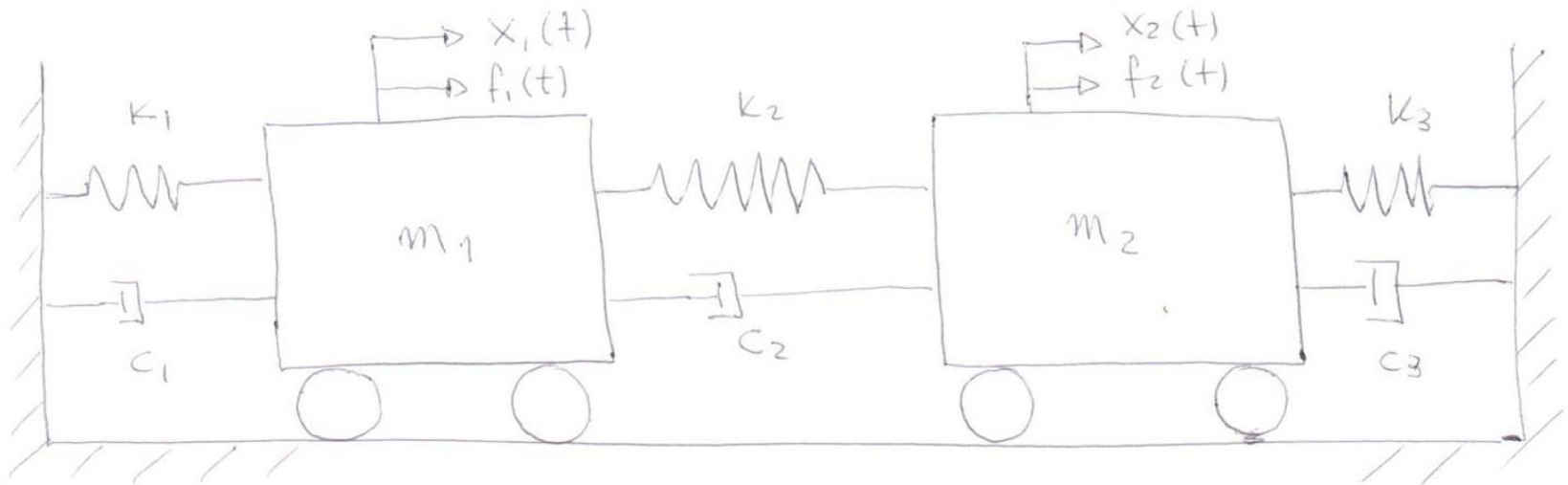


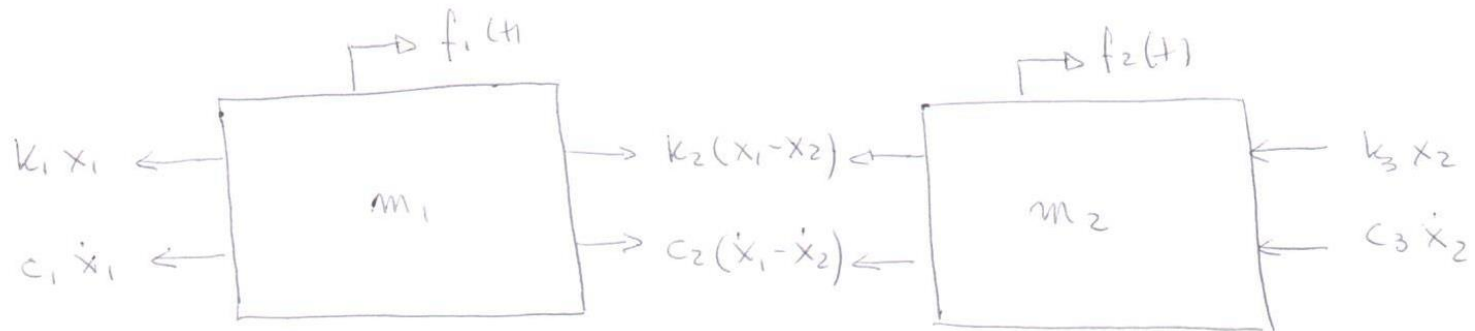
◆ 5.1 – Multi degree of freedom system

Given a two degree of freedom system,



◆ 5.1 – Multi degree of freedom system

Doing the free body diagram



and applying the Newton's second law

$$f_1(t) - k_1 x_1 - k_2 (x_1 - x_2) - c_1 \dot{x}_1 - c_2 (\dot{x}_1 - \dot{x}_2) = m \ddot{x}_1(t)$$
$$f_2(t) - k_3 x_2 - c_3 \dot{x}_2 + k_2 (x_1 - x_2) + c_2 (\dot{x}_1 - \dot{x}_2) = m \ddot{x}_2(t)$$

◆ 5.1 – Multi degree of freedom system

The system equations can be rewritten as

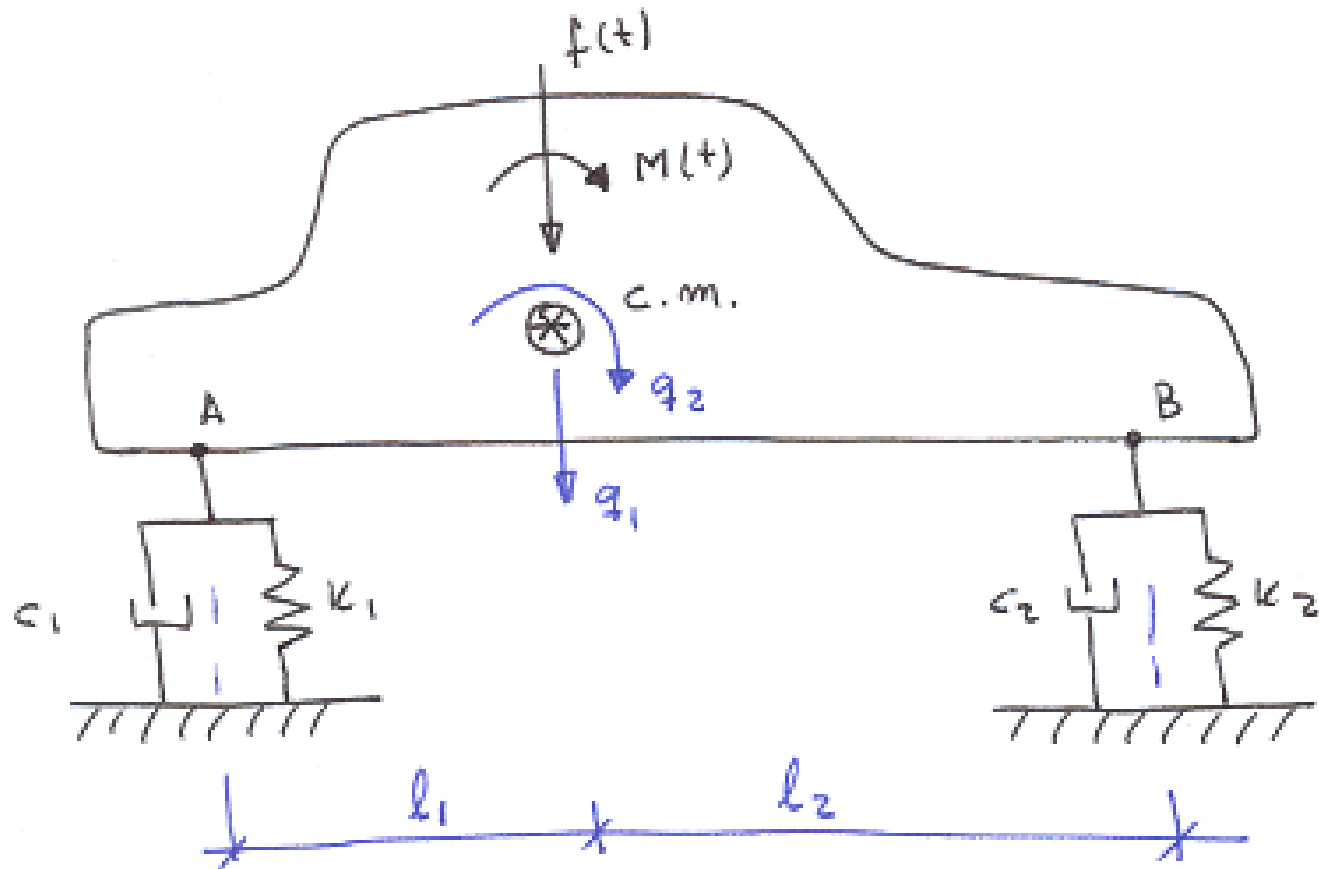
$$\begin{aligned} m\ddot{x}_1(t) + (k_1 + k_2)x_1(t) + (c_1 + c_2)\dot{x}_1(t) - k_2x_2(t) - c_2\dot{x}_2(t) &= f_1(t) \\ m\ddot{x}_2(t) + (k_2 + k_3)x_2(t) + (c_2 + c_3)\dot{x}_2(t) - k_2x_1(t) - c_2\dot{x}_1(t) &= f_2(t) \end{aligned}$$

Or in matrix form

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \{\ddot{x}\} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix} \{x\} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 + c_3 \end{bmatrix} \{\dot{x}\} = \{f\}$$

