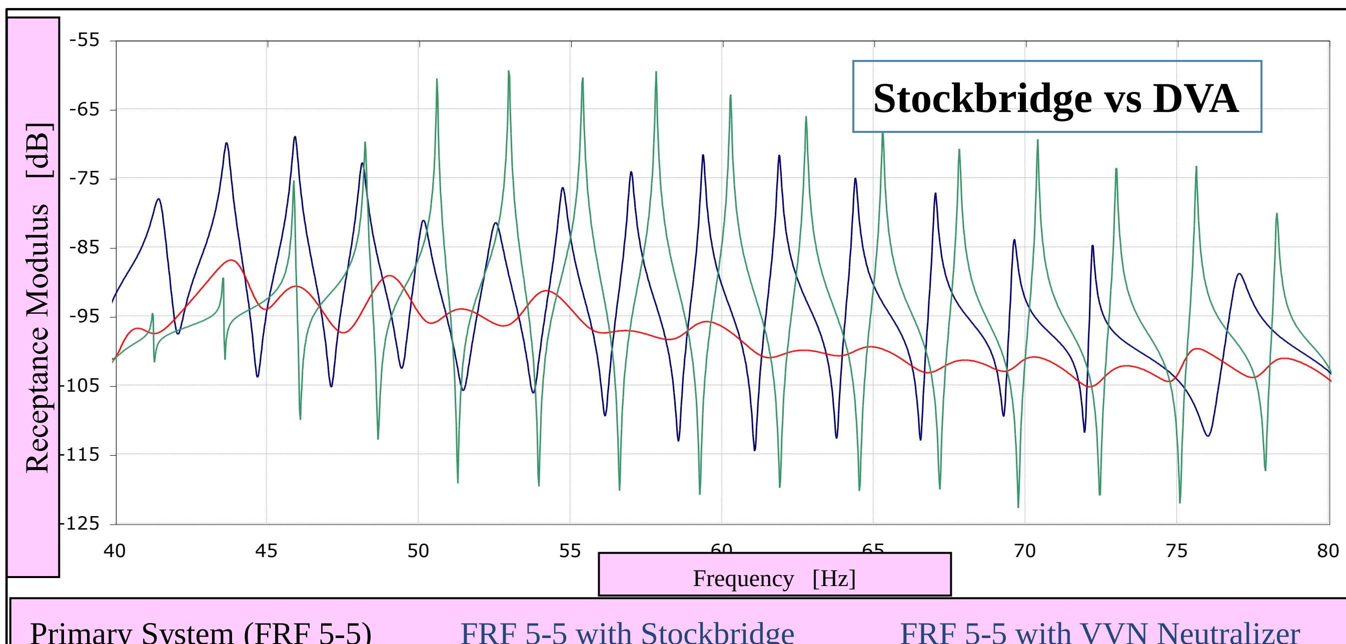
PISA/CNPq group



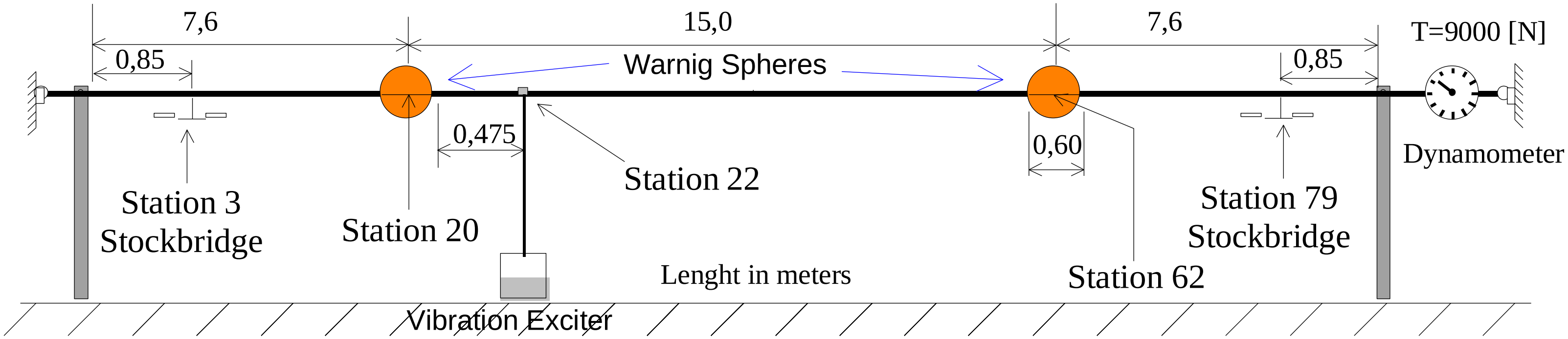


A power transmission line



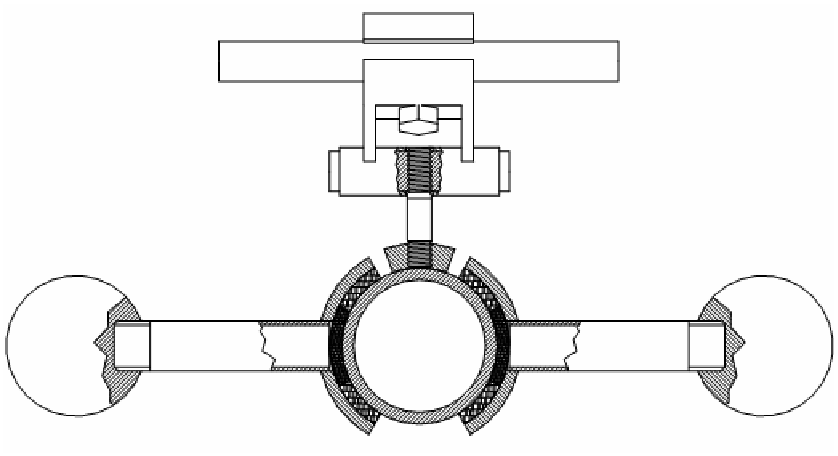


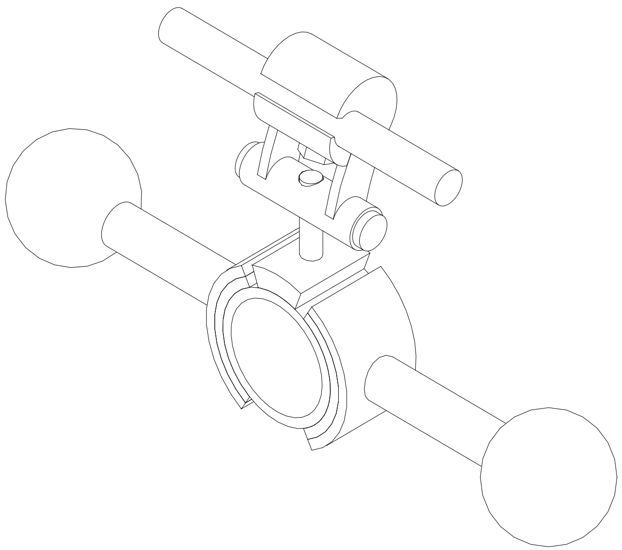
A power transmission line



Stockbridge *vs* DVA

A power transmission line

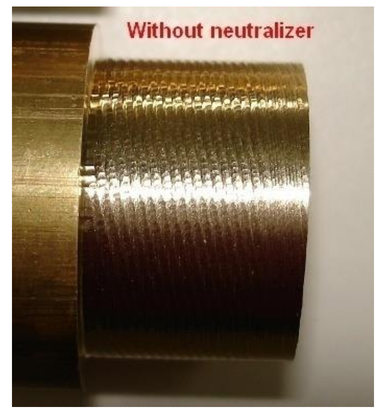




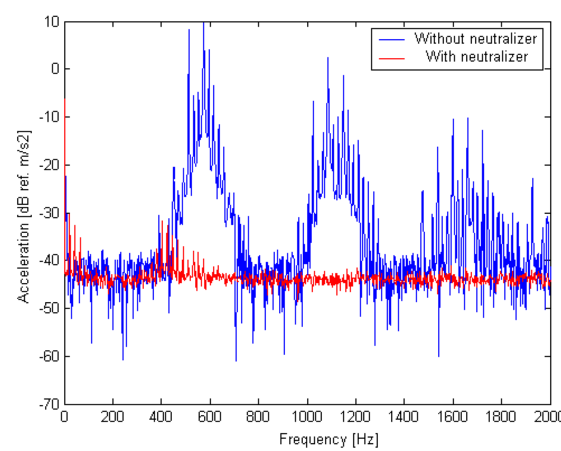
- Dynamic viscoelastic absorber
  - total additional mass (2,5% a 10%);
  - efficiently in a large frequency band;
  - allows more axial force on the transmission line;
  - then minor curve of the line;
  - Low towers.

◆ Instability-Chatter in Turning Processes

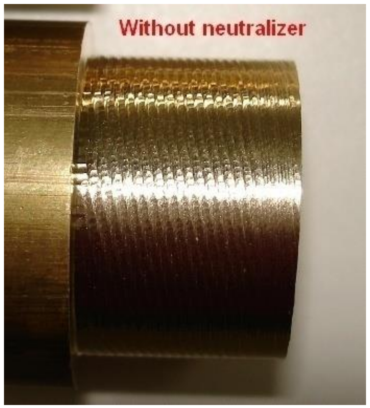
Set-up for the Tool Holder with Dynamic Absorber to eliminate Instability-Chatter in Turning Processes



Surface profile of the work piece machined

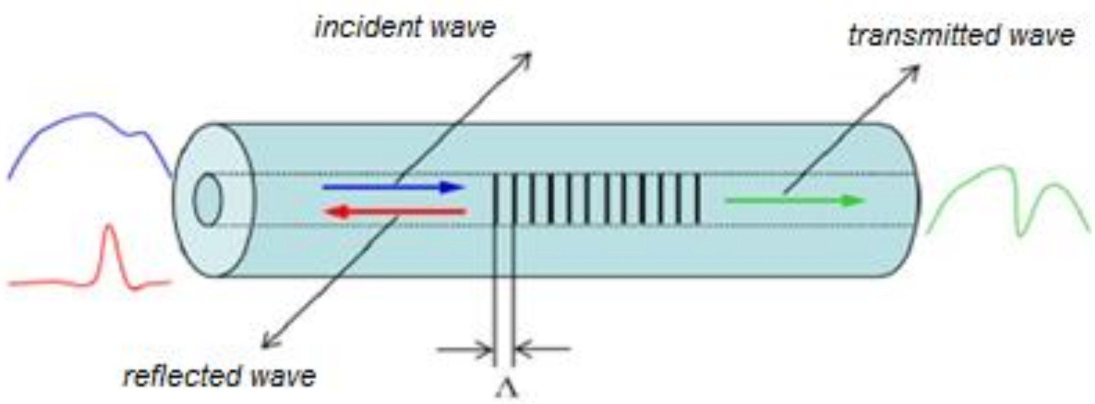


◈ Instability-Chatter in Turning Processes



Surface profile of the workpiece machined

◆ The Numerical example and experimental setup



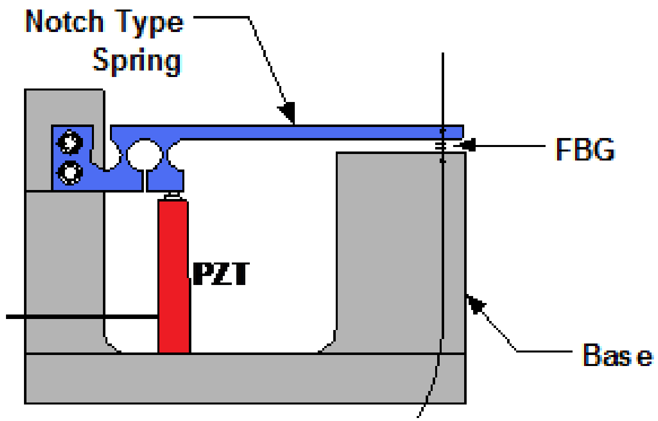
A typical Fiber Bragg Grating (FBG) has a central wavelength reflected spectrum given by

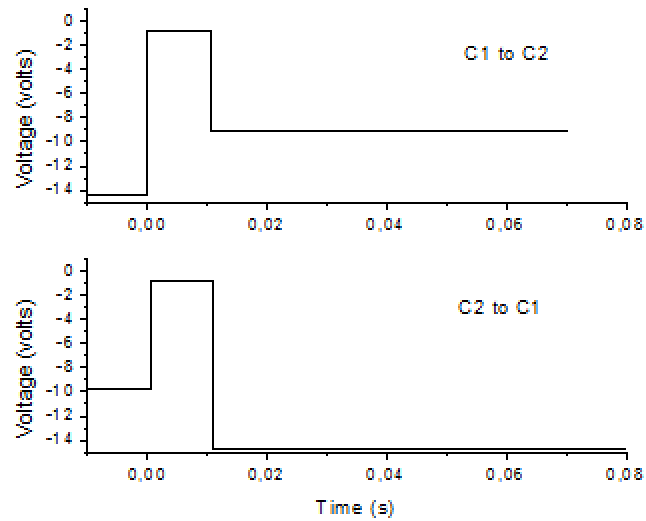
$$\lambda_B = 2n_{eff}\Lambda$$

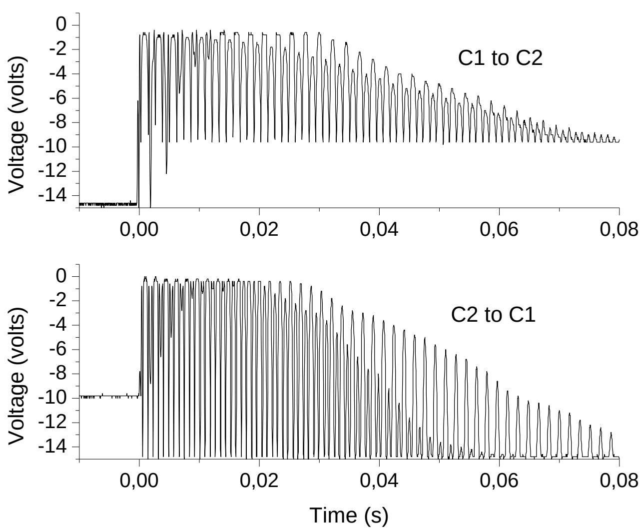
Disregarding the variation of temperature ΔT, which occurred in the present work, the change in Bragg wavelength due to application of a longitudinal strain is

$$\Delta\lambda_B = \lambda_B(1 - p_e)\varepsilon_{xx}$$

- The Numerical example and experimental setup



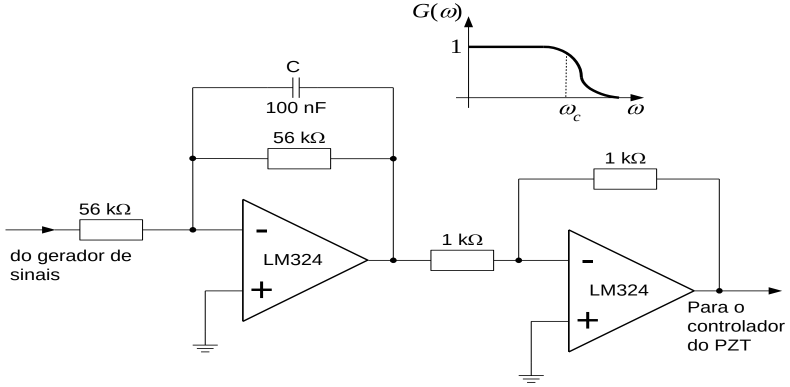


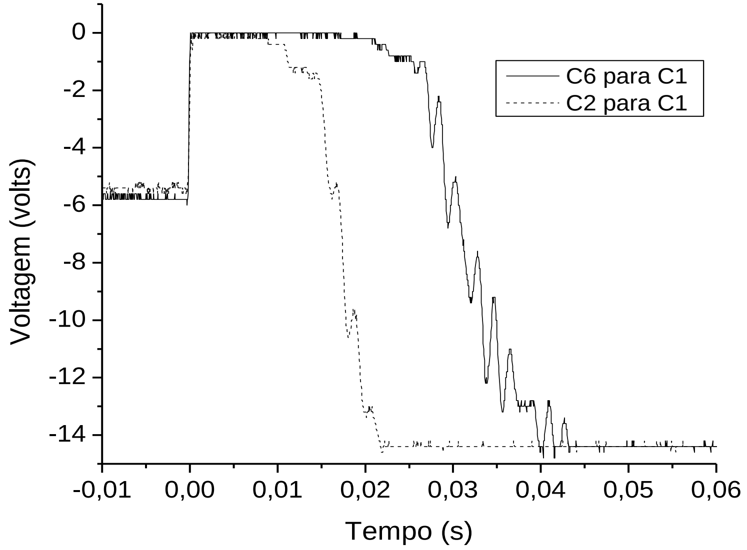


Tuning of channels 1 and 2. Ideal response and its experimental equivalent one

◈ The Numerical example and experimental setup

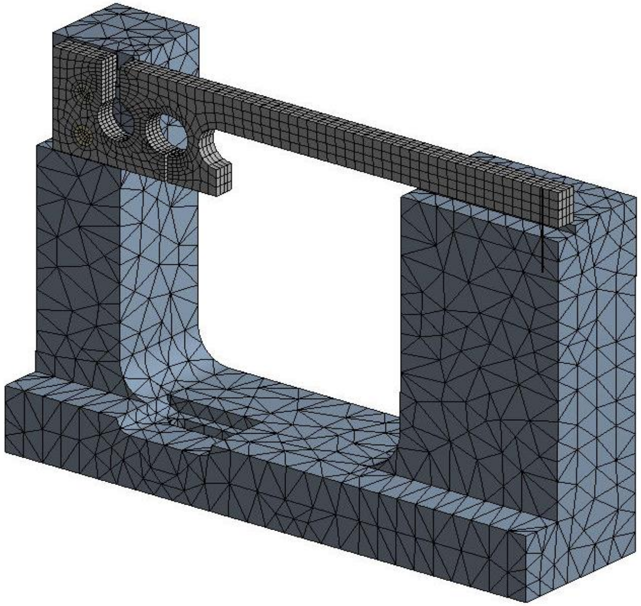
Electronic filter to reduce unwanted vibrations



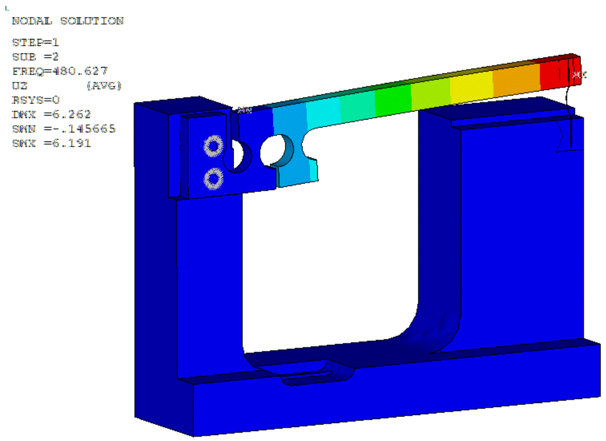


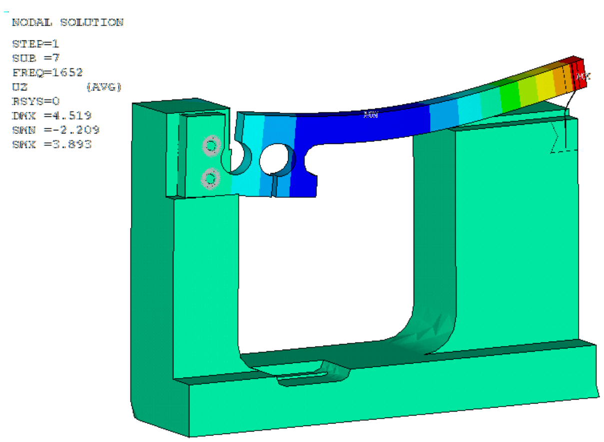
- The Numerical example and experimental setup