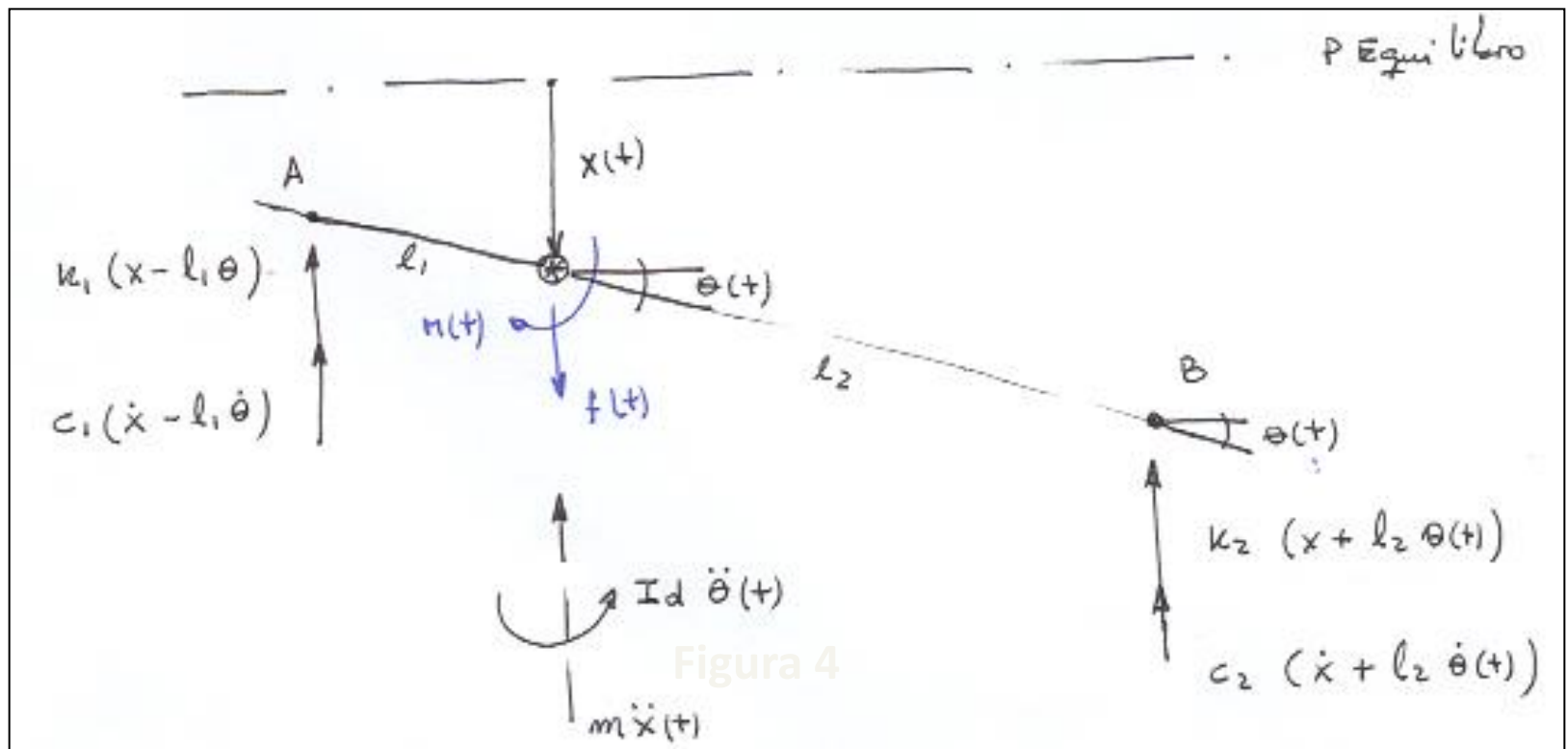
◆ 5.1 – Multi degree of freedom system: An Application

The two degree of freedom model can represent a simple model of the suspension car, the rotor and isotropic support and the isolation system, among others.



◆ 5.1 – Multi degree of freedom system: An Application

The free body diagram is



◆ 5.1 – Multi degree of freedom system: An Application

and applying the Newton's second law

$$f(t) - k_1(x - l_1\theta) - c_1(\dot{x} - l_1\dot{\theta}) - k_2(x + l_2\theta) - c_2(\dot{x} + l_2\dot{\theta}) = m\ddot{x}(t)$$

$$k_1(x - l_1\theta)l_1 + c_1(\dot{x} - l_1\dot{\theta})l_1 + M(t) - k_2(x + l_2\theta)l_2 - c_2(\dot{x} + l_2\dot{\theta})l_2 = I_d \ddot{\theta}(t)$$

◆ 5.1 – Multi degree of freedom system: An Application

and in matrix form

$$\begin{bmatrix} m & 0 \\ 0 & I_d \end{bmatrix} \{\ddot{q}\} + \begin{bmatrix} c_1 + c_2 & c_2 l_2 - c_1 l_1 \\ c_2 l_2 - c_1 l_1 & c_2 l_2^2 + c_1 l_1^2 \end{bmatrix} \{\dot{q}\} + \begin{bmatrix} k_1 + k_2 & k_2 l_2 - k_1 l_1 \\ k_2 l_2 - k_1 l_1 & k_2 l_2^2 + k_1 l_1^2 \end{bmatrix} \{q\} = \{f(t)\}$$

$$\{q\} = \begin{Bmatrix} x(t) \\ \theta(t) \end{Bmatrix}$$

$$\{f(t)\} = \begin{Bmatrix} f(t) \\ M(t) \end{Bmatrix}$$

◆ 5.1 – Influence Coefficient Method

Stiffness: The reaction force introducing by the elastic properties is, as saw, given by

$$Q_i = \sum_{j=1}^n k_{ij} q_j$$





◆ 5.1 – Influence Coefficient Method

If we supposed that the “s” coordinates is 1 and another ones, with  $j \neq s$ , are 0, the resultant force to produce such situation it will, numerically, equals to the column of stiffness matrix

$$q_s = 1, \text{ and } q_{j \neq s} = 0 \quad \rightarrow \quad Q_i = k_{is}$$

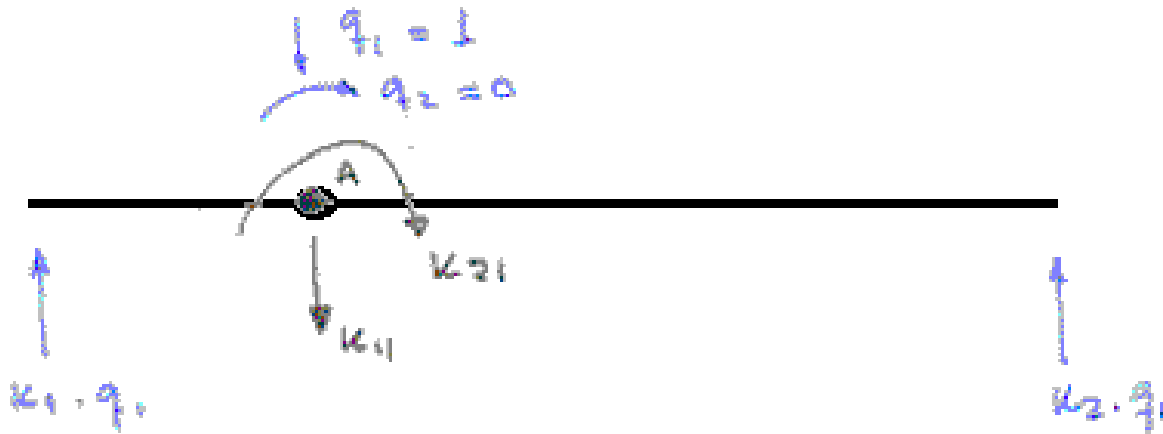
This procedure allows us to find the K matrix.

The same concept can be applied to the damping and inertial matrix. In these cases we use the velocity and acceleration coordinates, instead the displacement coordinates.

◆ 5.1 – Influence Coefficient Method: Example 1

Use this method for the example above, considering

a)  $q_1 = 1$  and  $q_2 = 0$ ;



◆ 5.1 – Influence Coefficient Method: Example 1

The resultant force is given by

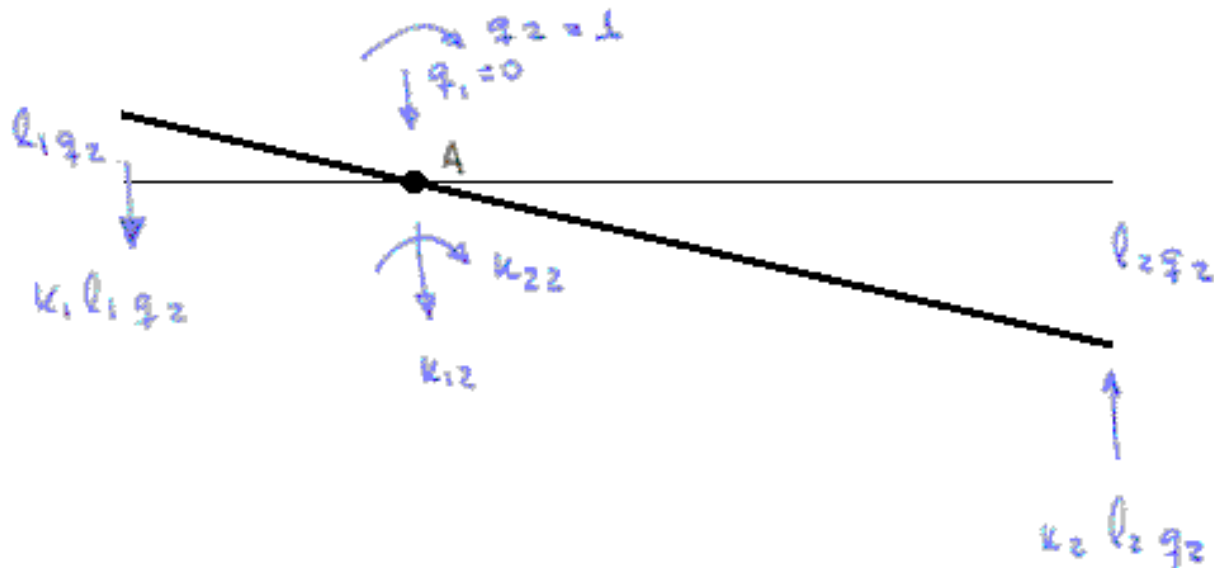
$$\sum F_{vertical} = 0 \Rightarrow -k_1 - k_2 + k_{11} = 0 \Rightarrow k_{11} = k_1 + k_2$$

$$\sum M_A = 0 \Rightarrow k_1 l_1 - k_2 l_2 + k_{21} = 0 \Rightarrow k_{21} = k_2 l_2 - k_1 l_1$$

Then we find the first column of the stiffness matrix

◆ 5.1 – Influence Coefficient Method: Example 1

b)  $q_2 = 1$  and  $q_1 = 0$ ;



◆ 5.1 – Influence Coefficient Method: Example 1

The resultant force is given by

$$\sum F_{vertical} = 0 \Rightarrow -k_{12} + k_1 l_1 \cdot 1 - k_2 l_2 \cdot 1 = 0 \Rightarrow k_{12} = k_2 l_2 - k_1 l_1$$

$$\sum M_A = 0 \Rightarrow k_1 l_1 l_1 + k_2 l_2 l_2 + k_{22} = 0 \Rightarrow k_{22} = k_1 l_1^2 - k_2 l_2^2$$

$$K = \begin{bmatrix} k_1 + k_2 & k_2 l_2 - k_1 l_1 \\ k_2 l_2 - k_1 l_1 & k_2 l_2^2 + k_1 l_1^2 \end{bmatrix}$$

◆ 5.1 – Influence Coefficient Method: Example 1

In a similar way, doing

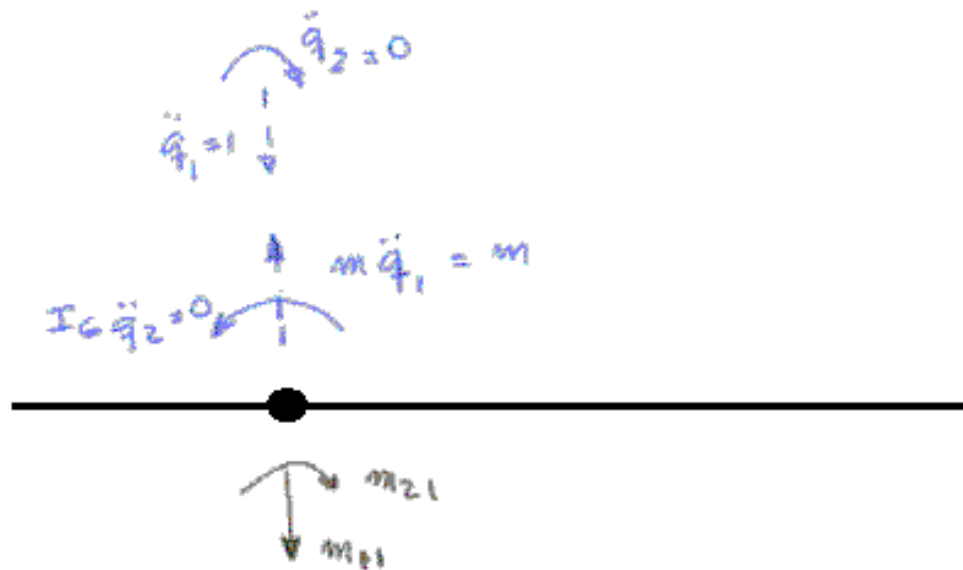
$$\begin{array}{l} a) \dot{q}_1 = 1 \text{ and } \dot{q}_2 = 0; \text{ and} \\ b) \dot{q}_2 = 1 \text{ and } \dot{q}_1 = 0, \end{array}$$

$$C = \begin{bmatrix} c_1 + c_2 & c_2 l_2 - c_1 l_1 \\ c_2 l_2 - c_1 l_1 & c_2 l_2^2 + c_1 l_1^2 \end{bmatrix}$$

◆ 5.1 – Influence Coefficient Method: Example 1

Lastly, for the inertial matrix

a)  $\ddot{q}_1 = 1$  and  $\ddot{q}_2 = 0$ ;



◆ 5.1 – Influence Coefficient Method: Example 1

Then, the first column of the inertial matrix

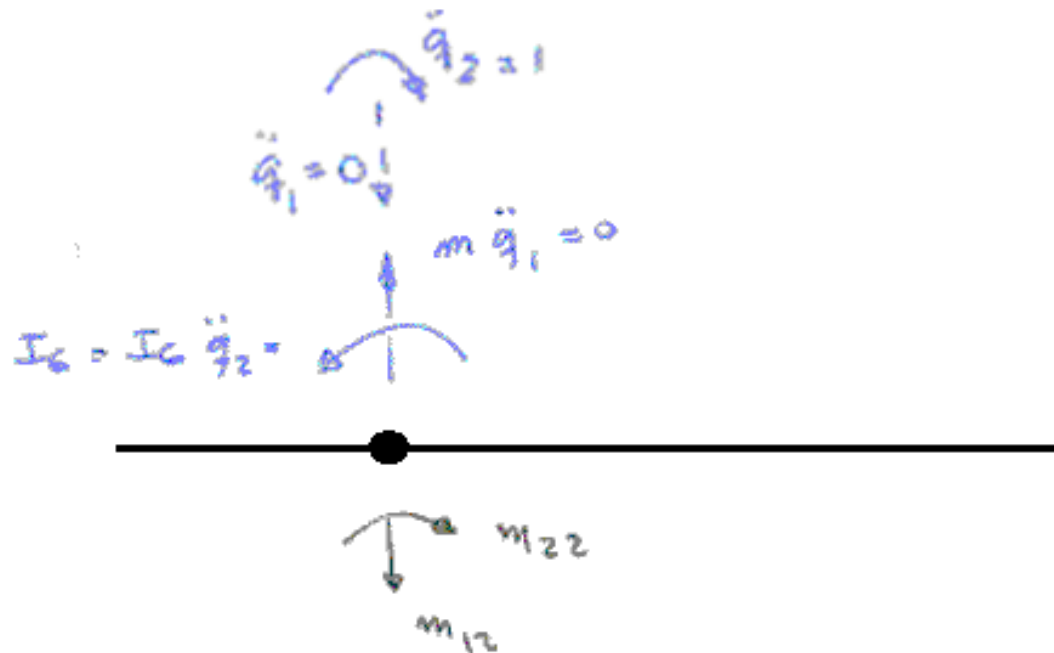
$$\sum F_v = m_{11} - m\ddot{q}_1 = 0 \Rightarrow m_{11} = m$$

$$\sum M_A = m_{21} = 0 \Rightarrow m_{21} = 0$$



◆ 5.1 – Influence Coefficient Method: Example 1

b)  $\ddot{q}_2 = 1$  and  $\ddot{q}_1 = 0$ ;



◆ 5.1 – Influence Coefficient Method: Example 1

$$\sum F_v = m_{12} = 0 \Rightarrow m_{12} = 0$$

$$\sum M_A = 0 = m_{22} - I_G \ddot{q}_2 = m_{22} - I_G = 0 \Rightarrow m_{22} = I_G$$

$$M = \begin{bmatrix} m & 0 \\ 0 & I_G \end{bmatrix}$$

◆ 5.1 – Influence Coefficient Method: Example 1

Then, the equation of motion is, in matrix simplified notation

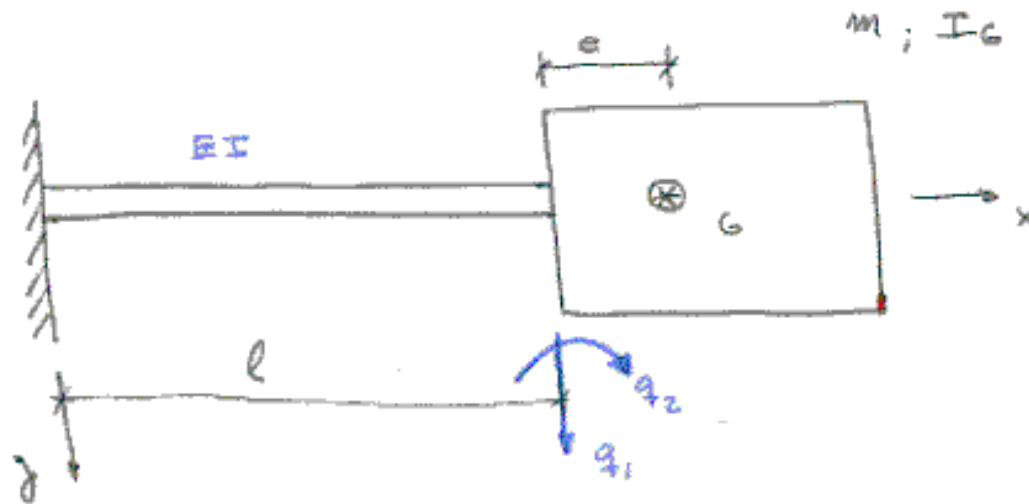
$$M \{\ddot{q}(t)\} + C \{\dot{q}(t)\} + K \{q(t)\} = f(t)$$

where

$$\begin{bmatrix} m & 0 \\ 0 & I_G \end{bmatrix} \begin{Bmatrix} \ddot{q}_1(t) \\ \ddot{q}_2(t) \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 & c_2 l_2 - c_1 l_1 \\ c_2 l_2 - c_1 l_1 & c_2 l_2^2 + c_1 l_1^2 \end{bmatrix} \begin{Bmatrix} \dot{q}_1(t) \\ \dot{q}_2(t) \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & k_2 l_2 - k_1 l_1 \\ k_2 l_2 - k_1 l_1 & k_2 l_2^2 + k_1 l_1^2 \end{bmatrix} \begin{Bmatrix} q_1(t) \\ q_2(t) \end{Bmatrix} = \begin{Bmatrix} f(t) \\ M(t) \end{Bmatrix}$$



◆ 5.1 – Influence Coefficient Method: Example 2

The second example represents a water tank in the cities, the middle of the part of the Stockbridge, among others real systems.