Finite Element Model



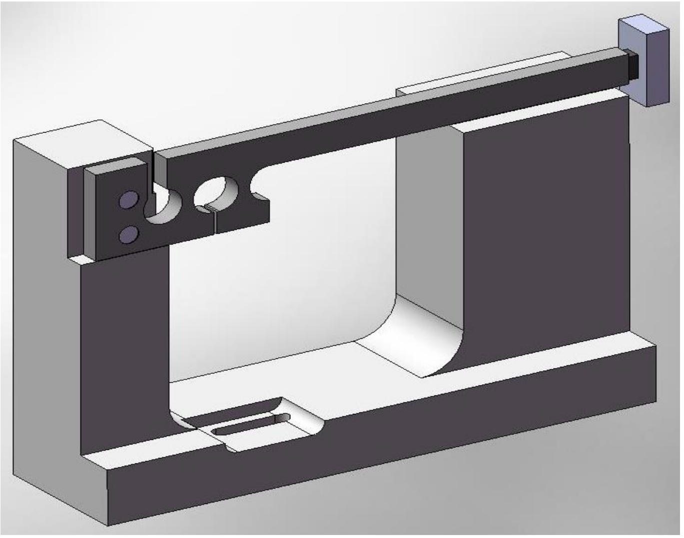
Modal Analysis of the dynamic system

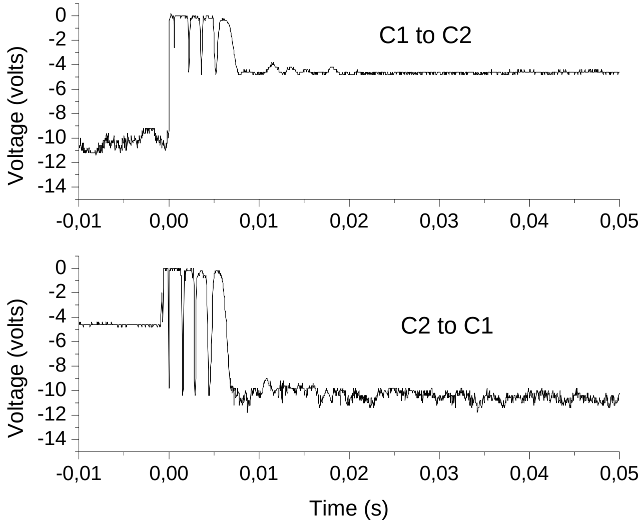




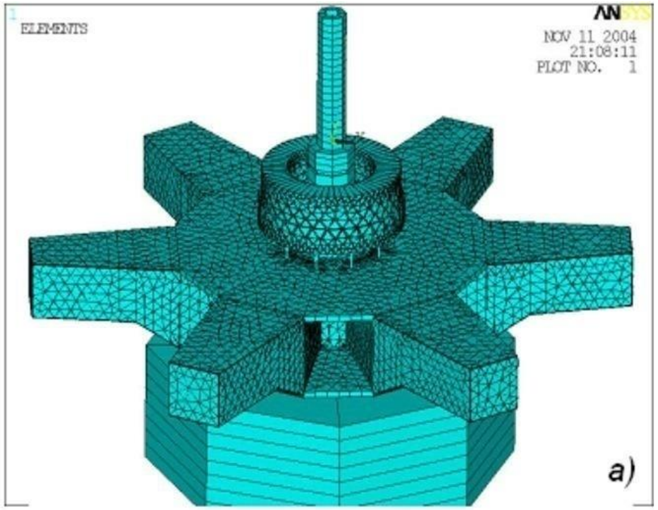
◈ The Numerical example and experimental setup

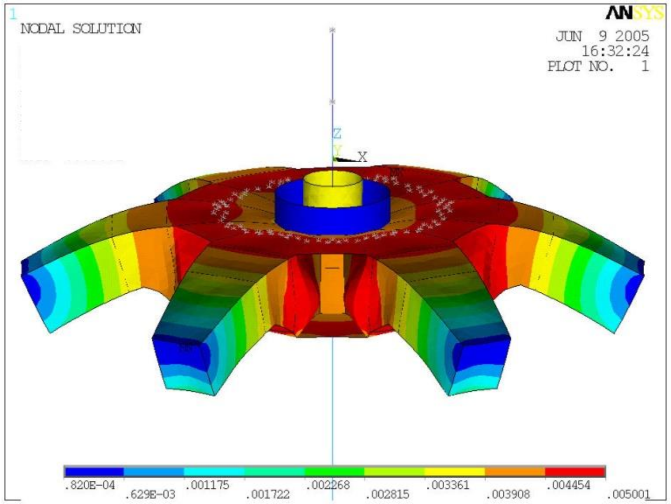
Optimal design of the dynamic viscoelastic neutralizer





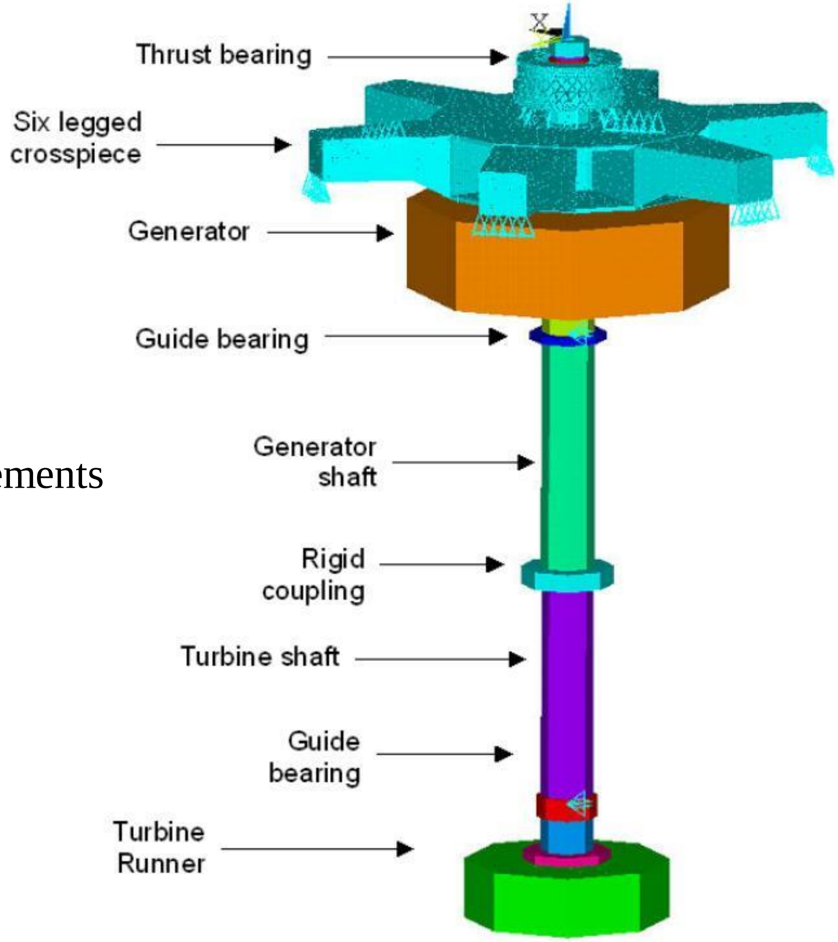
FEM of Hydroelectric Group



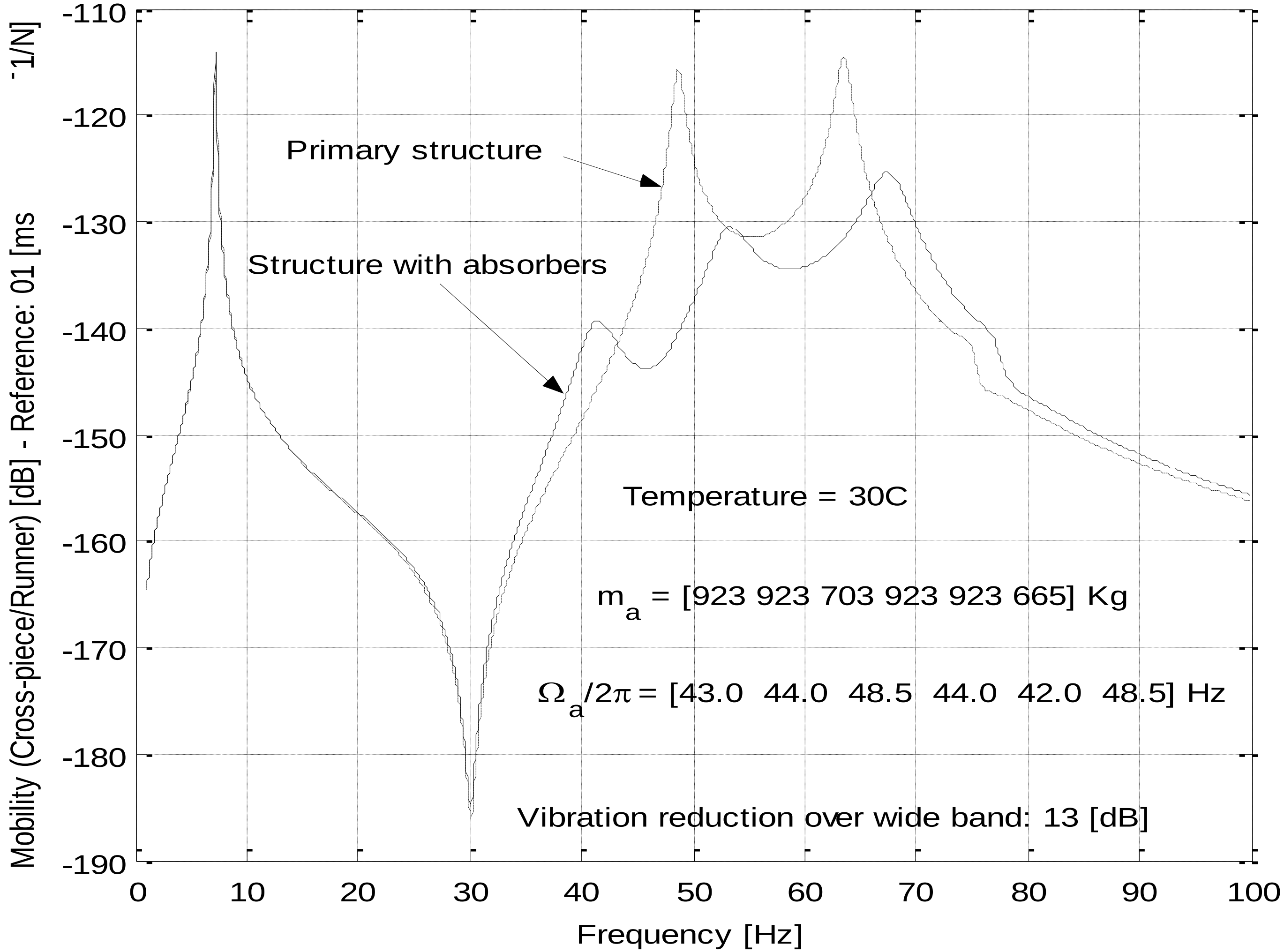


Finite Elements

Level of axial vibration = 16 mm/s



FEM of Hydroelectric Group



FEM of Hydroelectric Group

An Absorber in its Mounting Recess

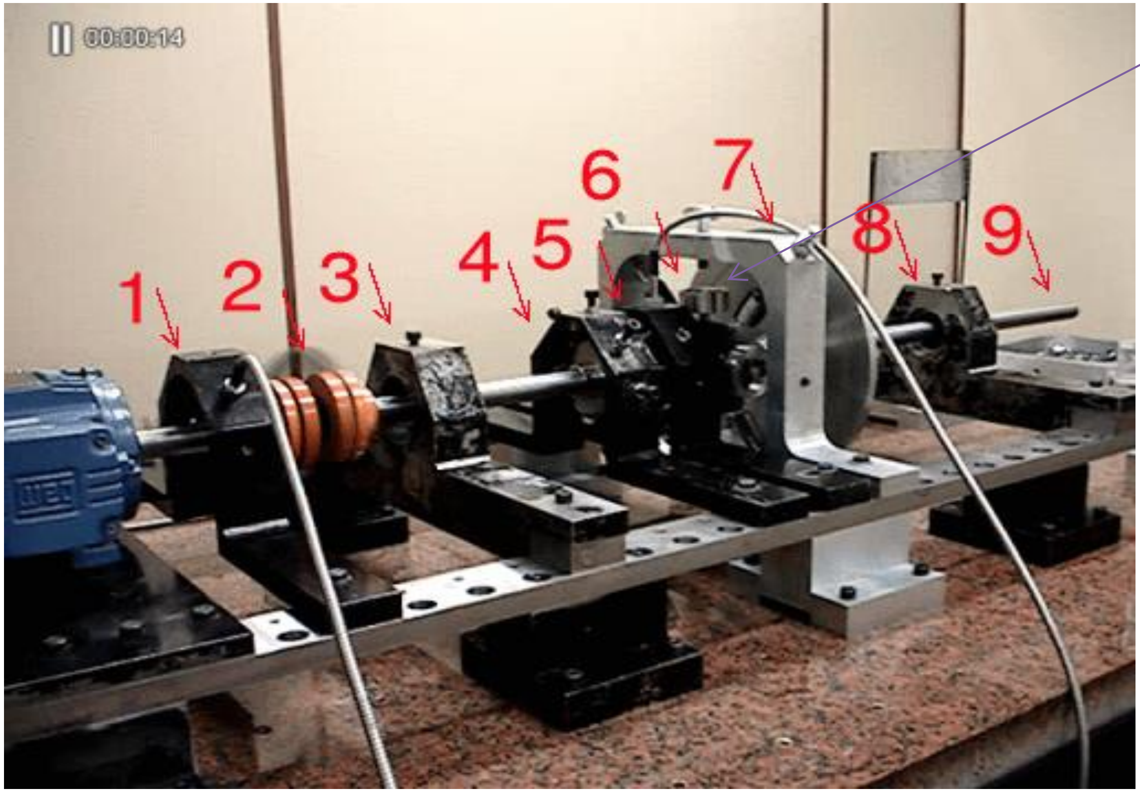
Panoramic View of the Six-Legged Crosspiece Showing an Dynamic Absorber Installed.

After DVA, the level of axial vibration

= 2.5 mm/s

8 - Rotating Systems

The rotor rig used in this work



Dynamic absorber

2, 4 e 5 – The steel disks

3 e 8 – The ball bearings

6 - The floating ball bearing

7 - The alloy disk

9 – The steel shaft

- Rotor Equations

In the frequency domain

$$\left(-\Omega^2 M + i\Omega\left(C + G\left(\Omega_{rpm}\right)\right) + K\right)Q(\Omega) = F(\Omega)$$

In term of the state variables

$$\left(i\Omega\begin{bmatrix} C_1 & M \\ M & 0 \end{bmatrix} + \begin{bmatrix} K & 0 \\ 0 & -M \end{bmatrix}\right)\{Y(\Omega)\} = \begin{Bmatrix} F(\Omega) \\ 0 \end{Bmatrix}$$

$$C_1 = C + G\left(\Omega_{rpm}\right)$$

Considering the associated eigenvalue problem

$$B\theta = \lambda A\theta$$

$$B^T\psi = \lambda A^T\psi$$

Solving the whole system for all speed range, it is possible to obtain the Campbell diagram.

- Rotor Equations

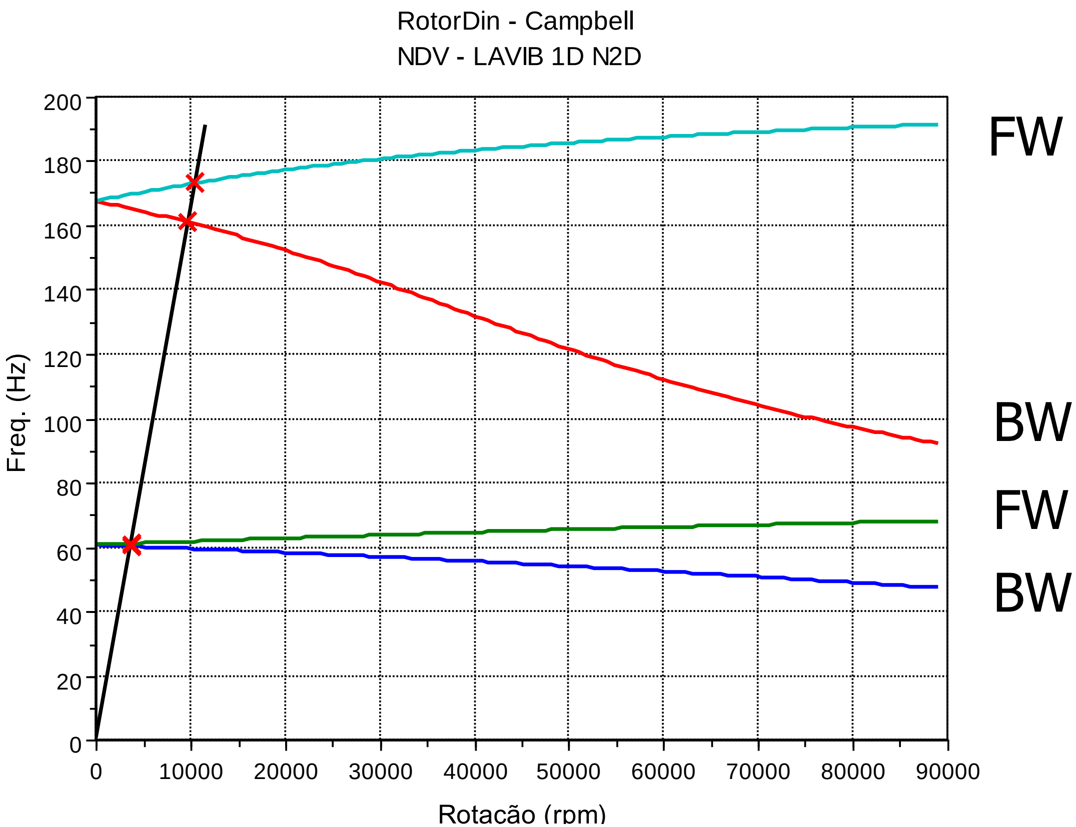
Campbell diagram

$$C_1 = C + G(\Omega_{rpm})$$

$$\Omega_{rpm}$$

$$B\theta = \lambda A\theta$$

$$B^T\psi = \lambda A^T\psi$$



- Rotating System with Dynamic Absorbers

Using the equivalent generalized concept

$$\left[\tilde{A}(\Omega)\right] = \begin{bmatrix} C_{eq}(\Omega) & M_{eq}(\Omega) \\ M_{eq}(\Omega) & 0 \end{bmatrix} \qquad \left[\tilde{B}(\Omega)\right] = \begin{bmatrix} 0 & 0 \\ 0 & -M_{eq}(\Omega) \end{bmatrix}$$

$$M_{eq}(\Omega) = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 0 & m_{eq_1}(\Omega) & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & m_{eq_p}(\Omega) & 0 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \qquad C_{eq}(\Omega) = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 0 & c_{eq_1}(\Omega) & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & c_{eq_p}(\Omega) & 0 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$\overline{A}(\Omega) = A + \tilde{A}(\Omega)$$

$$\overline{B}(\Omega) = B + \tilde{B}(\Omega)$$

- For a given Ω_{rpm}

$$\left(i\Omega\,\overline{A}(\Omega) + \overline{B}(\Omega)\right)Y(\Omega) = N(\Omega)$$

The Response of the Compound System

Using the eigenvector of the matrix transformation

$$Y(\Omega) = \Theta P(\Omega)$$

It is possible to find the response in the modal space state

$$P(\Omega) = \left[i\Omega\left(I + \Psi^T \tilde{A} \Theta\right) + \left(\Lambda + \Psi^T \tilde{B} \Theta\right)\right]^{-1} \Psi^T N(\Omega)$$

Then, the response q(t) can be obtained with a inverse Fourier transformation of the response in the space state using de transformation matrix = right eigenvector.