◆ 5.1 – Influence Coefficient Method: Example 2

It is known, for elasticity theory, that the external excitation applied in the free end of a beam elements produce the follow deformation:

	$\begin{cases} y_o = \frac{M_o l^2}{2EI} \\ \theta_o = \frac{M_o l}{EI} \end{cases}$
	$\begin{cases} y_o = \frac{P_o l^3}{3EI} \\ \theta_o = \frac{P_o l^2}{2EI} \end{cases}$

◆ 5.1 – Influence Coefficient Method: Example 2

$$a) q_1 = 1 \text{ and } q_2 = 0;$$



$$\left. \begin{aligned} \frac{k_{11}l^3}{3EI} + \frac{k_{21}l^2}{2EI} &= 1 \\ \frac{k_{11}l^2}{2EI} + \frac{k_{21}l}{EI} &= 0 \end{aligned} \right\}$$

$$k_{11} = \frac{12EI}{l^3}$$

$$k_{21} = -\frac{6EI}{l^2}$$

◆ 5.1 – Influence Coefficient Method: Example 2

b)  $q_2 = 1$  and  $q_1 = 0$ ;



$$\left. \begin{aligned} \frac{k_{12}l^2}{2EI} + \frac{k_{22}l}{EI} &= 1 \\ \frac{k_{12}l^3}{3EI} + \frac{k_{22}l^2}{2EI} &= 0 \end{aligned} \right\}$$

$$k_{12} = -\frac{6EI}{l^2}$$

$$k_{22} = \frac{4EI}{l}$$



$$K = \begin{bmatrix} 12EI/l^3 & -6EI/l^2 \\ -6EI/l^2 & 4EI/l \end{bmatrix}$$

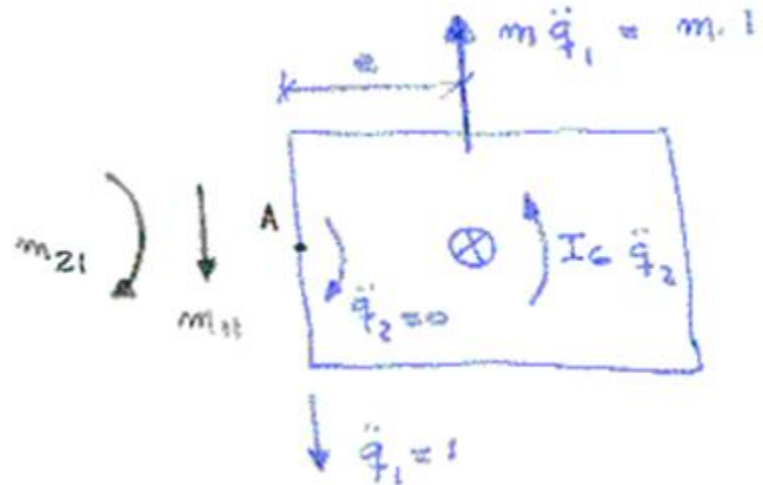
◆ 5.1 – Influence Coefficient Method: Example 2

For the inertial matrix

$$a) \ddot{q}_1 = 1 \text{ and } \ddot{q}_2 = 0;$$

$$\sum F_v = m_{11} - m\ddot{q}_1 = 0 \Rightarrow m_{11} = m$$

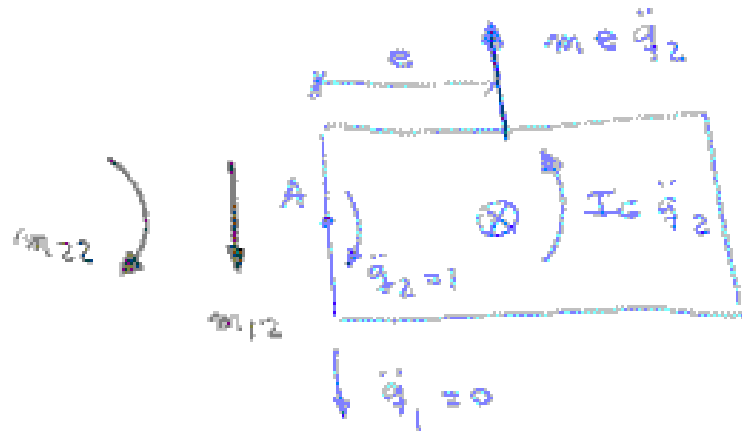
$$\sum M_A = m_{21} - m\ddot{q}_1 \cdot e = 0 \Rightarrow m_{21} = me$$





◆ 5.1 – Influence Coefficient Method: Example 2

$$b) \ddot{q}_2 = 1 \text{ and } \ddot{q}_1 = 0;$$



$$\sum F_v = m_{12} - me\ddot{q}_2 = 0 \Rightarrow m_{12} = me$$

$$\sum M_A = 0 = m_{22} - I_G\ddot{q}_2 - me^2\ddot{q}_2 = 0 \Rightarrow m_{22} = I_G + me^2$$

◆ 5.1 – Influence Coefficient Method: Example 2

Then, the equation of motion is

$$\begin{bmatrix} m & me \\ me & I_G + me^2 \end{bmatrix} \begin{Bmatrix} \ddot{q}_1(t) \\ \ddot{q}_2(t) \end{Bmatrix} + \begin{bmatrix} 12EI/l^3 & -6EI/l^2 \\ -6EI/l^2 & 4EI/l \end{bmatrix} \begin{Bmatrix} q_1(t) \\ q_2(t) \end{Bmatrix} = \begin{Bmatrix} f(t) \\ m(t) \end{Bmatrix}$$

or

$$M \{\ddot{q}(t)\} + C \{\dot{q}(t)\} + K \{q(t)\} = f(t)$$

◆ 5.1 – Influence Coefficient Method of Flexibility

For mechanical systems, the calculation of the stiffness matrix, through the influence coefficients of stiffness, requires the resolution of an equation system.

This leads, in general, to an excessive computational cost.

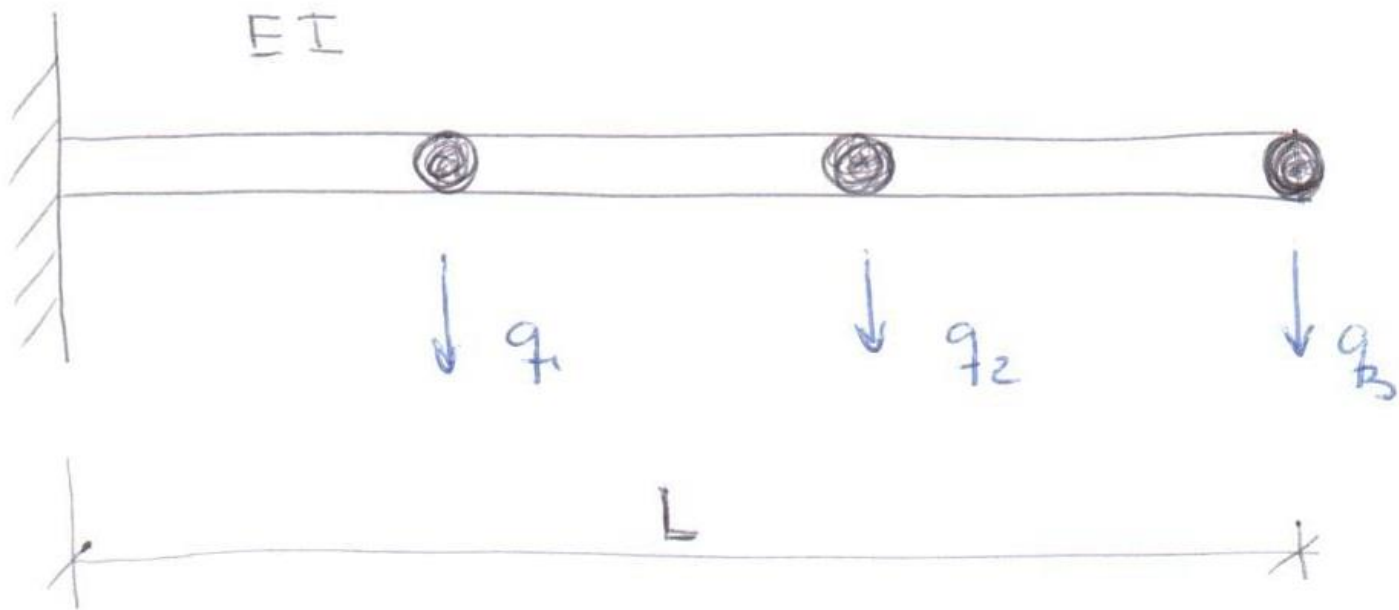
For another hands,  $K$  can be calculated by the inverse of the flexible matrix.

$$AF = AKq \qquad AF = q$$

$$q_j = \sum_{i=1}^n \alpha_{ji} f_i = \sum_{i=1}^n a_{ji} f_i$$

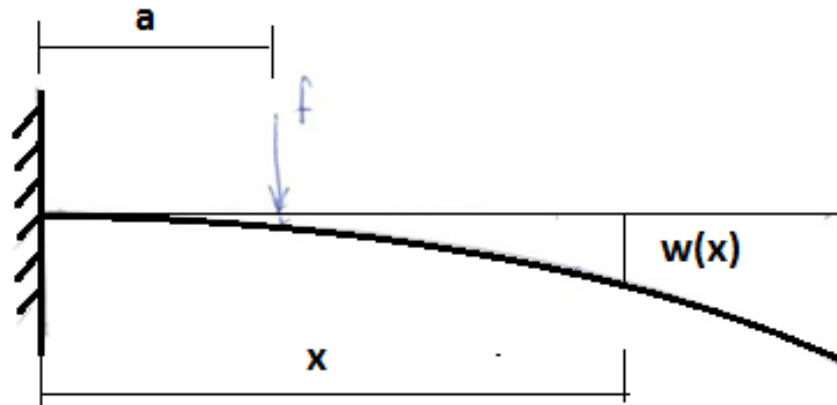
◆ 5.1 – Influence Coefficient Method of Flexibility

Considering the case below



◆ 5.1 – Influence Coefficient Method of Flexibility

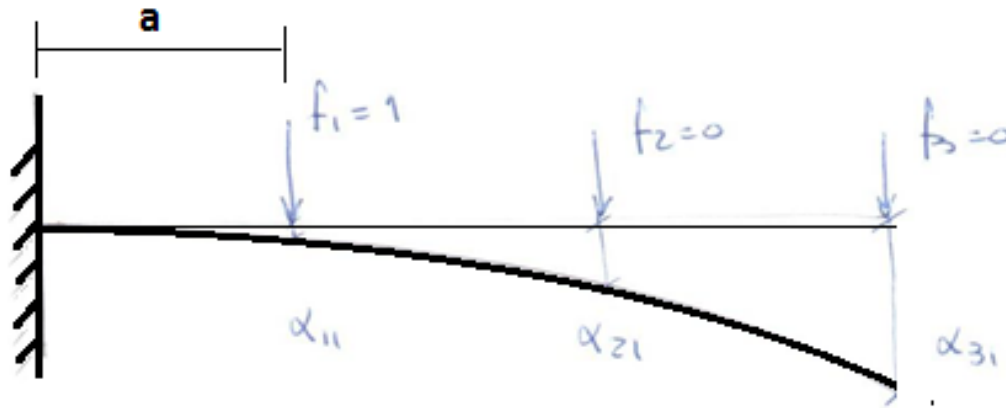
As known



$$y(x) = \frac{1}{EI} \left[ \frac{1}{6} (x-a)^3 \mu(x-a) - \frac{x^3}{6} + a \frac{x^2}{2} \right]$$

◆ 5.1 – Influence Coefficient Method of Flexibility

a)  $F_1 = 1$  and  $F_{j \neq s} = 0$



◆ 5.1 – Influence Coefficient Method of Flexibility

The first column of the flexibility matrix will be

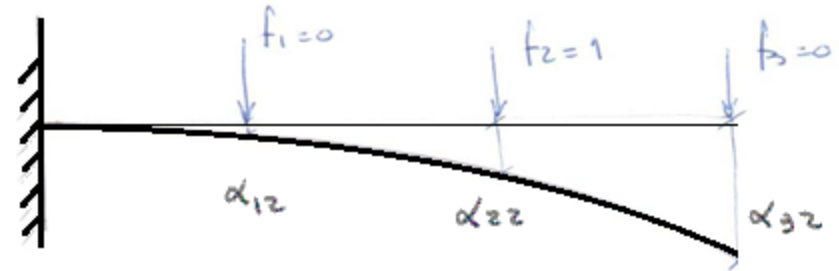
$$\alpha_{11}\left(x = \frac{L}{3}\right) = \frac{1}{EI} \left[ -\frac{\left(\frac{L}{3}\right)^3}{6} + \frac{L}{3} \frac{\left(\frac{L}{3}\right)^2}{2} \right] = \frac{1}{EI} \left( \frac{L^3}{81} \right)$$

$$\alpha_{21}\left(x = \frac{2}{3}L\right) = \frac{1}{EI} \left[ \frac{1}{6} \left(\frac{L}{3}\right)^3 - \left(\frac{2L}{3}\right)^3 \frac{1}{6} + \frac{L}{3} \left(\frac{\left(2L/3\right)^2}{2}\right) \right] = \frac{L^3}{EI} \frac{5}{162}$$

$$\alpha_{31}(x = L) = \frac{1}{EI} \left[ \frac{1}{6} \left(\frac{2L}{3}\right)^3 - \frac{L^3}{6} + \frac{L}{3} \frac{L^2}{2} \right] = \frac{L^3}{EI} \frac{4}{81}$$

◆ 5.1 – Influence Coefficient Method of Flexibility

$$b) F_2 = 1 \text{ and } F_{j \neq s} = 0$$



$$\alpha_{12}\left(x = \frac{L}{3}\right) = \frac{1}{EI} \left[ -\frac{\left(\frac{L}{3}\right)^3}{6} + \frac{2L}{3} \frac{\left(\frac{L}{3}\right)^2}{2} \right] = \frac{1}{EI} L^3 \left( \frac{5}{162} \right)$$

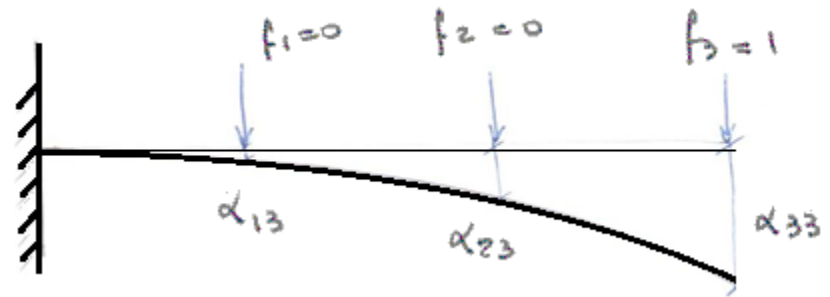
$$\alpha_{22}\left(x = \frac{2L}{3}\right) = \frac{1}{EI} \left[ -\frac{\left(\frac{2L}{3}\right)^3}{6} + \frac{2L}{3} \frac{\left(\frac{2L}{3}\right)^2}{2} \right] = \frac{1}{EI} L^3 \left( \frac{8}{81} \right)$$

$$\alpha_{32}(x = L) = \frac{1}{EI} \left[ \frac{1}{6} \left(\frac{L}{3}\right)^3 - \left(\frac{L}{3}\right)^3 + \frac{2L}{3} \frac{L^2}{2} \right] = \frac{1}{EI} L^3 \left( \frac{14}{81} \right)$$



◆ 5.1 – Influence Coefficient Method of Flexibility

$$b) F_3 = 1 \text{ and } F_{j \neq s} = 0$$



$$\alpha_{13} \left( x = \frac{L}{3} \right) = \frac{1}{EI} \left[ -\frac{\left( \frac{L}{3} \right)^3}{6} + L \frac{\left( \frac{L}{3} \right)^2}{2} \right] = \frac{1}{EI} L^3 \left( \frac{4}{81} \right)$$

$$\alpha_{23} \left( x = \frac{2L}{3} \right) = \frac{1}{EI} \left[ -\frac{\left( \frac{2L}{3} \right)^3}{6} + L \frac{\left( \frac{2L}{3} \right)^2}{2} \right] = \frac{1}{EI} L^3 \left( \frac{14}{81} \right)$$

$$\alpha_{33} (x = L) = \frac{1}{EI} \left[ -\frac{L^3}{6} + L \frac{L^2}{2} \right] = \frac{1}{EI} L^3 \left( \frac{1}{3} \right)$$

◆ 5.1 – Influence Coefficient Method of Flexibility

And the matrix K and M are

$$M = \begin{bmatrix} m_e & 0 & 0 \\ 0 & m_e & 0 \\ 0 & 0 & m_e/2 \end{bmatrix}$$

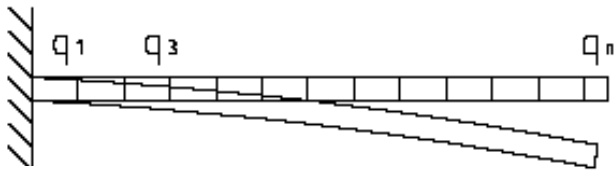
$$K = A^{-1}$$

$$A = \frac{L^3}{EI} \begin{bmatrix} 1/81 & 5/162 & 4/81 \\ 5/162 & 8/81 & 14/81 \\ 4/81 & 14/81 & 1/3 \end{bmatrix}$$

$$M_{3 \times 3} \ddot{q}(t)_{3 \times 1} + A_{3 \times 3}^{-1} q(t)_{3 \times 1} = f(t)_{3 \times 1}$$