Optimal Design of the Dynamic Absorbers

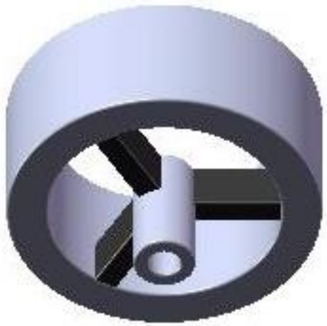
- The objective function is defined by

$$f_{cost}(x) = \left\| \max_{\Omega_1 \langle \Omega \langle \Omega_2} \left| \hat{P}(\Omega, x) \right| \right\|$$

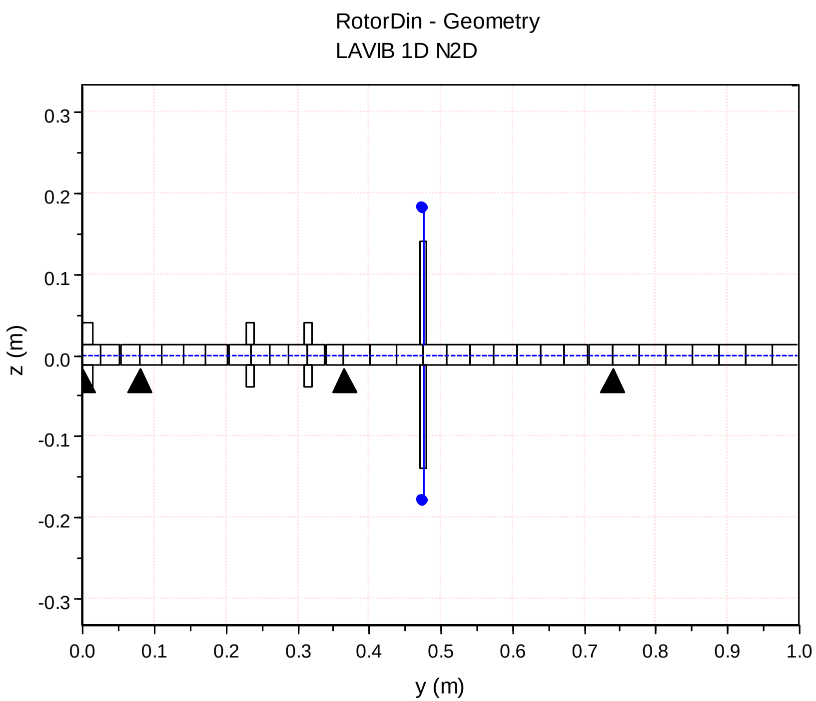
- where

$$x = \left(\Omega_{a1}, \Omega_{a2}, \ldots, \Omega_{ap} \right)$$

- After optimization procedure, the DVA's natural frequencies Ω_n are known. Then, it is possible to do a physical realization.

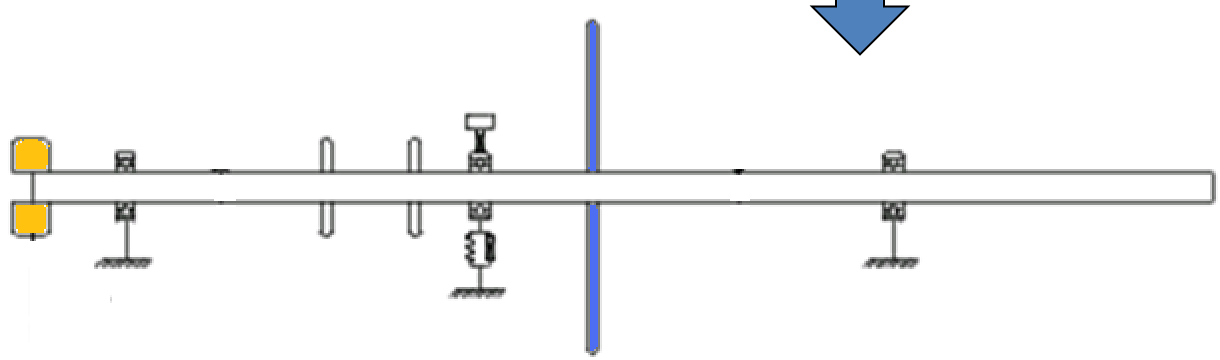


- Numerical Simulation

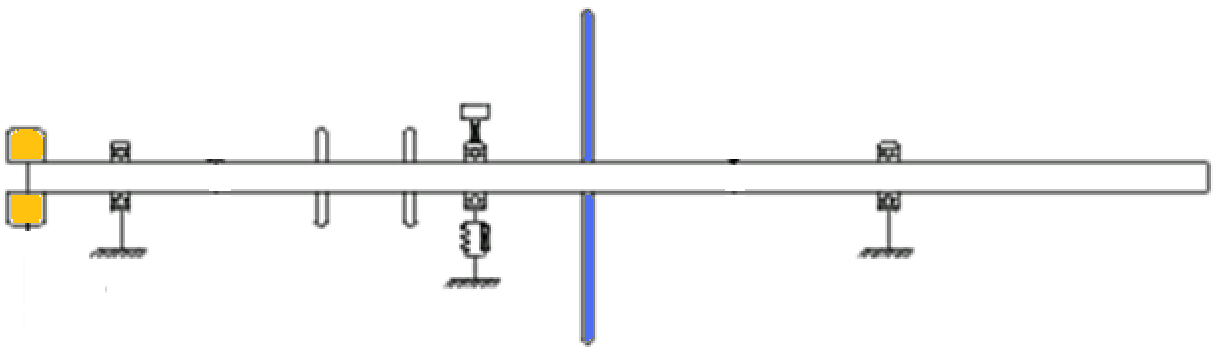


FEM model. Modal Analysis. Modal Parameters of the Rotor System

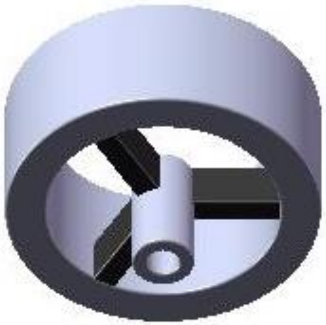
Compound System Model. Optimal Design of the Dynamic Absorber



- Numerical Simulation

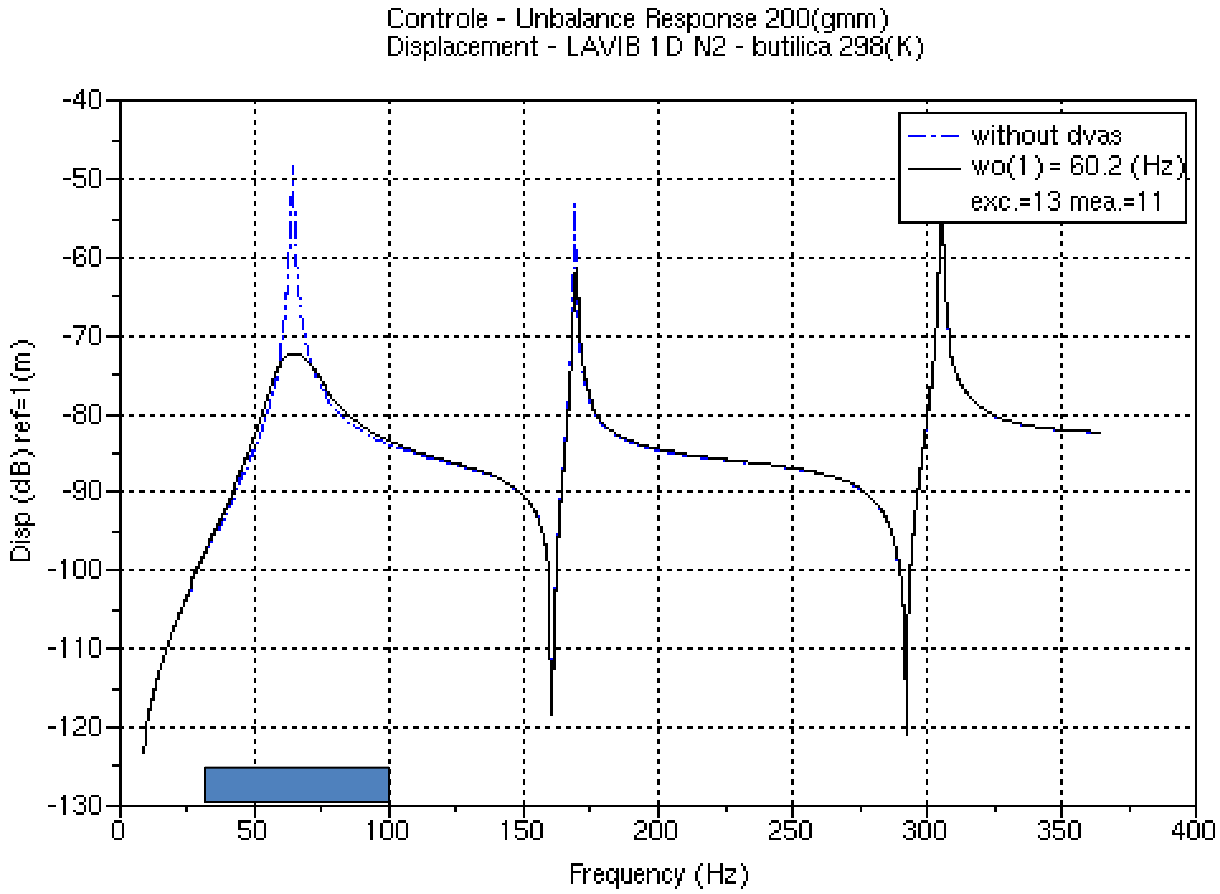


- After optimization procedure, the DVA's natural frequencies Ω_n are known. Then, it is possible to do a physical realization.



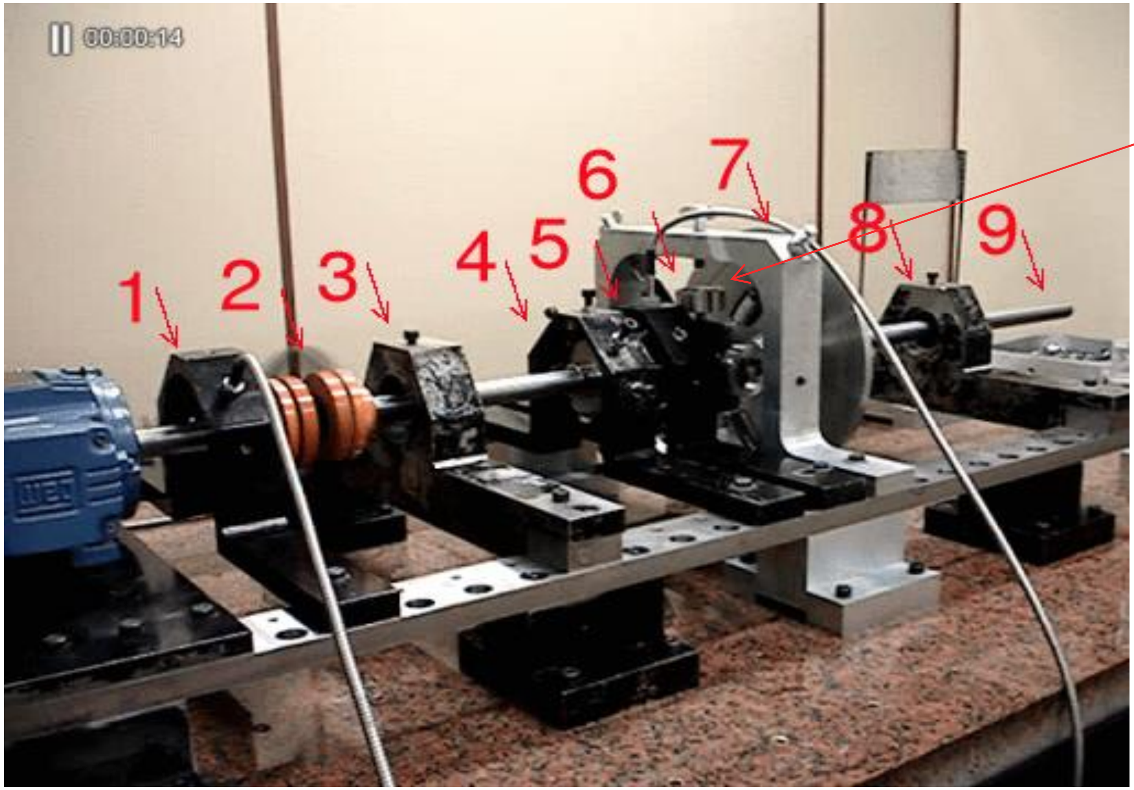
Numerical Solution

Then the first 16 eigenvalues have been used. The barrier frequencies were 33 and 100 Hz and the DVA's initial natural frequency was adopted to be 59 Hz.



The optimal natural frequency of the four absorbers is Ω_a = 60.2 Hz.

- Experimental Setup



The rotor rig

Dynamic absorber

2, 4 e 5 – The steel disks

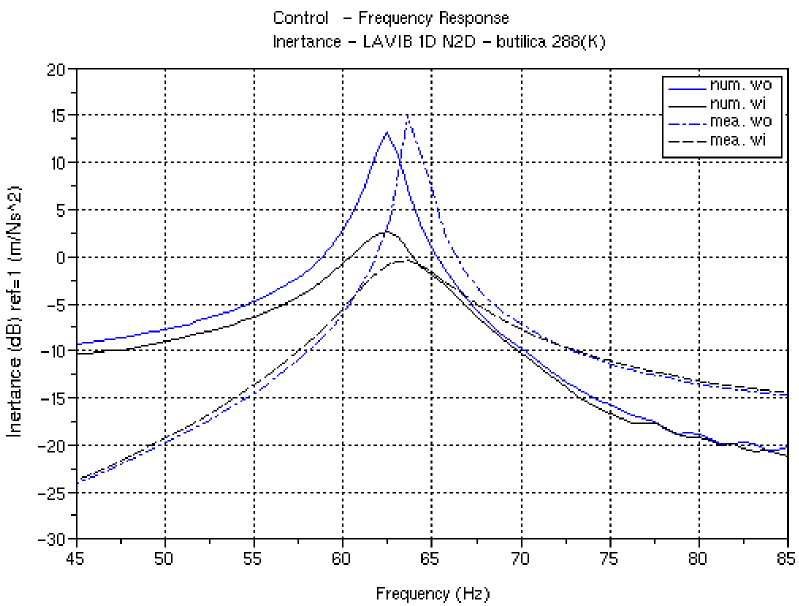
3 e 8 – The ball bearings

6 - The floating ball bearing

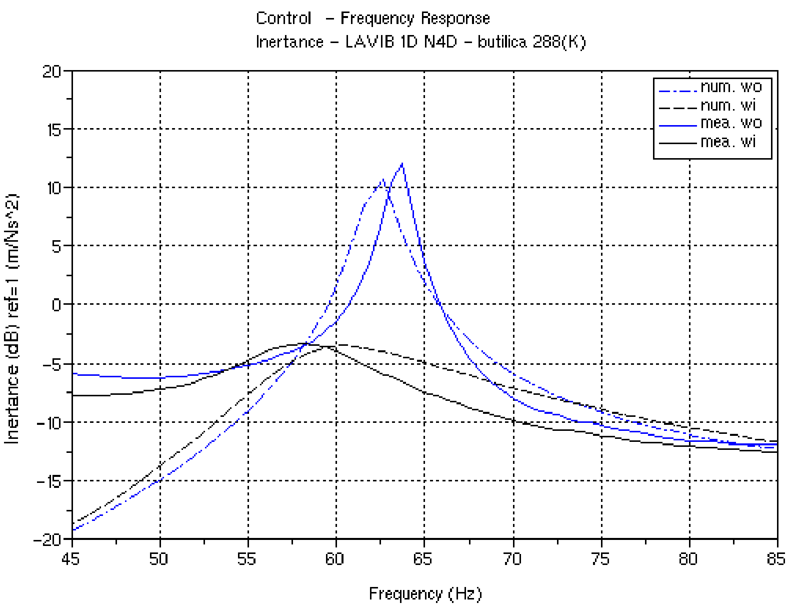
7 - The alloy disk

9 – The steel shaft

- Frequency Response Function



with 2 dynamic absorbers



with 4 dynamic absorbers

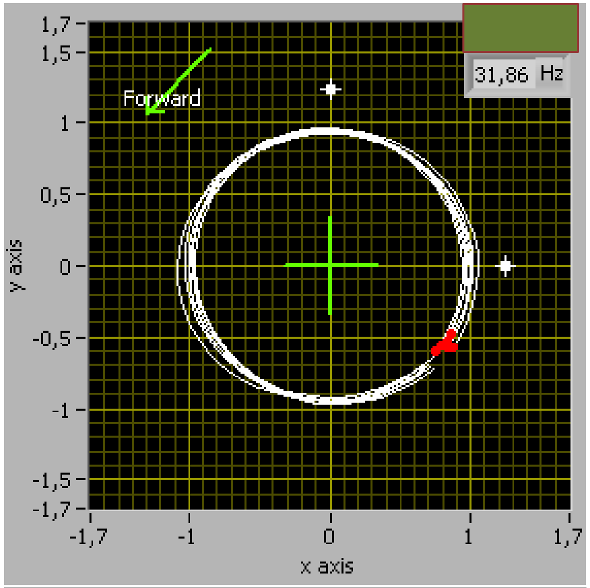
mea = measured curves (mea)
wi = with absorbers
wo = without absorbers

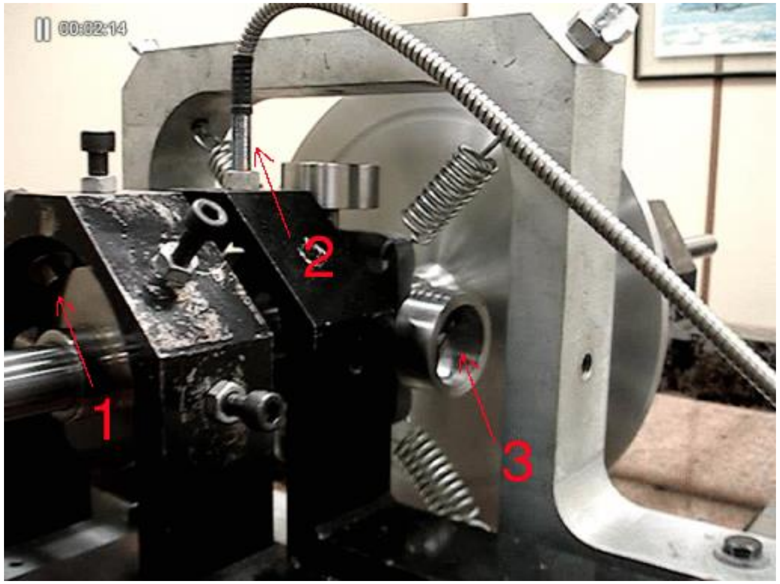
num = numerical curves
exp = experimental curves

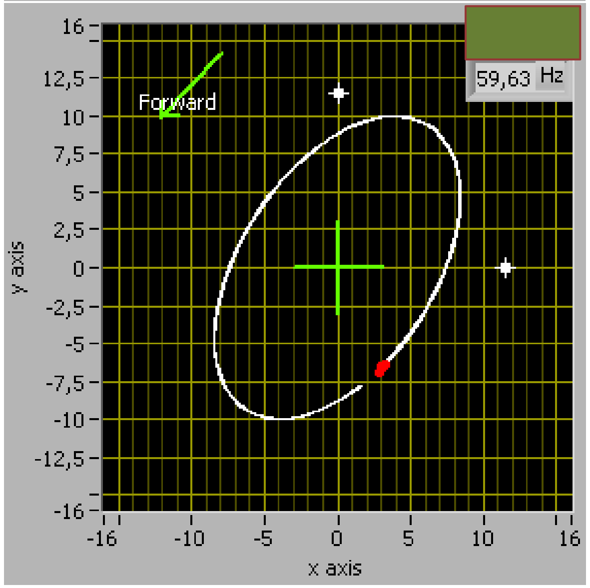
Experimental Setup

2 - The dynamic orbit measuring has been done using a 90 (degrees) proximeters set

Reference







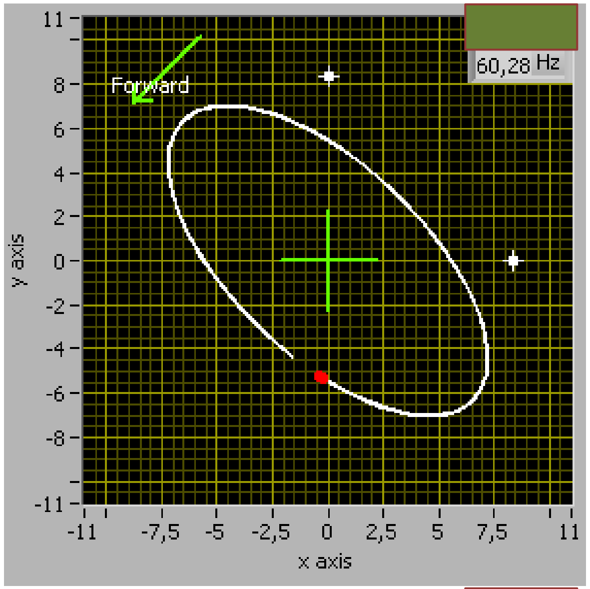
Without Absorbers

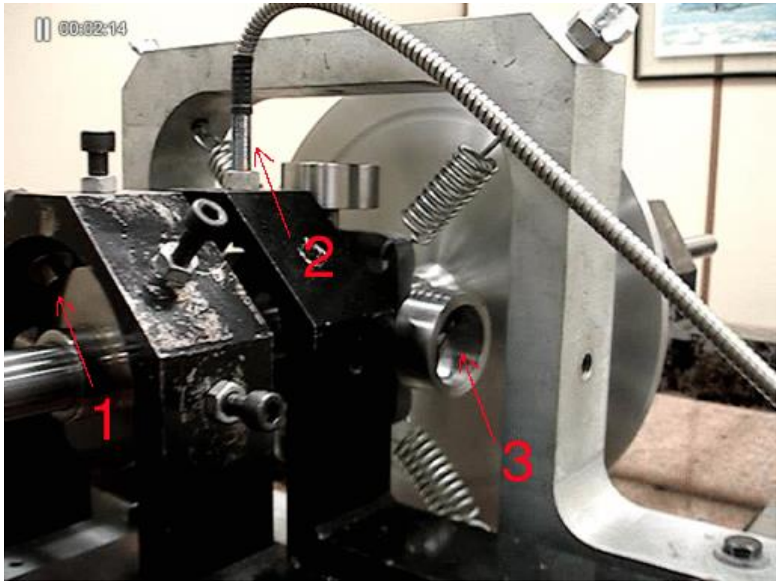
1 - Kind of excursion limiters
2 - Proximiter set
3 - Dynamic Viscoelastic Absorbers

Experimental Setup

2 - The dynamic orbit measuring has been done using a 90 (degrees) proximeters set

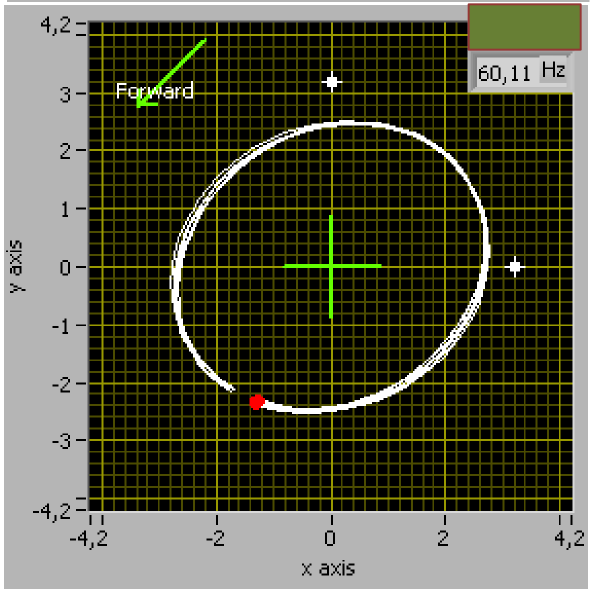
2 Absorbers





1 - Kind of excursion limiters
2 - Proximiter set
3 - Dynamic Viscoelastic Absorbers

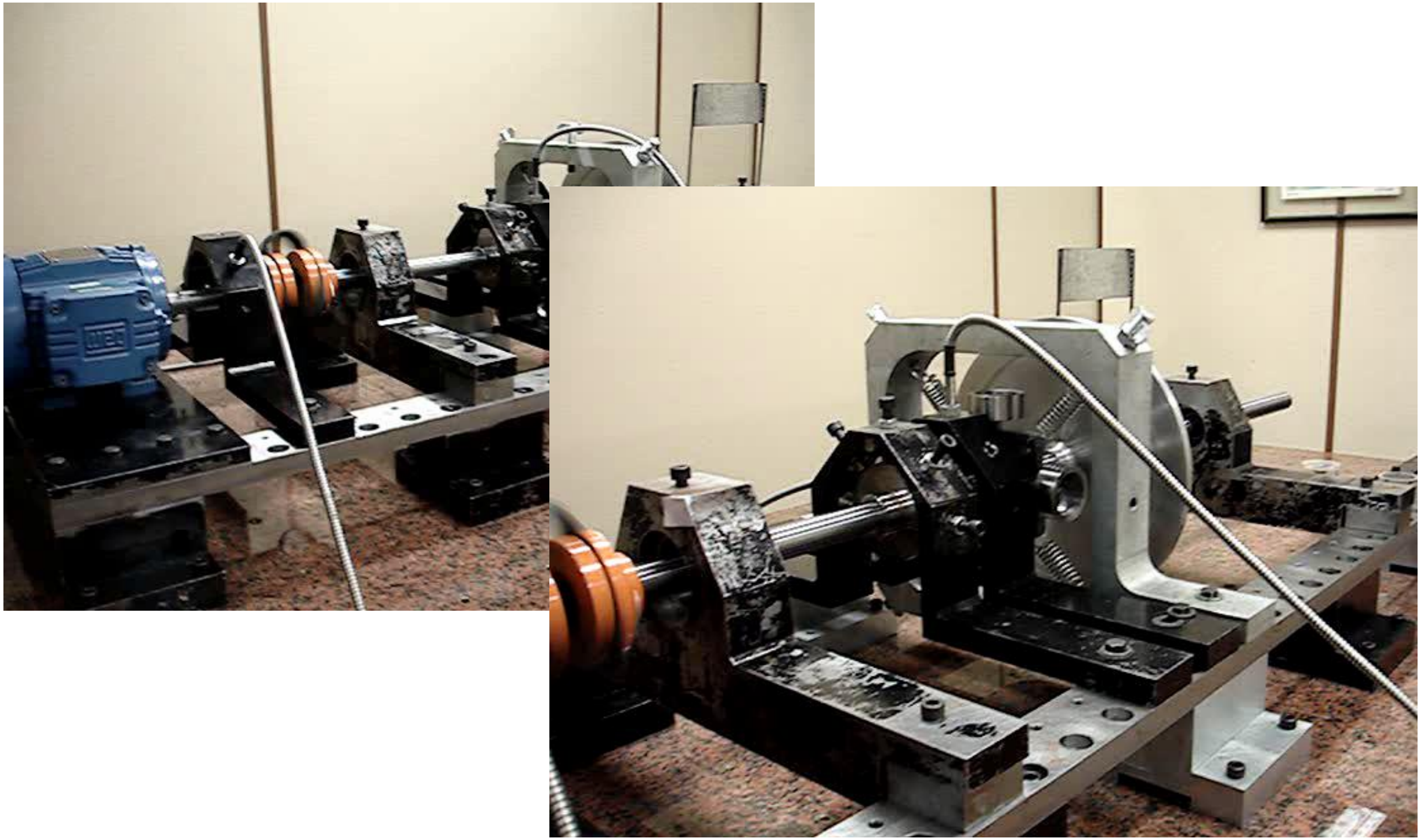
With 4 Absorbers



- Experimental Setup

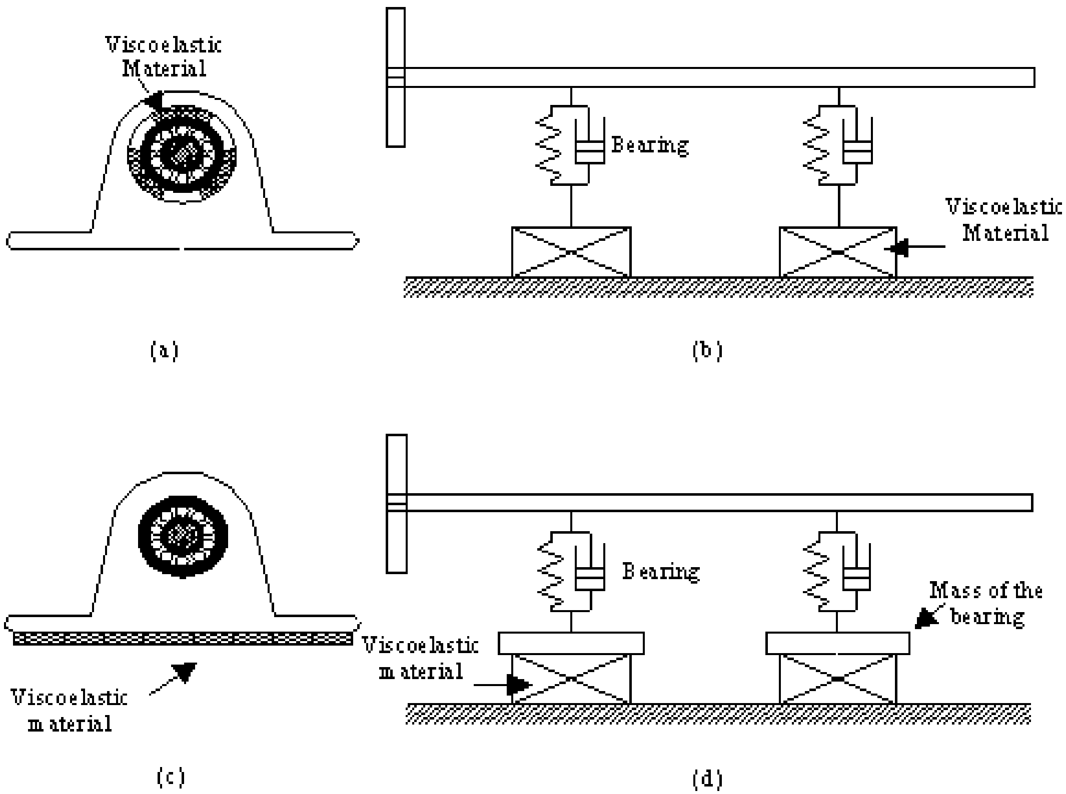
the excursion limiters

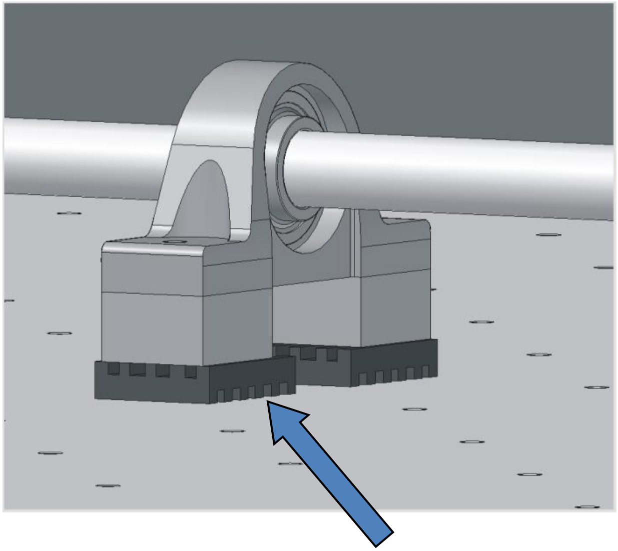
- Experimental Setup



Rotors with Flexible Bearings

The instability problems, when working at high rotations, can be solved by including damping in the bearings.

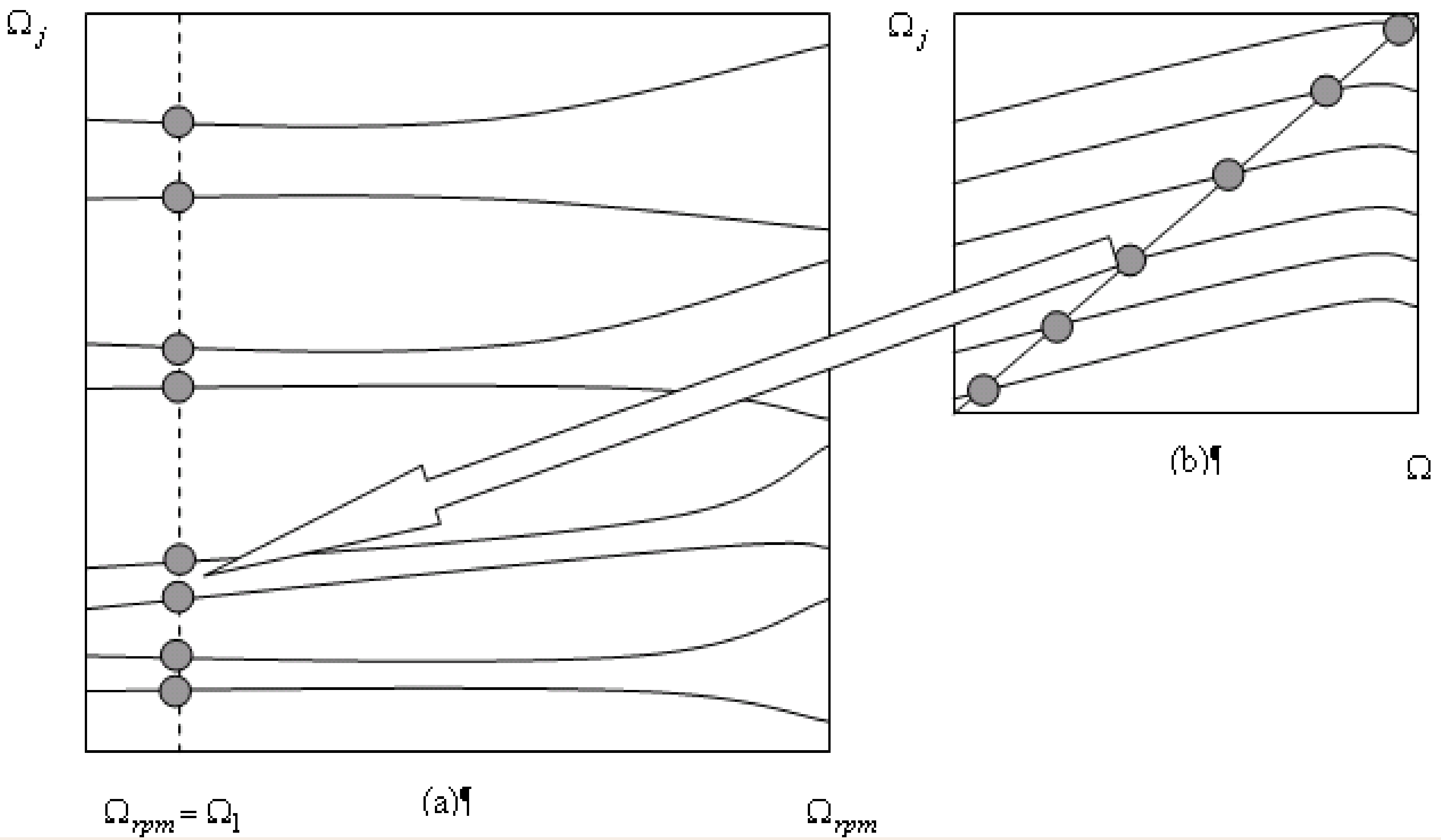




Viscoelastic Material

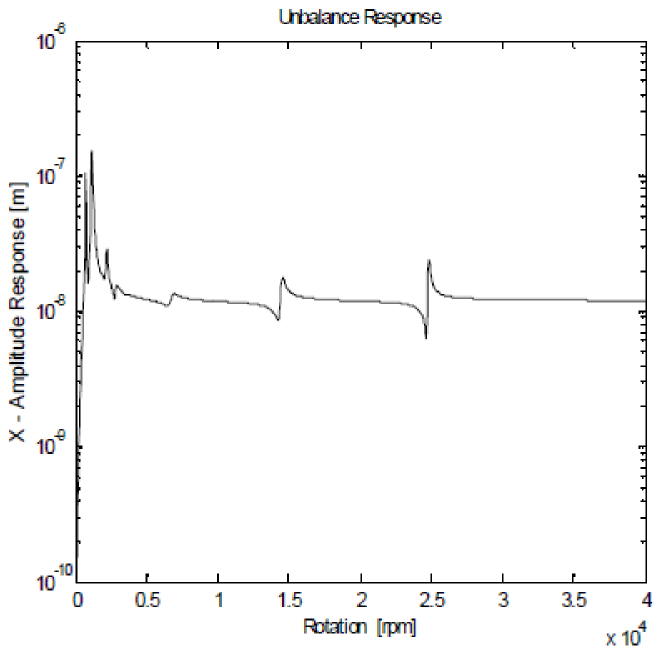
- Rotors with Flexible Bearings

Ω_{rpm}=cte

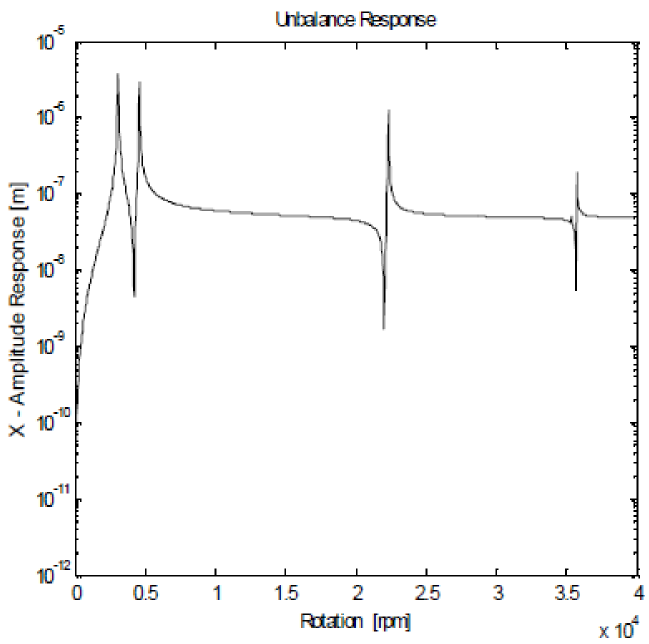


- Rotors with Flexible Bearings

The instability problems, when working at high rotations, can be solved by including damping in the bearings.