◆ 6 - Mathematical Model of Non Rotating Systems

The equation of motion of the multi-degree systems is given by



$$M \ddot{q}(t) + C \dot{q}(t) + K q(t) = f(t)$$

$q(t)$  generalized coordinates, nx1

$f(t)$  generalized forces, nx1

$M$  generalized mass matrix, nxn

$K$  generalized stiffness matrix, nxn

$C$  generalized damping matrix, nxn

◆ Mathematical Model of Non Rotating Systems



In the frequency domain

$$[-\Omega^2 M + i\Omega C + K]Q(\Omega) = F(\Omega)$$

Using the below transformation matrix, and taking only a few eigenvectors who are into the frequency range of interest

$$Q(\Omega)_{nx1} = \hat{\Phi}_{n \times \hat{n}} \hat{P}(\Omega)_{\hat{n} \times 1} \quad \text{and pre-multiplying by } \hat{\Phi}_{\hat{n} \times 1}^T$$

◆ Mathematical Model of Non Rotating Systems

Considering a orthogonal properties of the eigenvectors and the orthonormalized characteristic, it is possible to obtain

$$\Theta^T M \Theta = I$$

$$\Theta^T K \Theta = \Lambda$$

$$\Theta^T C \Theta = \begin{bmatrix} \backslash & & \\ & 2\xi_j \Omega_j & \\ & & \backslash \end{bmatrix}$$

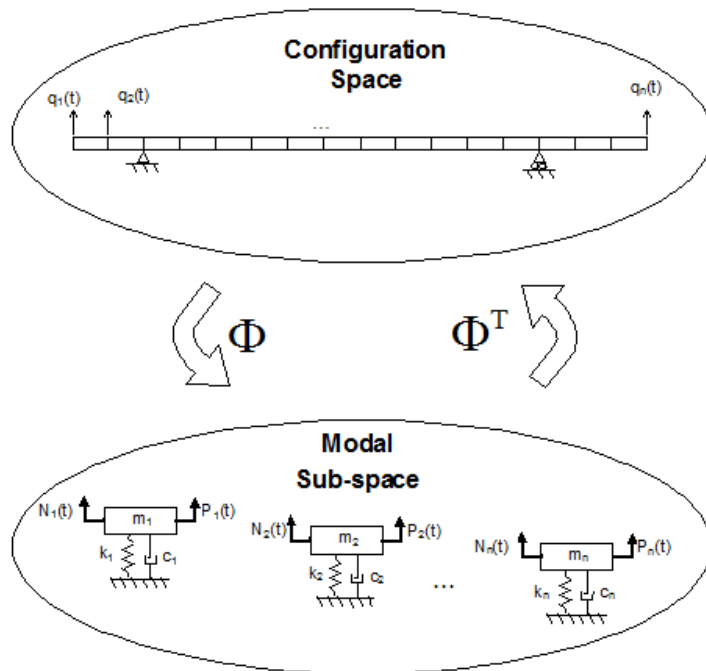
The equation of motion in the modal sub-space is given by

$$\left\{ -\Omega^2 [I_{\hat{n}\hat{n}}] + i\Omega [\Gamma_{\hat{n}\hat{n}}] + \Lambda_{\hat{n}\hat{n}} \right\} \hat{P}(\Omega) = \hat{N}(\Omega) = \hat{\Theta}^T F(\Omega)$$

◆ Mathematical Model of Non Rotating Systems

So, it is possible to obtain the response  $\hat{P}(\Omega) \rightarrow Q(\Omega)$

$$\hat{P}(\Omega) = [\hat{D}(\Omega)]^{-1} \hat{\Theta}^T F(\Omega)$$



Then, the solution in the configuration space,  $q(t)$ , is:

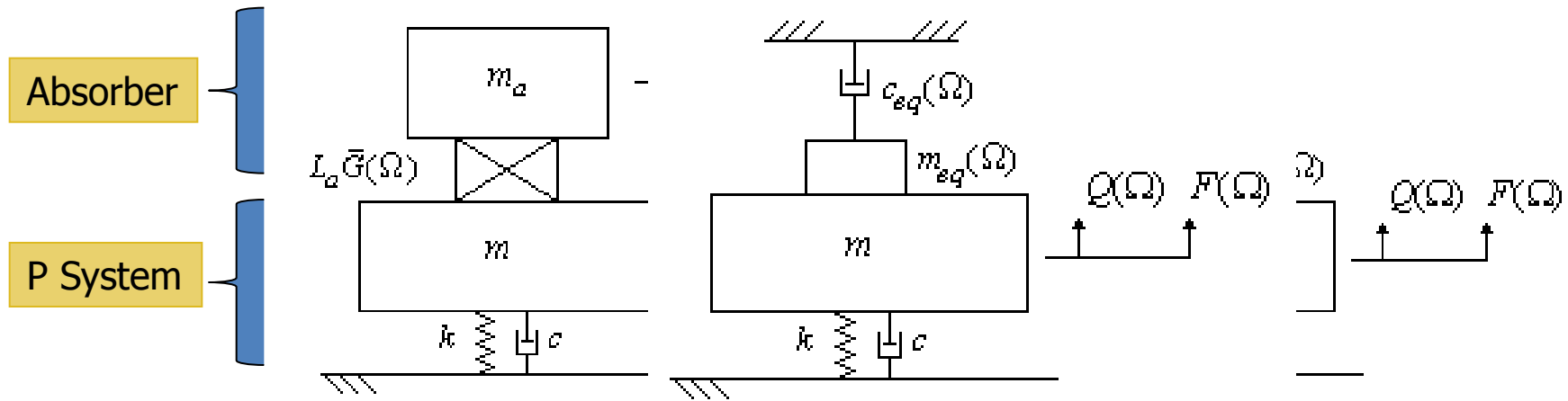
$$Q(\Omega) = \hat{\Theta} [\hat{D}(\Omega)]^{-1} \hat{\Theta}^T F(\Omega)$$

In the modal sub-space of the primary system the equations of motion are uncoupled.

Each line can be considered as one degree of freedom system

# ◆ Equivalent Generalized Parameters

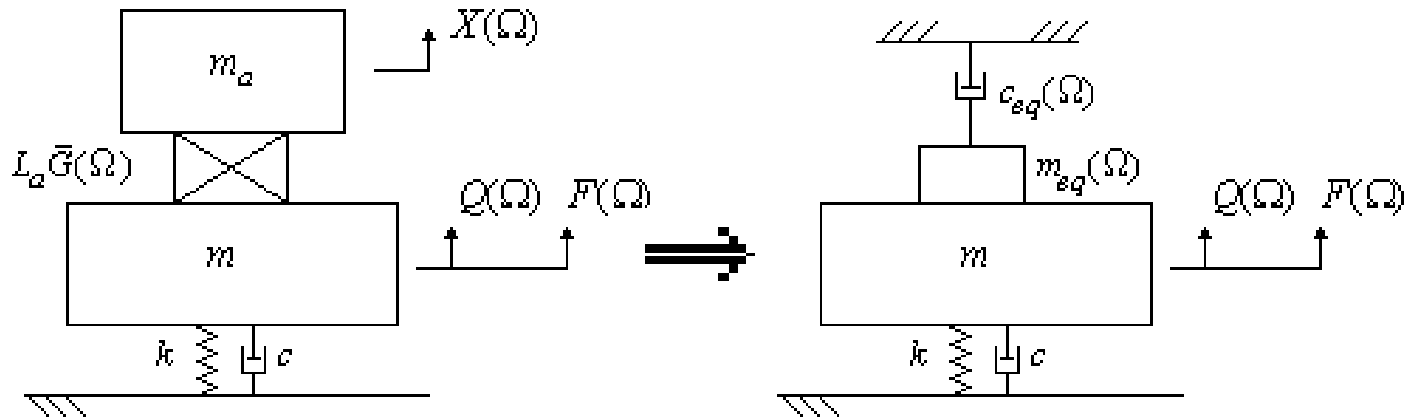
How to model the dynamic absorbers, such that, when attached to the structure to be controlled, the model of the compound system will be simple and inexpensive of the computational point of view



$$Z(\Omega) = \frac{F(\Omega)}{i\Omega X_b(\Omega)}$$

$$Z(\Omega) = c_{eq}(\Omega) + i\Omega m_{eq}(\Omega)$$

# ◆ Equivalent Generalized Parameters



$$c_{eq}(\Omega) = m \Omega_a \frac{r(\Omega) \eta(\Omega) \varepsilon^3}{(\varepsilon^2 - r(\Omega))^2 + (r(\Omega) \eta(\Omega))^2}$$

$$m_{eq}(\Omega) = -m \frac{r(\Omega) \{ \varepsilon^2 - r(\Omega) [1 + \eta(\Omega)^2] \}}{(\varepsilon^2 - r(\Omega))^2 + (r(\Omega) \eta(\Omega))^2}$$

■ where

$$r(\Omega) = \frac{G(\Omega)}{G(\Omega_a)} \quad \varepsilon = \frac{\Omega}{\Omega_a} \quad \eta(\Omega) = \frac{\text{Im}(G(\Omega))}{\text{Re}(G(\Omega))}$$