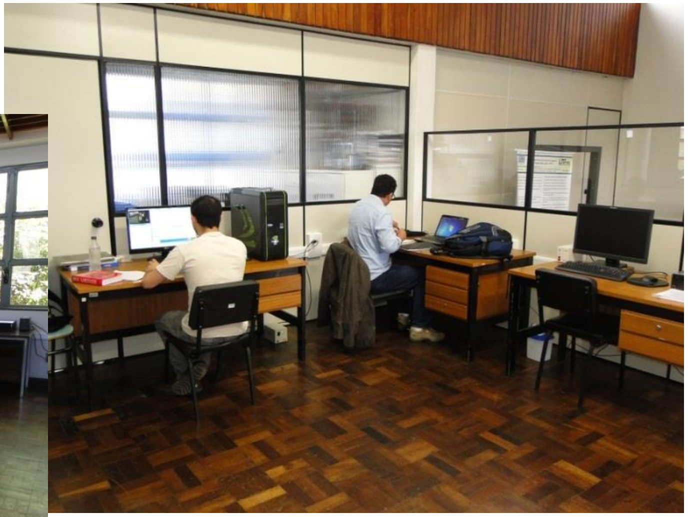


with flexible bearings



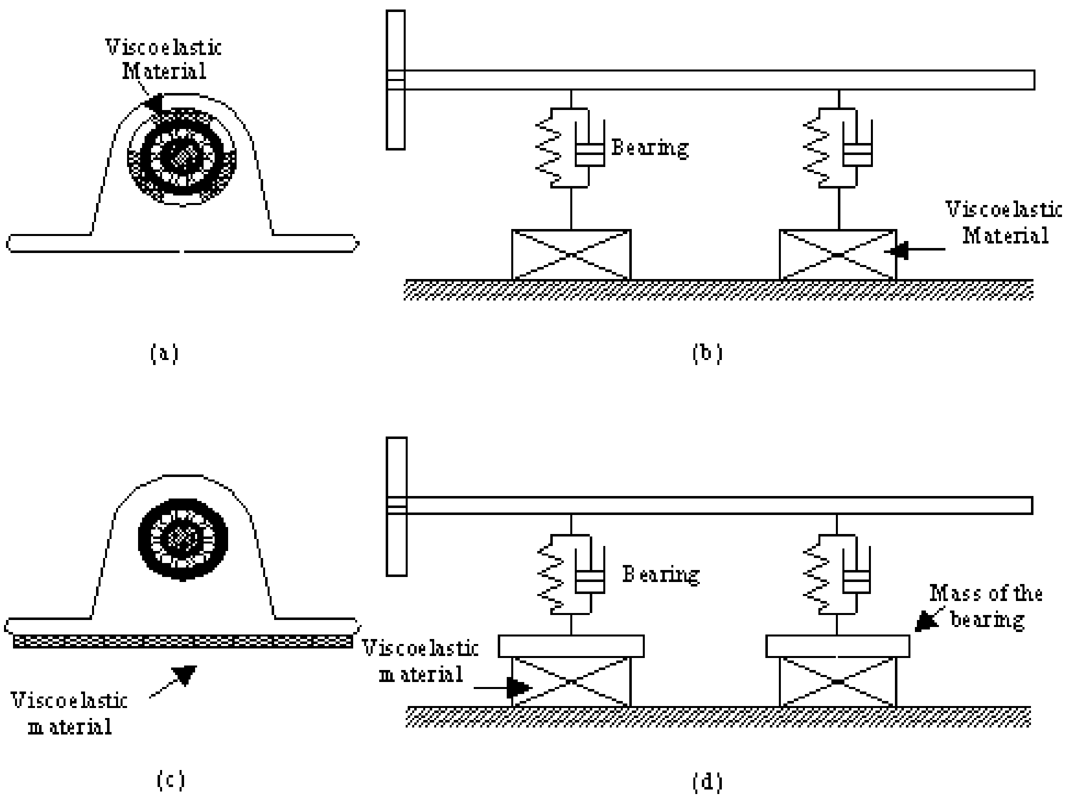
rigid ball bearings

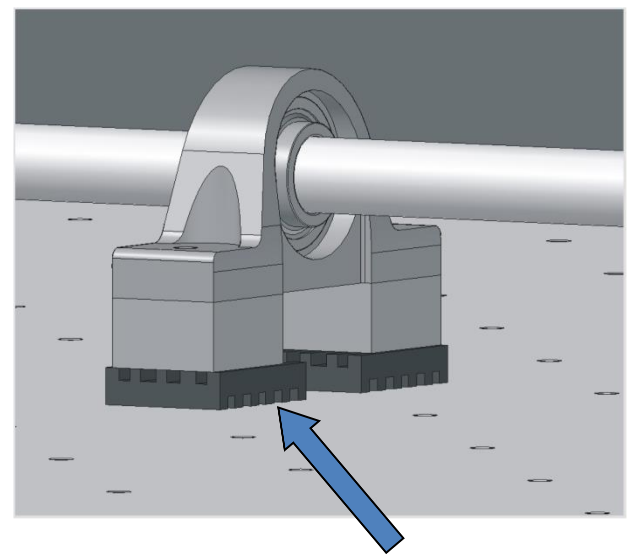
◈ Works in Progress – Laboratory of Sound and Vibration



Rotors with Flexible Bearings

The instability problems, when working at high rotations, can be solved by including damping in the bearings.

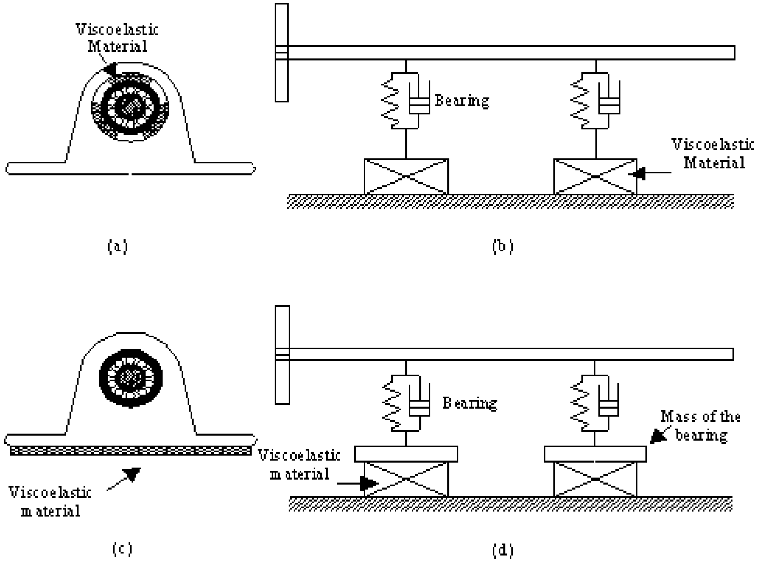


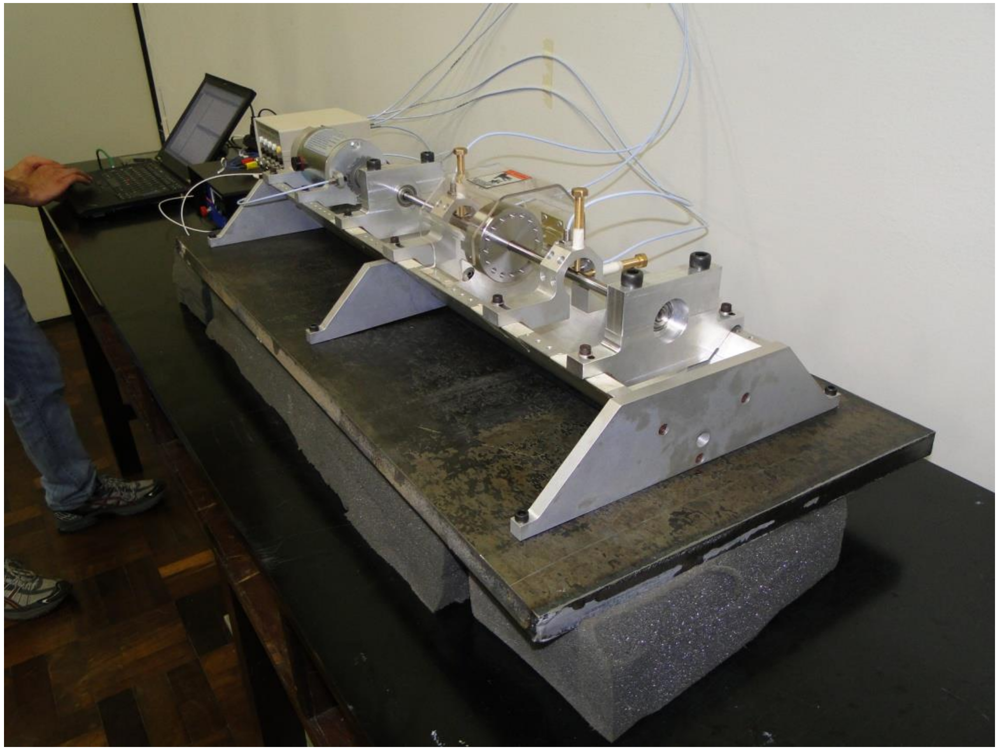


Viscoelastic Material

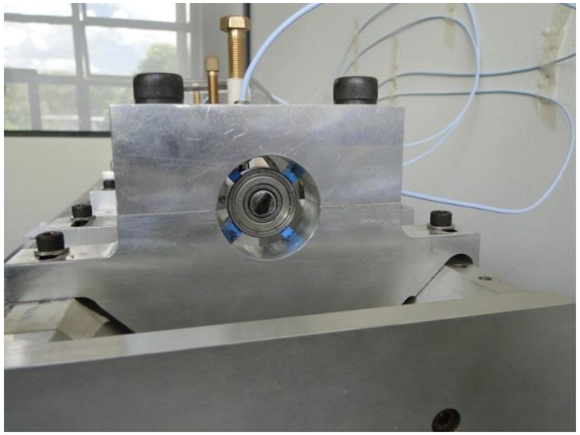
◆ Rotors with Flexible Bearings

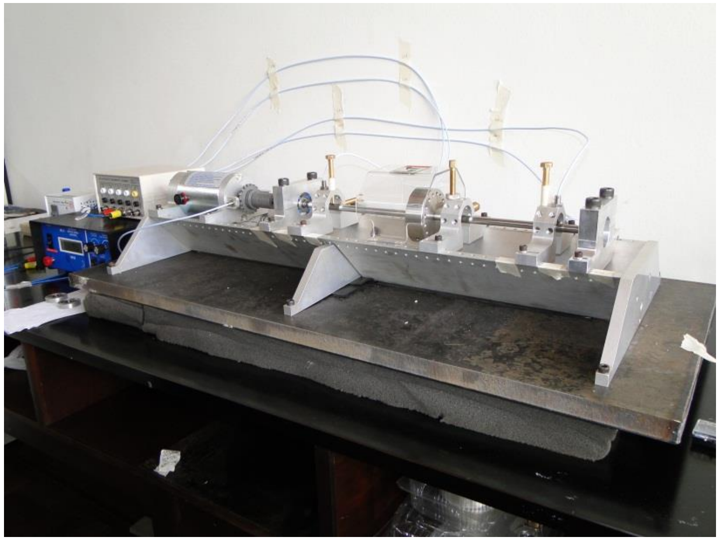
The instability problems, when working at high rotations, can be solved by including damping in the bearings.

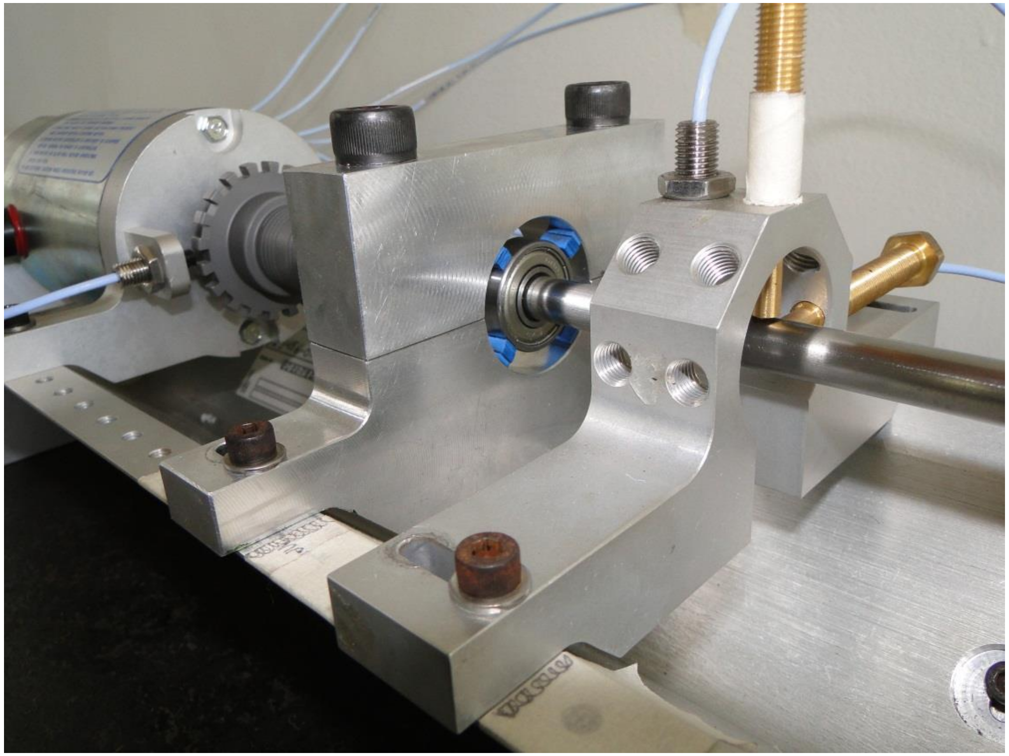




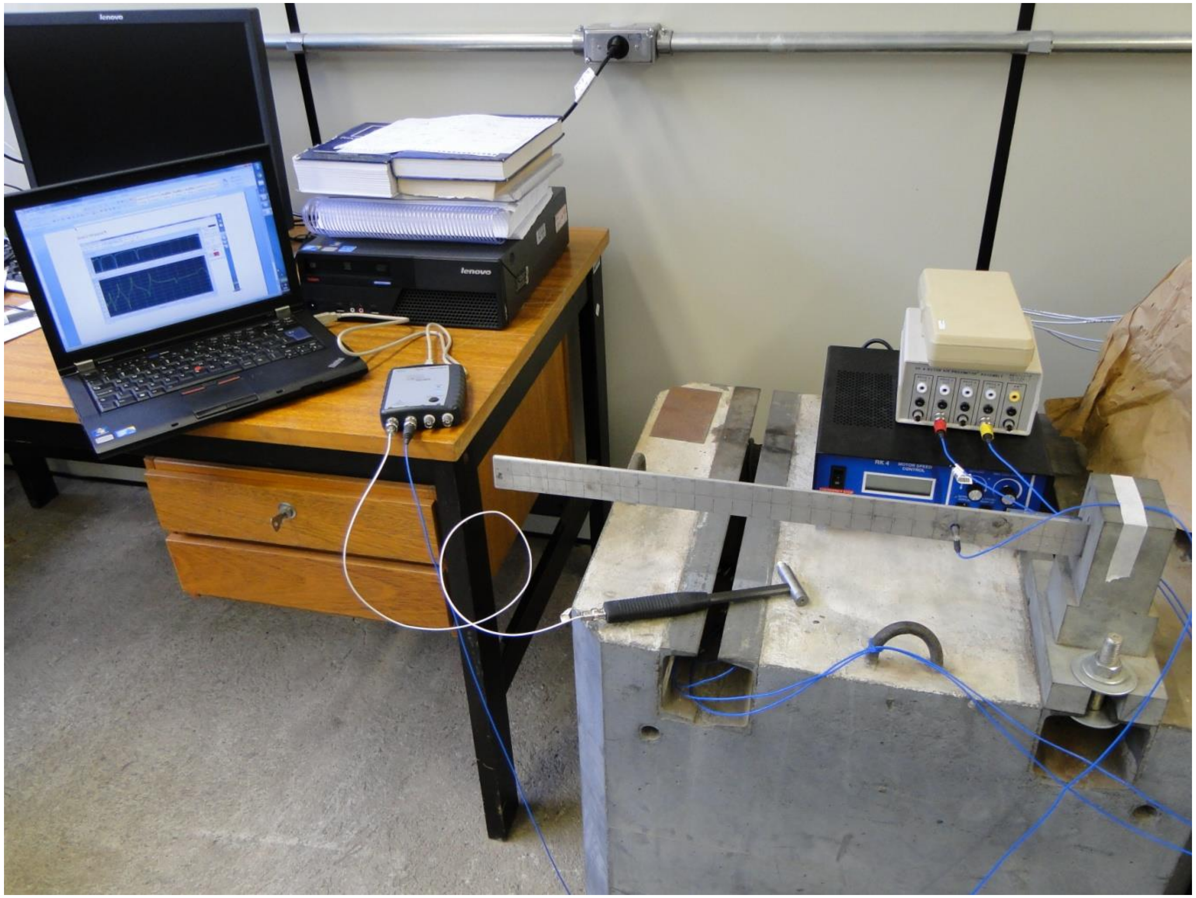
- Rotors with Flexible Bearings



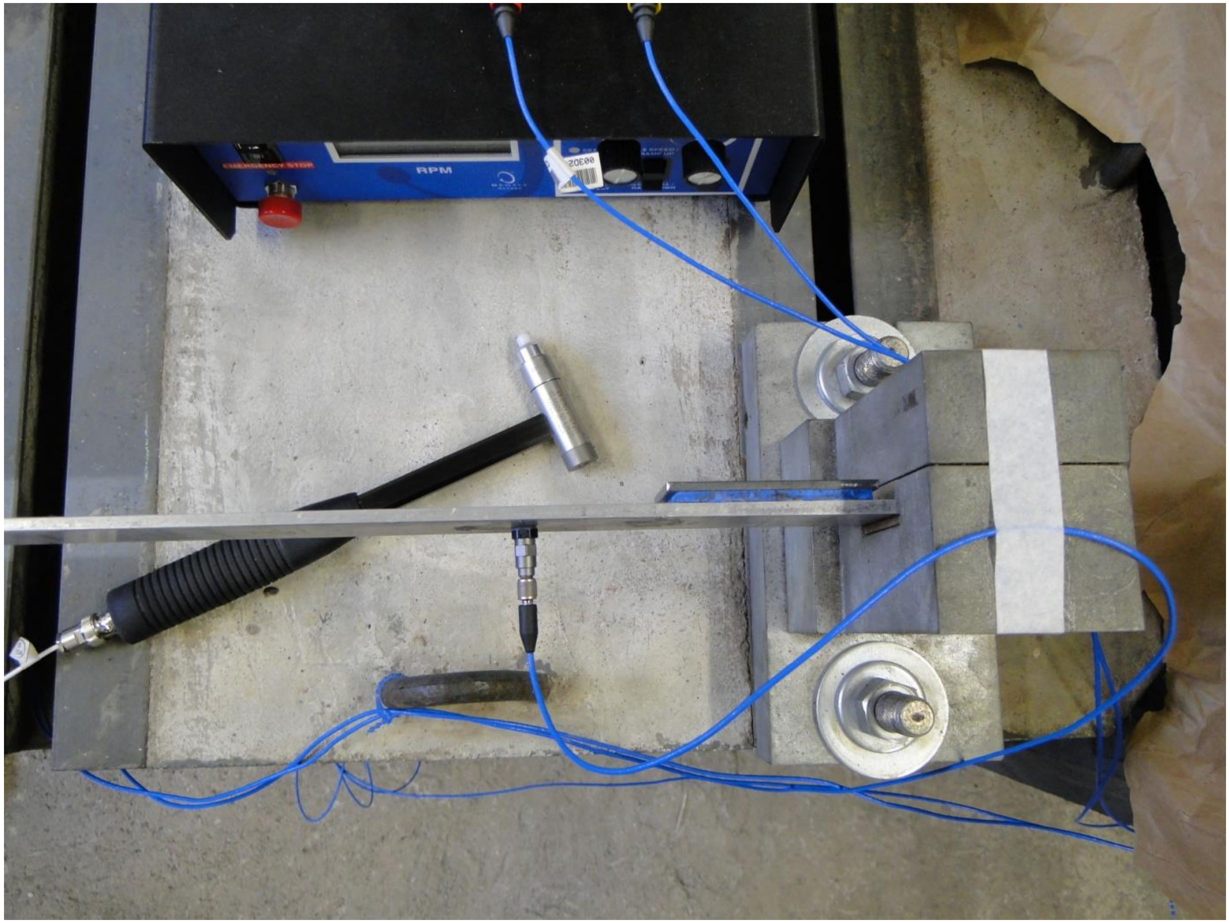




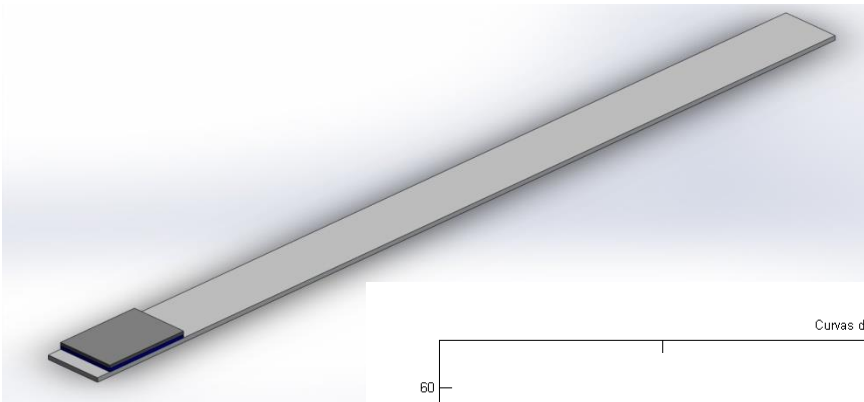
- The Numerical example and experimental setup of Constrained Layers and Sandwich beams

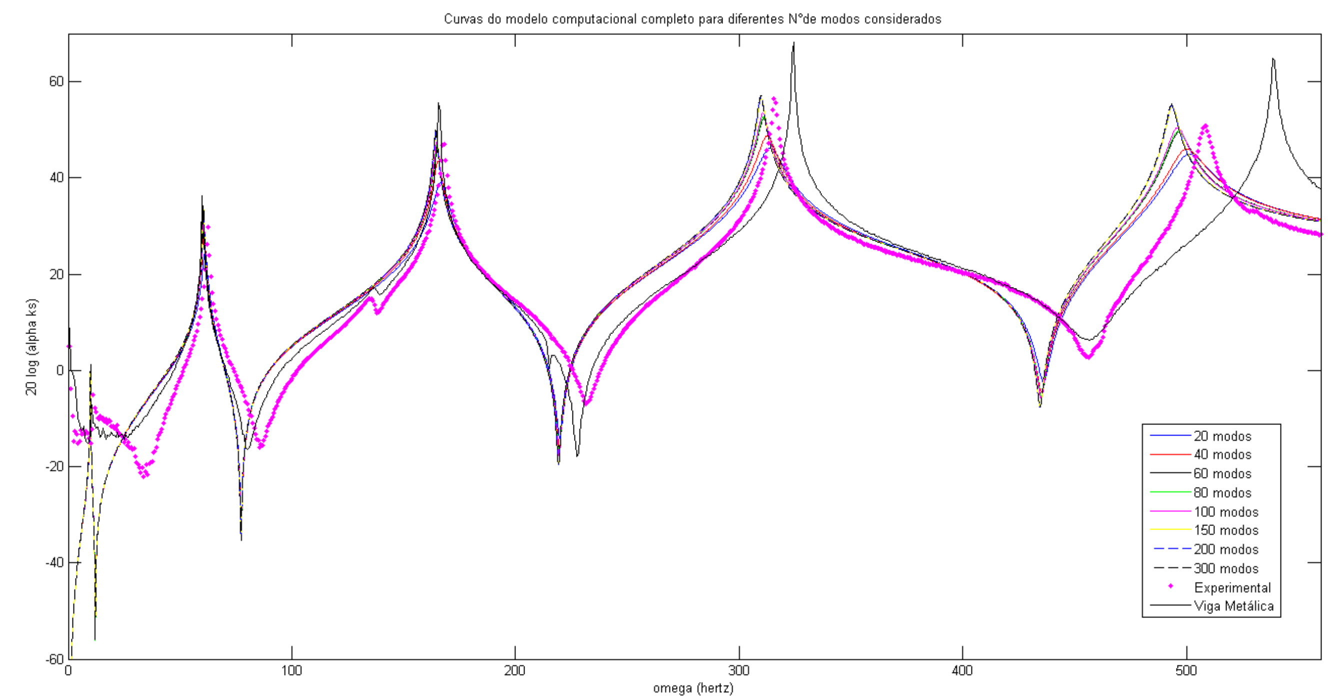


- The Numerical example and experimental setup of Constrained Layers



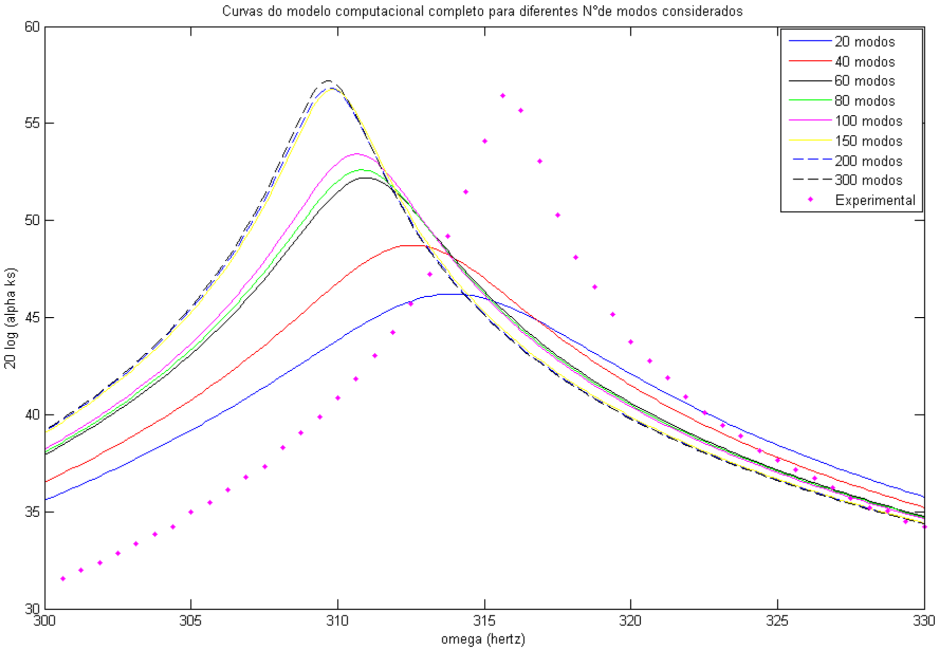
- The Numerical example and experimental setup of Constrained Layers



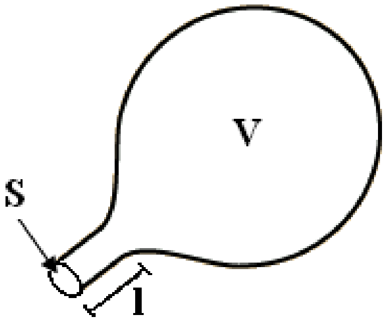


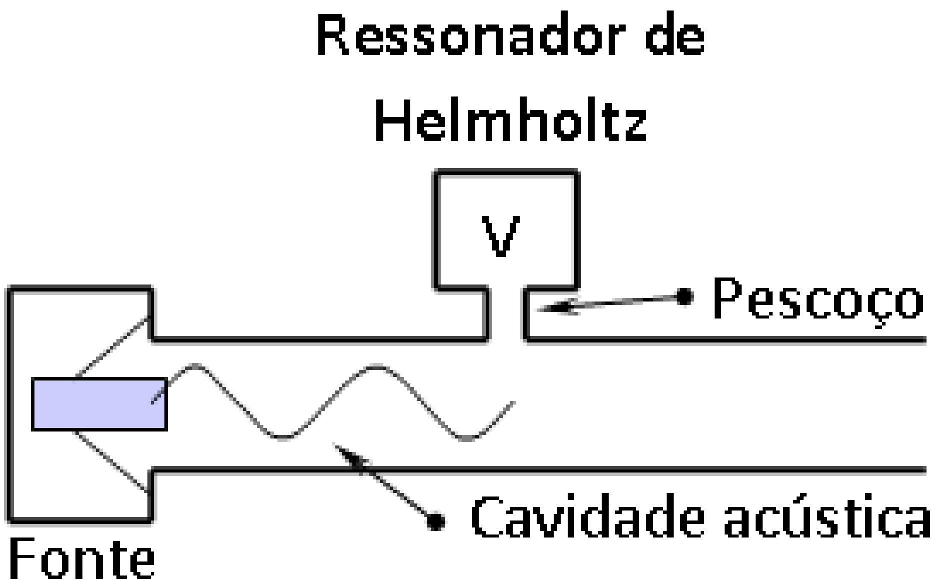
◈ The Numerical example and experimental setup of Constrained Layers

Zoom da FRF medida e obtida numericamente
ao redor do quarto modo



- Helmholtz ressonators



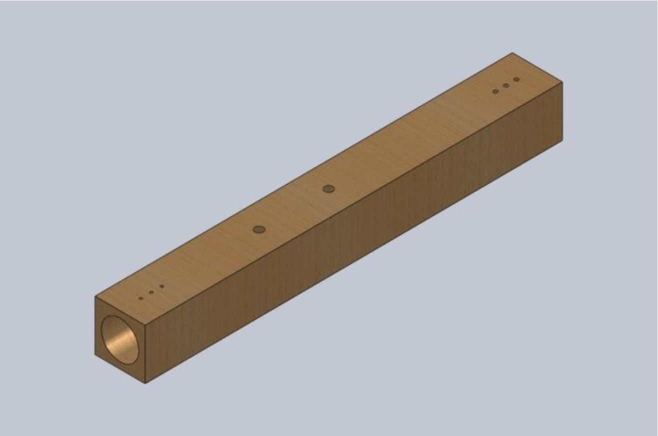


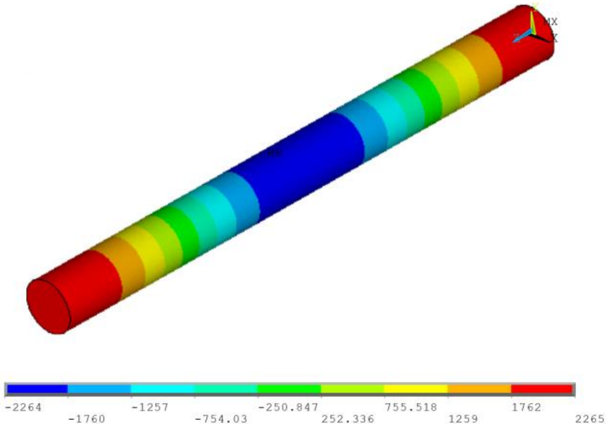
- Helmholtz ressonators

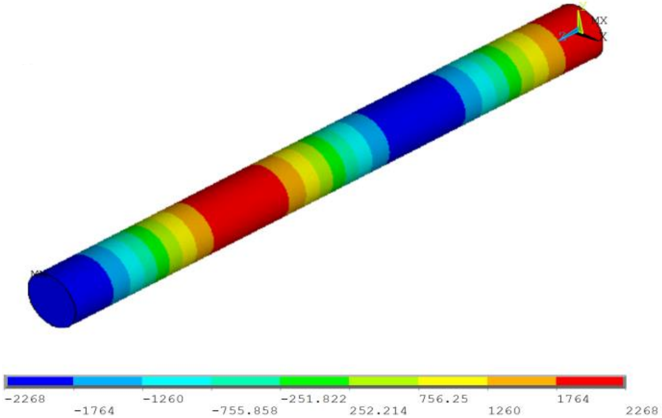
◆ Modal Analysis

- Modal Parameters

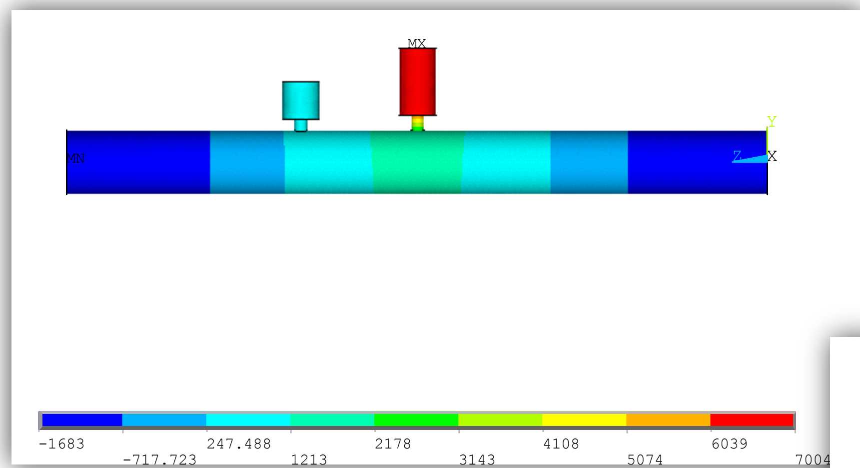
Modo	Frequência [Hz]
1	88,8974
2	177,92
3	267,163
4	356,792
5	446,811
6	537,518
7	628,859
8	721,273
9	814,658
10	909,115