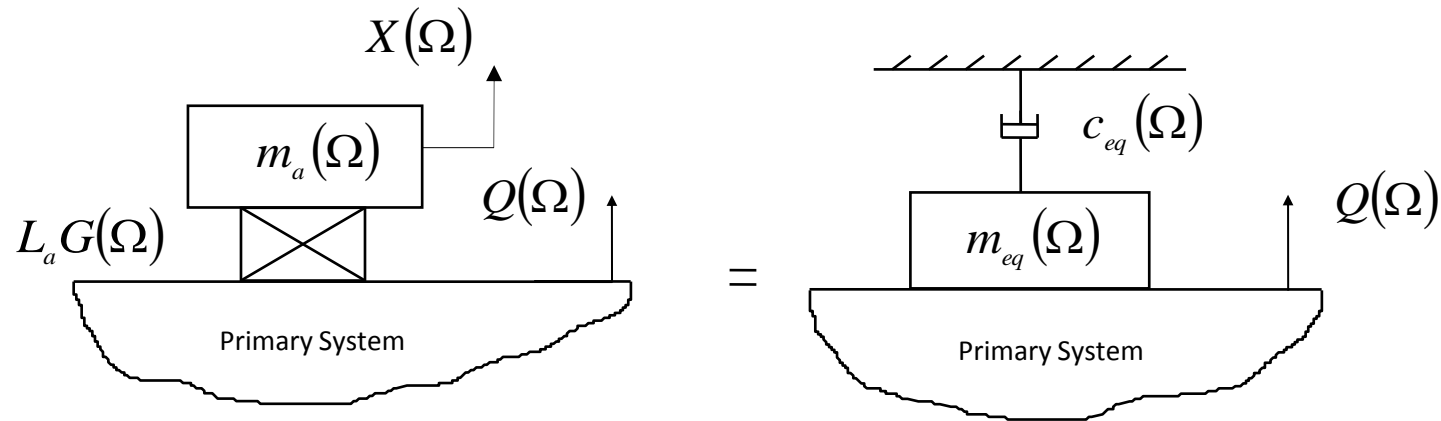


# ◆ Equivalent Generalized Parameters

Different types of model to different applications

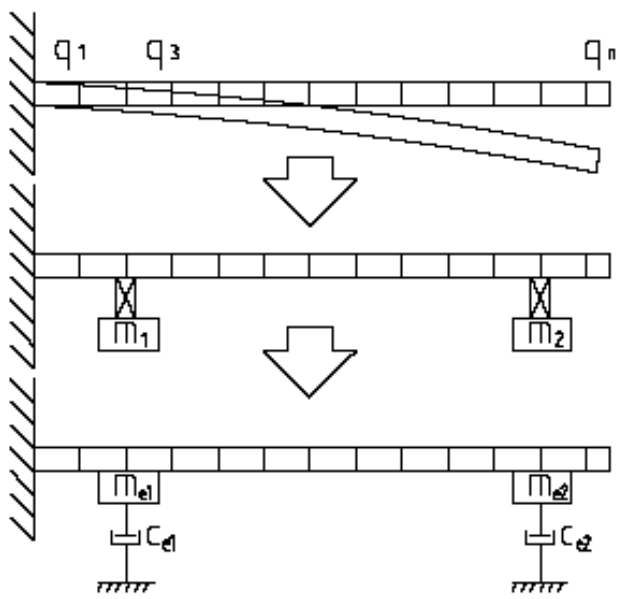
Type of Model	$m_e(\Omega)$	$c_e(\Omega)$	$\Omega_a$
Viscous Model	$-m \frac{\{\varepsilon^2 - [1 + (2\zeta\varepsilon)^2]\}}{(\varepsilon^2 - 1)^2 + (2\zeta\varepsilon)^2}$	$m\Omega_a \frac{2\zeta\varepsilon^4}{(\varepsilon^2 - 1)^2 + (2\zeta\varepsilon)^2}$	$\sqrt{\frac{k}{m}}$
Viscoelastic	$-m \frac{r(\Omega)\{\varepsilon^2 - r(\Omega)[1 + \eta(\Omega)^2]\}}{(\varepsilon^2 - r(\Omega))^2 + (r(\Omega)\eta(\Omega))^2}$	$m\Omega_a \frac{r(\Omega)\eta(\Omega)\varepsilon^3}{(\varepsilon^2 - r(\Omega))^2 + (r(\Omega)\eta(\Omega))^2}$	$\sqrt{\frac{LG_r(\Omega_a)}{m}}$
Hydraulic	$-m \frac{\{\varepsilon^2 - [1 + (2\zeta\varepsilon)^2]\}}{(\varepsilon^2 - 1)^2 + (2\zeta\varepsilon)^2}$	$m\Omega_a \frac{2\zeta\varepsilon^4}{(\varepsilon^2 - 1)^2 + (2\zeta\varepsilon)^2}$	$\sqrt{\frac{\gamma g S_1 \left(1 + \frac{S_1}{S_2}\right)}{m_t}}$
Electromechanical	$T^2 C \frac{(1 - \varepsilon)^2}{(1 - \varepsilon^2)^2 + (2\zeta_e \varepsilon)^2}$	$T^2 C \frac{2\zeta_e \varepsilon^2}{(1 - \varepsilon^2)^2 + (2\zeta_e \varepsilon)^2}$	$\sqrt{\frac{1}{LC}}$

## ◆ Equivalent Generalized Parameters



The primary system "feels" the absorber as being a equivalent mass  $m_{eq}(\Omega)$  attached to the generalized coordinate  $q_j(t)$  and a equivalent viscous damper with constant  $c_{eq}(\Omega)$ , connected to the ground. Therefore, the dynamics of the resultant system (primary + absorbers) can be formulated in terms of the generalized coordinates of the primary system, where  $Q(\Omega)$  is representative, despite the new system now having added degrees of freedom. **This is the main advantage of the generalized equivalent quantities concept.**

# ◆ The Mathematical Model of Compound System



In the stationary system, the equation of motion for the compound system is

$$[-\Omega^2 \tilde{M} + i\Omega \tilde{C} + K] Q(\Omega) = F(\Omega)$$

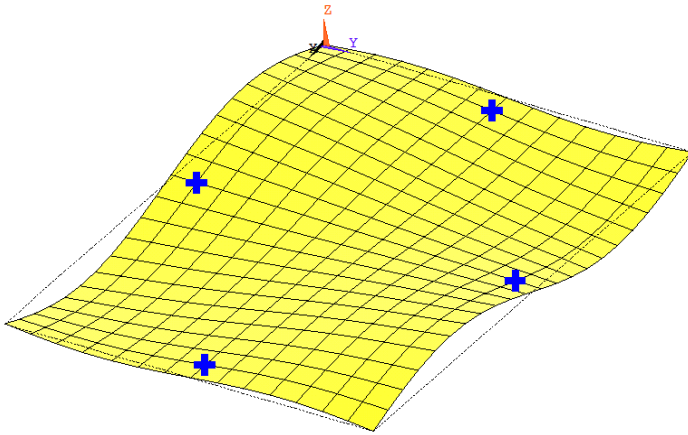
$$\tilde{M} = M + \begin{bmatrix} 0 & & & \\ & m_{eq}^{(1)}(\Omega) & & 0 \\ & & \ddots & \\ & & & m_{eq}^{(p)}(\Omega) \\ & & & & 0 \end{bmatrix} = M + M_{eq}(\Omega)$$

$$\tilde{C} = C + \begin{bmatrix} 0 & & & \\ & c_{eq}^{(1)}(\Omega) & & 0 \\ & & \ddots & \\ & & & c_{eq}^{(p)}(\Omega) \\ & & & & 0 \end{bmatrix} = C + C_{eq}(\Omega)$$

Then, the solution in the modal sub-space for the primary system

$$Q(\Omega) = \Phi \tilde{P}(\Omega)$$

# ◆ The Mathematical Model of Compound System



Pre-multiplying the equation of motion by the transpose eigenvectors, considering the orthonormalized characteristic, it is possible to obtain

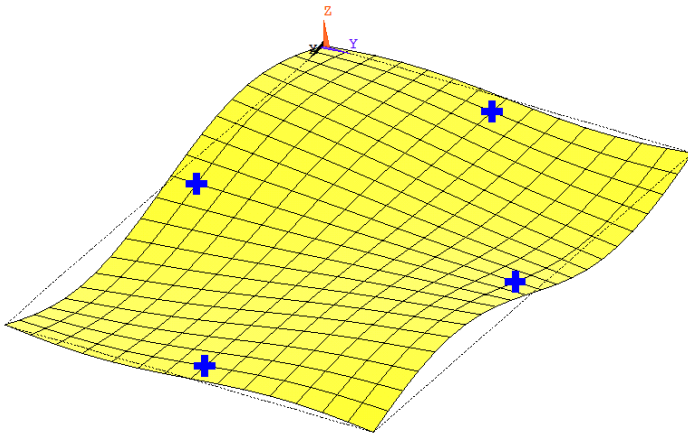
$$\Theta^T \tilde{M} \Theta = I + M_A(\Omega) \quad \Theta^T K \Theta = \Lambda$$

$$\Theta^T \tilde{C} \Theta = \left[ \begin{array}{c} \backslash \\ 2\xi_j \Omega_j \\ \backslash \end{array} \right] + C_A(\Omega)$$