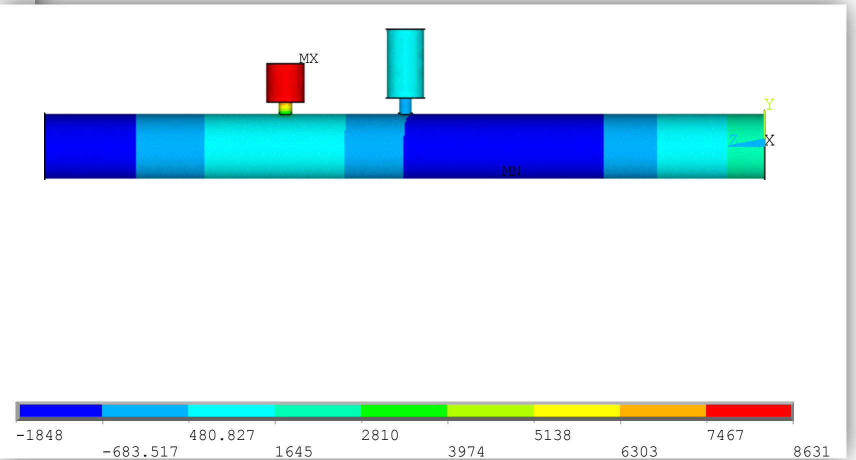


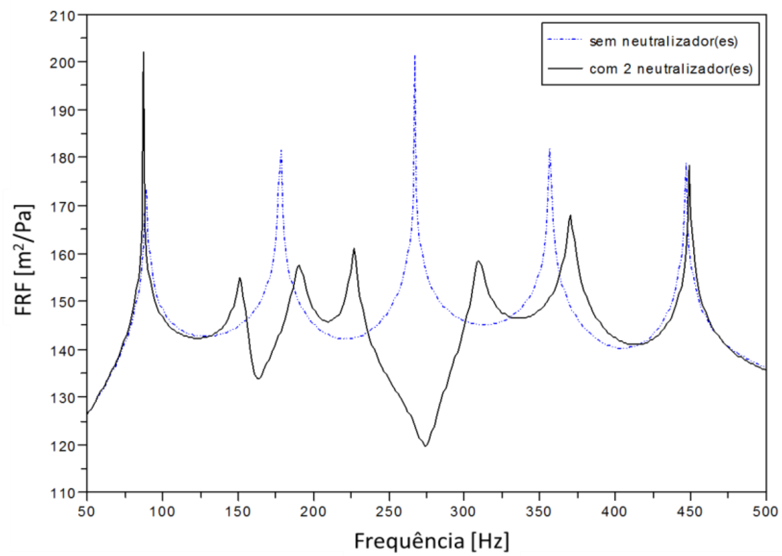




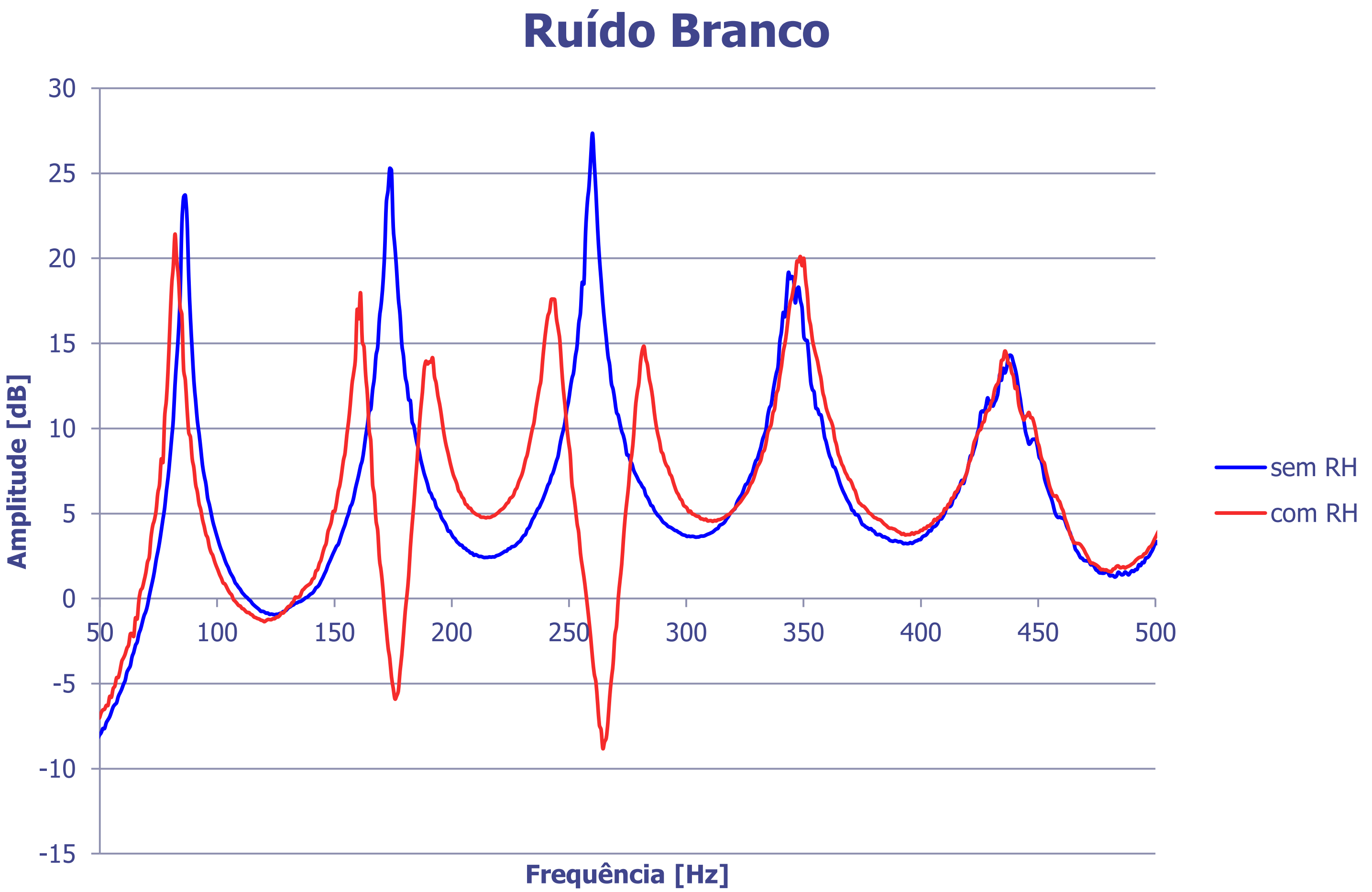
Numerical simulations



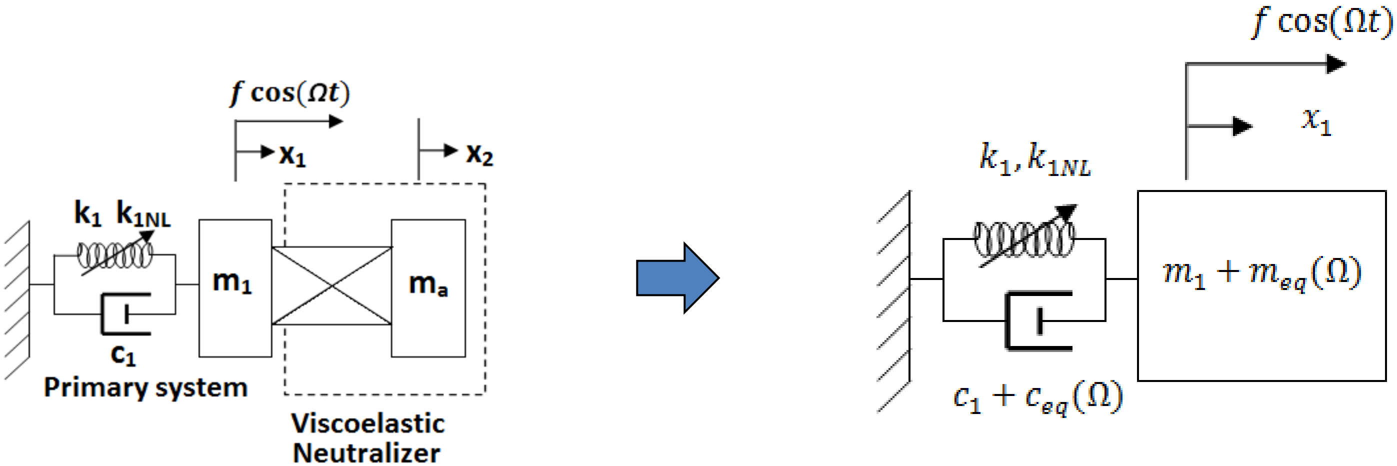




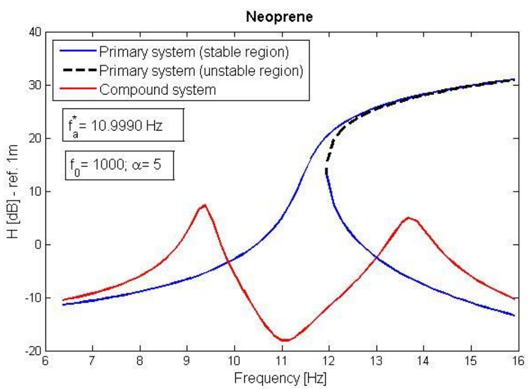
Experimental Results

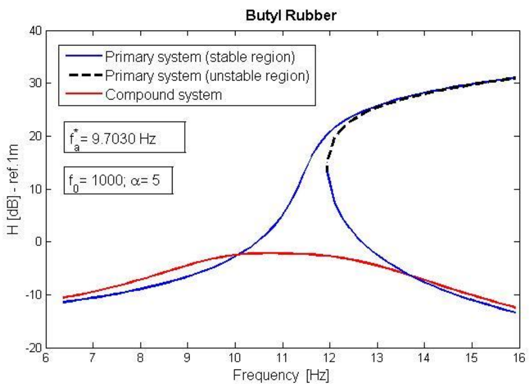


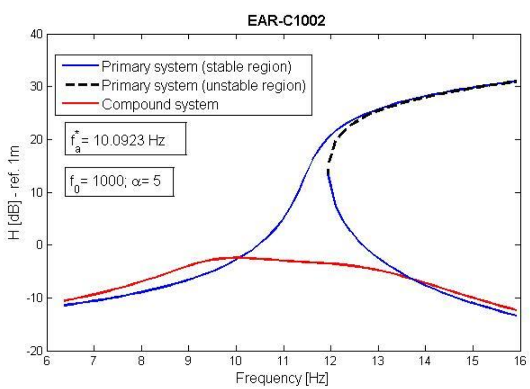
Optimal design of the viscoelastic neutralizer applied a non linear systems



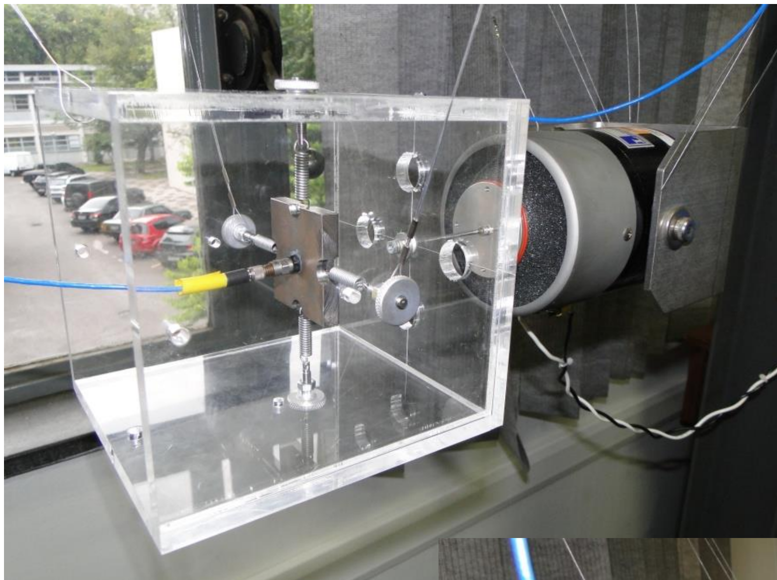
$$(m_1 + m_e(\Omega))\ddot{x}_1 + k_1 x_1 + k_{1NL} x_1^3 + (c_1 + c_e(\Omega))\dot{x}_1 = f \cos(\Omega t)$$

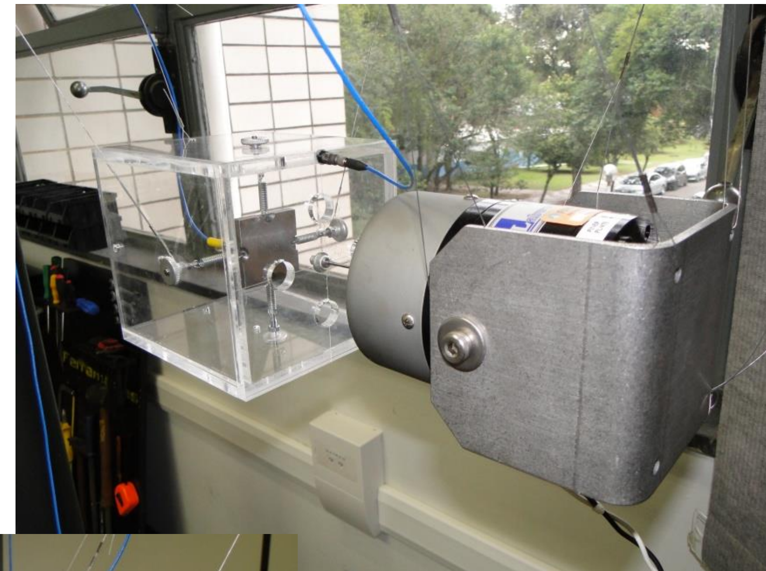


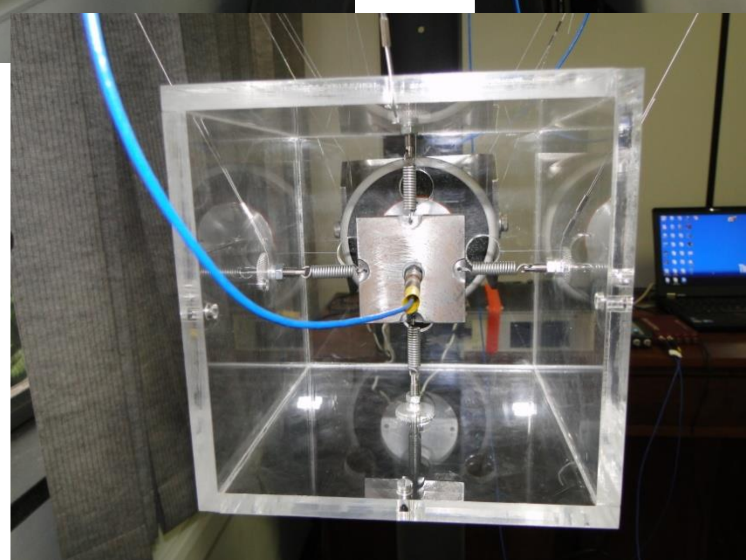




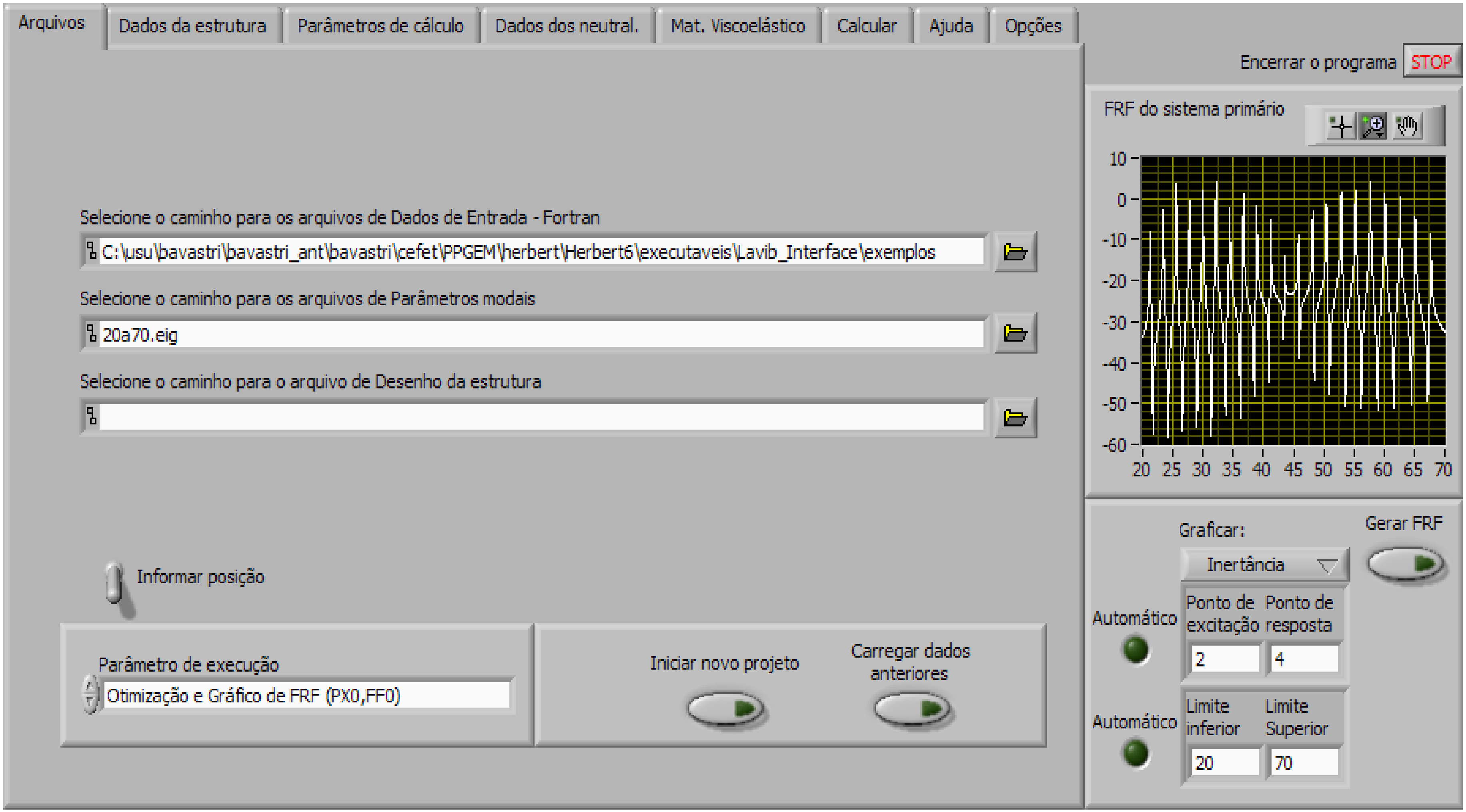
- Optimal design of the viscoelastic neutralizer applied a non linear systems



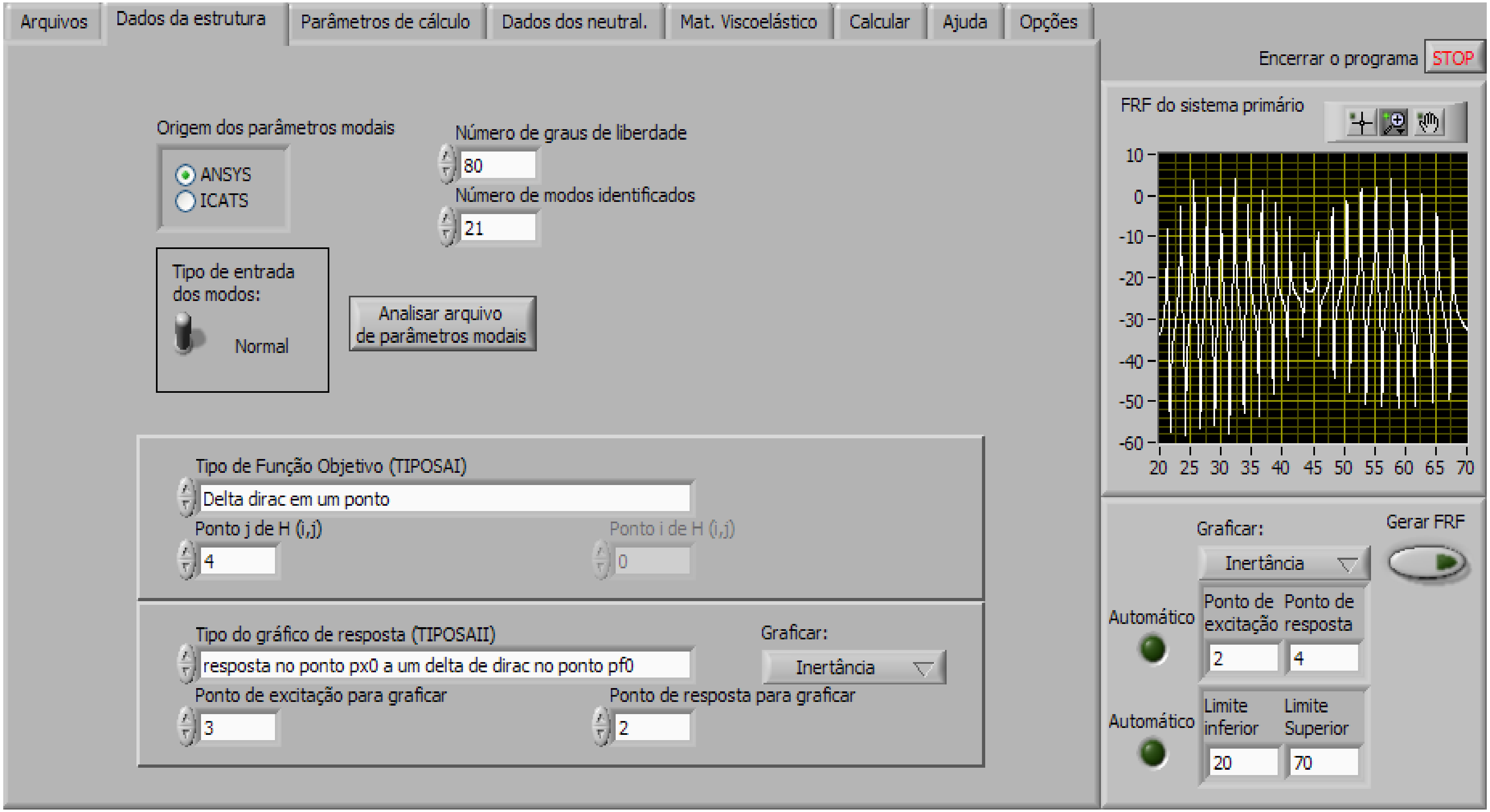




Projeto ótimo dos neutralizadores:



Projeto ótimo dos neutralizadores:



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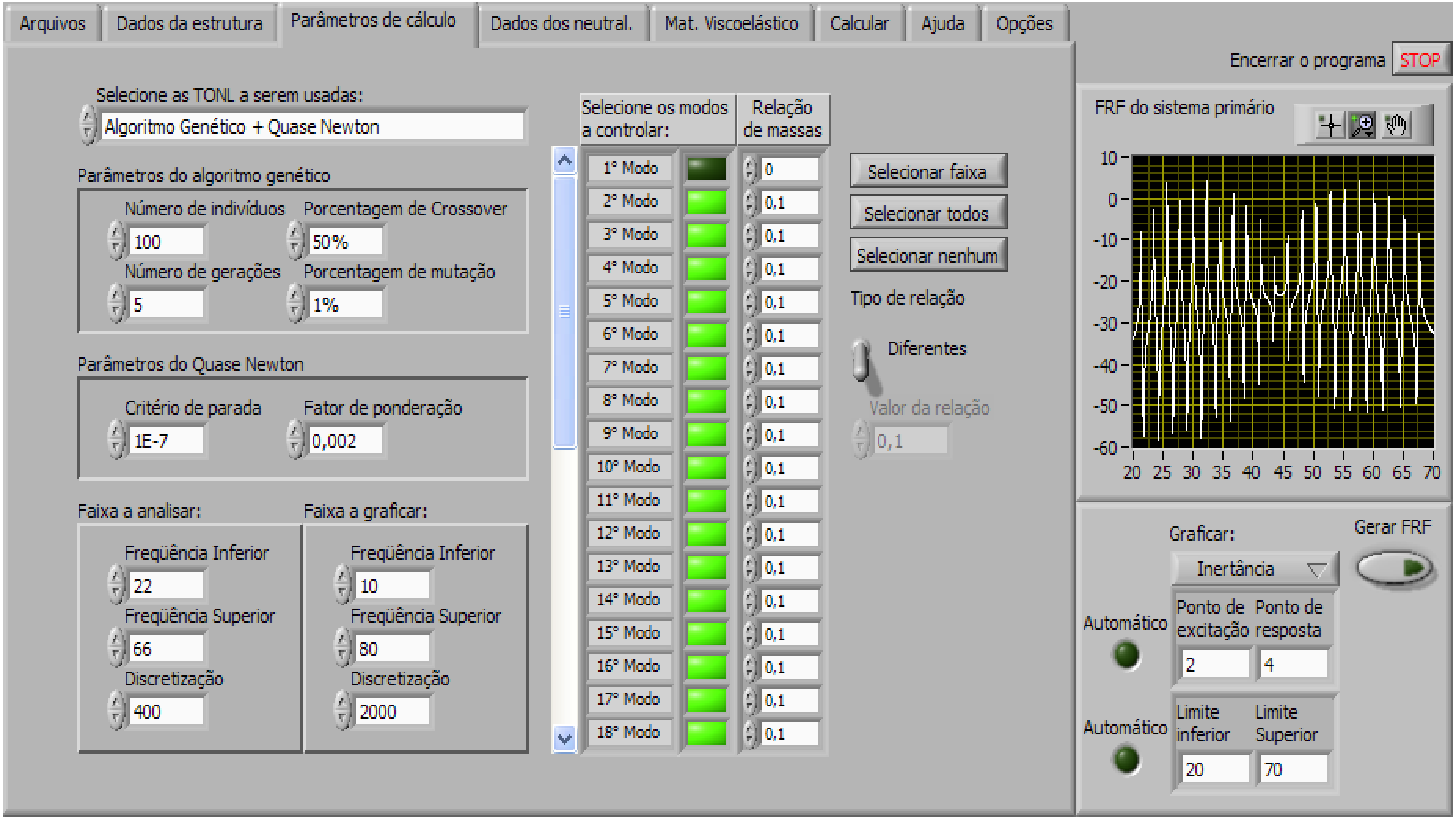

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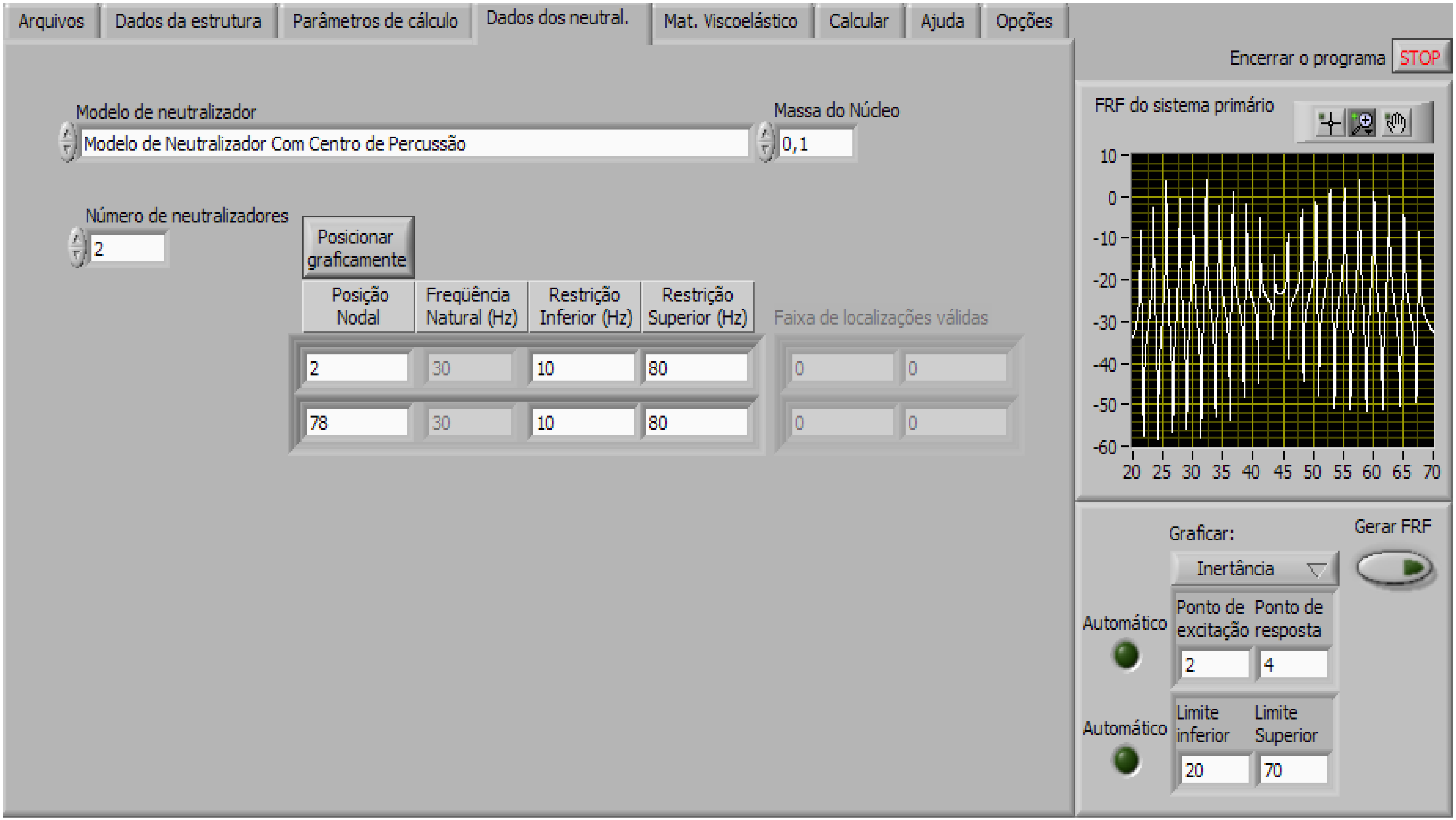

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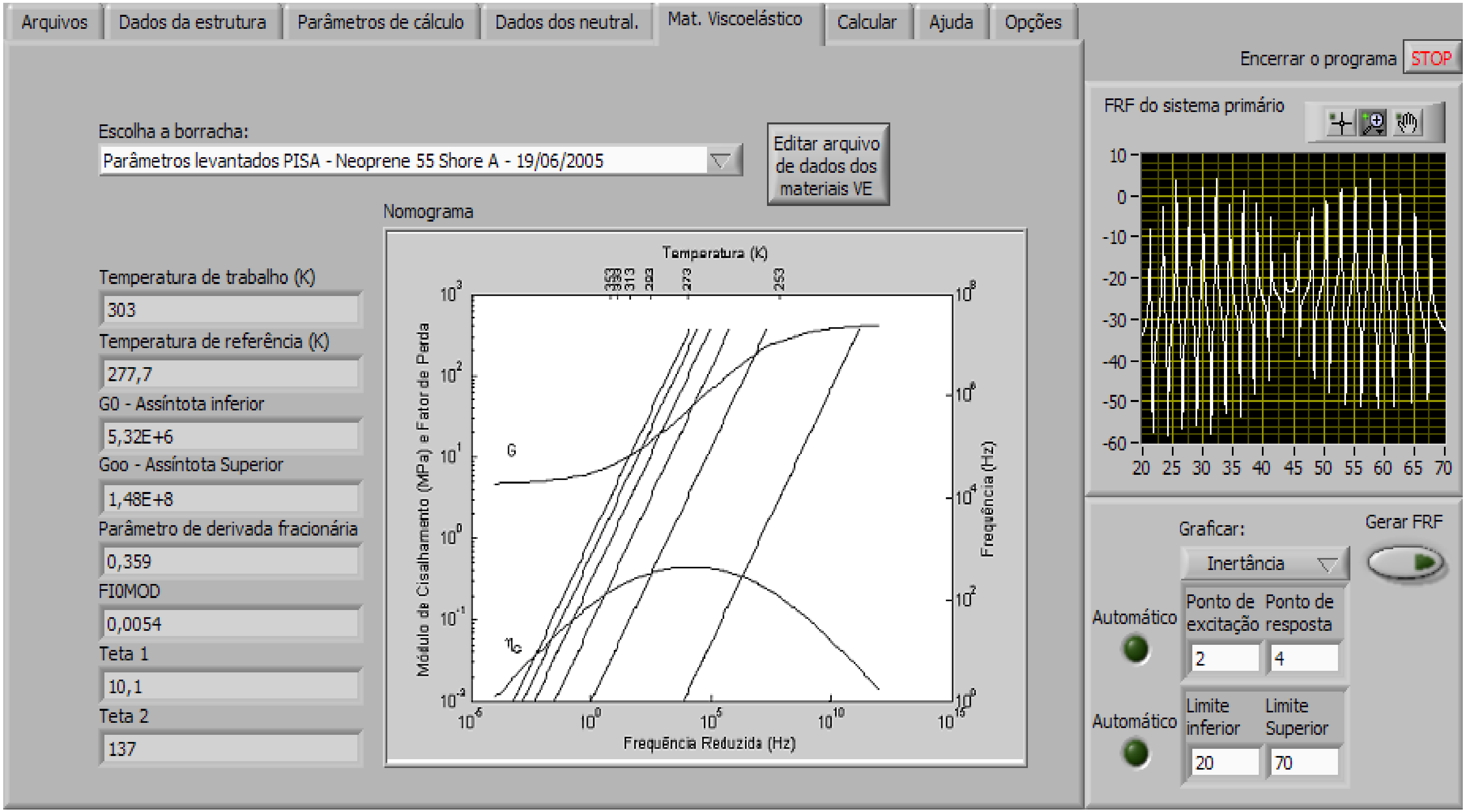

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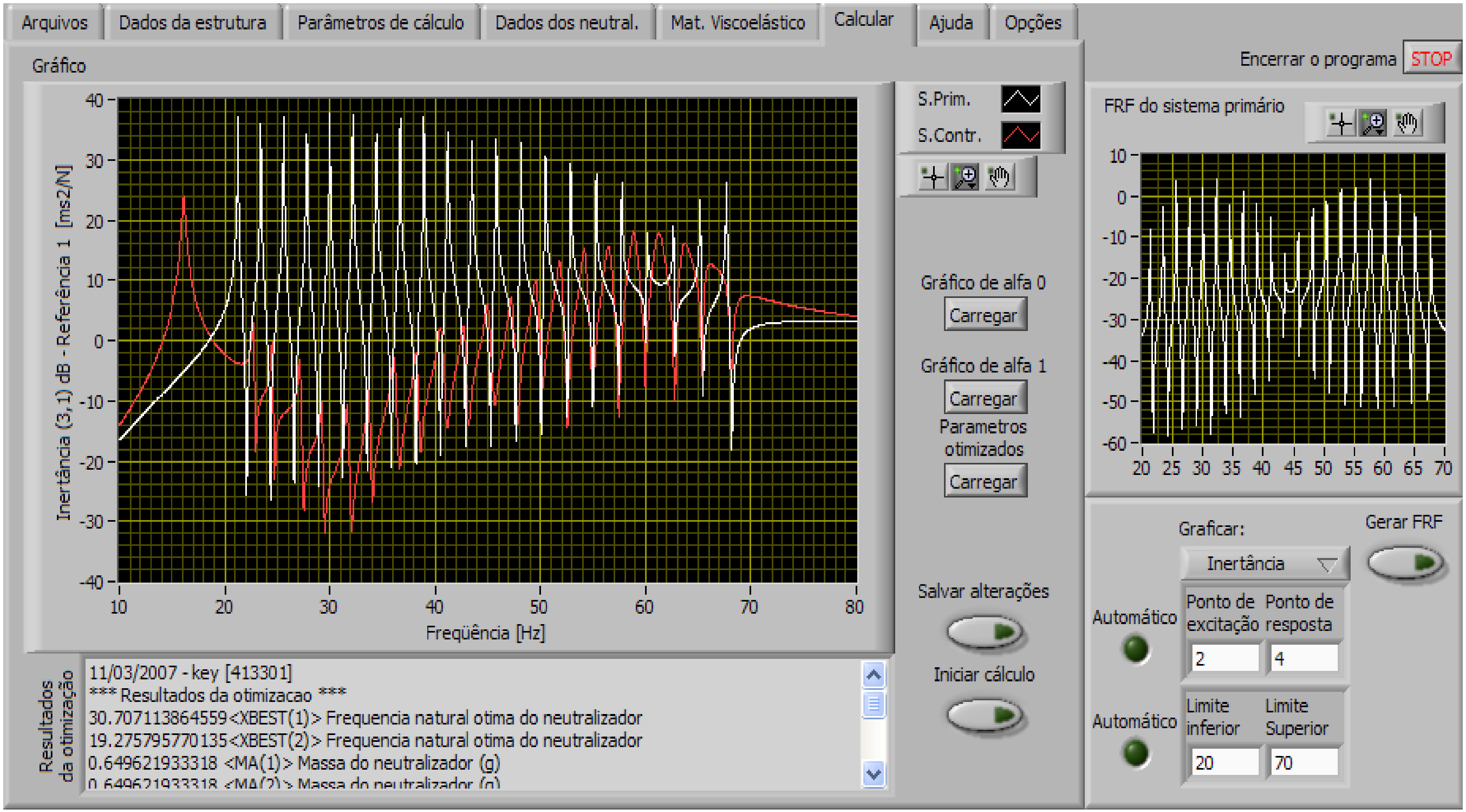

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PROGRAMA DE PÓS-GRADUAÇÃO EM ENGENHARIA MECÂNICA

Mestrado e Doutorado - *stricto sensu* - NOTA CAPES: 5

Áreas de Concentração:	Linhas de Pesquisa:
Manufatura	1 - Engenharia de Superfícies 2 - Engenharia de Materiais e Fabricação
Fenômenos de Transporte e Mecânica dos Sólidos	3 - Fenômenos de Transporte e Engenharia Térmica 4 - Mecânica dos Sólidos e Projeto Mecânico

Mechanics of Solids and Vibrations

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COORDENAÇÃO	
Coordenador	Prof. Dr. Paulo Victor Prestes Marcondes telefone: (41) 3361-3701 gabinete:Sala 7-02 B, Depto. Eng. Mecânica/UFPR
Vice-Coordenador	Prof. Dr. Carlos Alberto Bavastri telefone: (41) 3361-3309 gabinete:Sala 7-07 Depto. Eng. Mecânica/UFPR
Secretário	Sr. Marcio Brandani Tenório telefone: (41) 3361-3701

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SECRETARIA	
Local	Departamento de Engenharia Mecânica Setor de Tecnologia Centro Politécnico da UFPR Bloco IV Bairro Jardim das Américas Curitiba - PR
Endereço postal	Programa de Pós-Graduação em Engenharia Mecânica (PG-MEC) Depto. Engenharia Mecânica - UFPR Rua Francisco H. dos Santos, S/Nº Caixa postal 19011 81531-980, Curitiba, PR
Atendimento	Horário de expediente Interno: 2ª a 6ª das 07:30 às 11:30 horas Horário de atendimento Externo: 2ª a 6ª das 13:00 às 17:00 horas
e-mail	pgmec@ufpr.br
website	www.pgmec.ufpr.br
Telefone/Fax	(41) 3361-3701

COLEGIADO DO CURSO		
Titular	**Suplente**	**Representante**
Prof. Ana Sofia C. M. D'Oliveira	Prof. Carlos José de M. Siqueira	Linha de Pesquisa: Engenharia de Superfícies
Prof. Silvio Francisco Brunatto	Profª. Thaís Helena S. Flores-Sahagun	Linha de Pesquisa: Engenharia de Materiais e Fabricação
Prof. Christian J. Losso Hermes	Profª. Maria José J. de S. Ponte	Linha de Pesquisa: Fenômenos de Transporte e Engenharia Térmica
Prof. Eduardo M. de O. Lopes	Prof. Heraldo Nélio Cambraia	Linha de Pesquisa: Mecânica dos Sólidos e Projeto Mecânico
Cristiano José Scheuer	João do Carmo Lopes Gonçalves	Alunos
Prof. Paulo Victor P. Marcondes	Coordenador do PG-Mec	
Prof. Carlos Aberto Bavastri	Vice-Coordenador do PG-Mec	

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CURSOS	
Tipo	Mestrado acadêmico e Doutorado (stricto sensu)
Custo	Gratuito
Data de criação do Mestrado	Mestrado: 15 de março de 2000 Doutorado: 13 de março de 2006
Nota CAPES/MEC	Mestrado: 5 Doutorado: 5
Alunos regulares 2012/1	29 = Mestrado 36 = Doutorado
Alunos Disciplinas Isoladas 2012/1	62
Cota de Bolsas de estudo 2012/1	Mestrado: CAPES/PROAP= 12 CAPES/REUNI = 05 Doutorado: CAPES/PROAP= 05 CAPES/REUNI = 05 Fundação Araucária = 01
Defesas de Dissertação realizadas	131
Defesas de Tese realizadas	14
Períodos das aulas	diurno (manhã e tarde)
Períodos letivos	trimestrais
Professores Credenciados	22 doutores
Laboratórios	18
Grupos de pesquisa CNPq	10

Thank you for your attention

MUITO OBRIGADO