Then, the solution in the modal sub-space for the primary system

$$\left\{ -\Omega^2 \left[ I_{\tilde{n}\tilde{x}\tilde{n}} + \tilde{M}_A(\Omega) \right] + i\Omega \left[ \Gamma_{\tilde{n}\tilde{x}\tilde{n}} + \tilde{C}_A(\Omega) \right] + \Lambda_{\tilde{n}\tilde{x}\tilde{n}} \right\} \tilde{P}(\Omega) = \tilde{N}(\Omega)$$

## ◆ The Mathematical Model of Compound System



$$\tilde{P}(\Omega) = [D(\Omega)]^{-1} \Theta^T F(\Omega)$$

Then, the solution in the configuration space,  $q(t)$ , is:

$$Q(\Omega) = \Theta [D(\Omega)]^{-1} \Theta^T F(\Omega)$$

The FRF of the compound systems is:

$$H(\Omega) = \Theta [D(\Omega)]^{-1} \Theta^T$$

## ◆ 7 - Nonlinear Optimization Techniques

The optimization problem is defined by:

$$\min f(x) \quad \text{where } f : R^{nd} \rightarrow R$$

$$\text{with } x \in R^{nd}$$

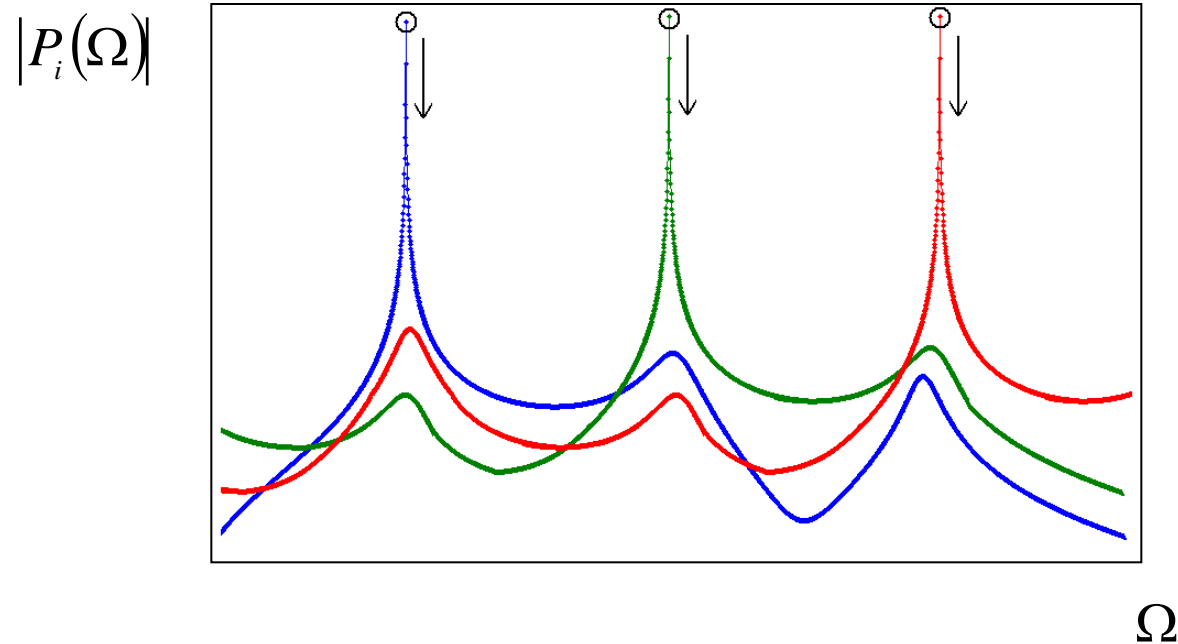
*subject to :*

$$h_i(x) = 0 \quad i = 1, 2, \dots, m_i$$

$$g_j(x) \geq 0 \quad j = m_i + 1, m_i + 2, \dots, l$$

◆ Nonlinear Optimization Techniques

Graphically:



$$\|P(\Omega)\|_2 = \sqrt{(\text{MÁX})^2 + (\text{MÁX})^2 + (\text{MÁX})^2 + \dots}$$

◆ Nonlinear Optimization Techniques

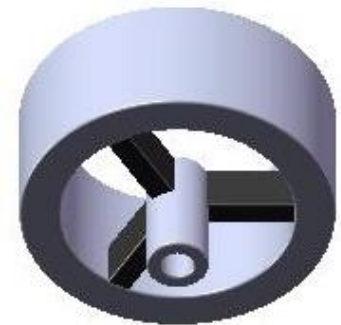
The objective function is defined by

$$f_{cost}(x) = \left\| \max_{\Omega_1 < \Omega < \Omega_2} \left| \hat{P}(\Omega, x) \right| \right\|$$

where

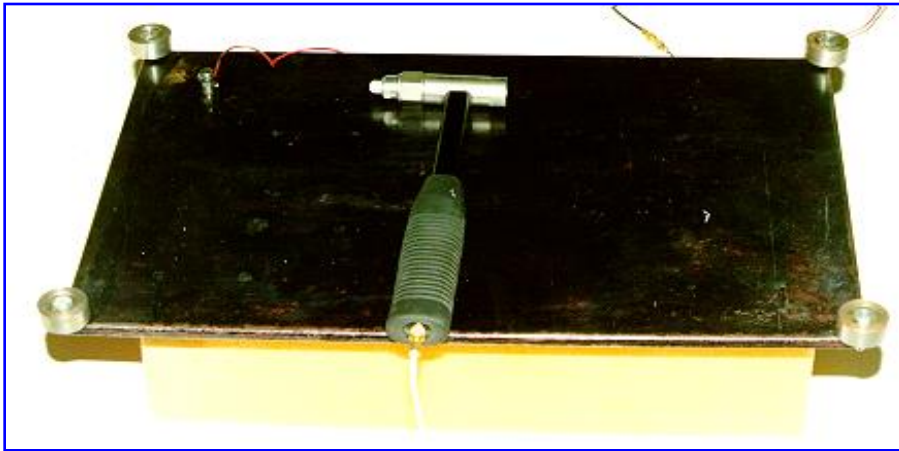
$$x = (\Omega_{a1}, \Omega_{a2}, \dots, \Omega_{ap})$$

After optimization procedure, the DVA's natural frequencies  $\Omega_n$  are known. Then, it is possible to do a physical realization.



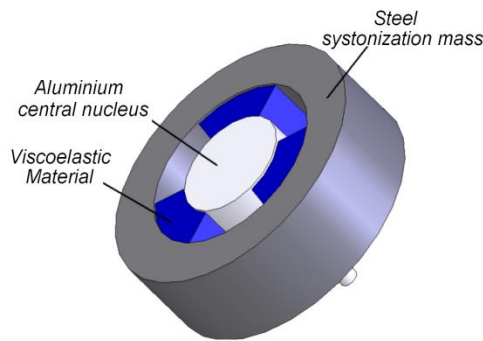
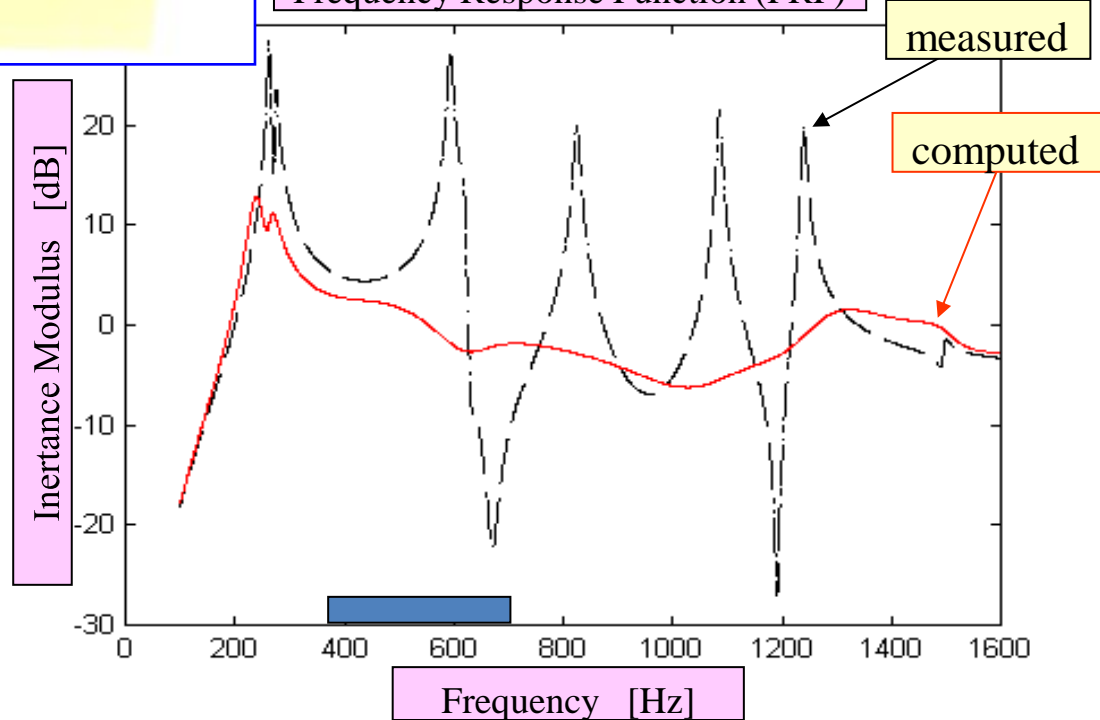


◆ A thin steel plate – doctoral work

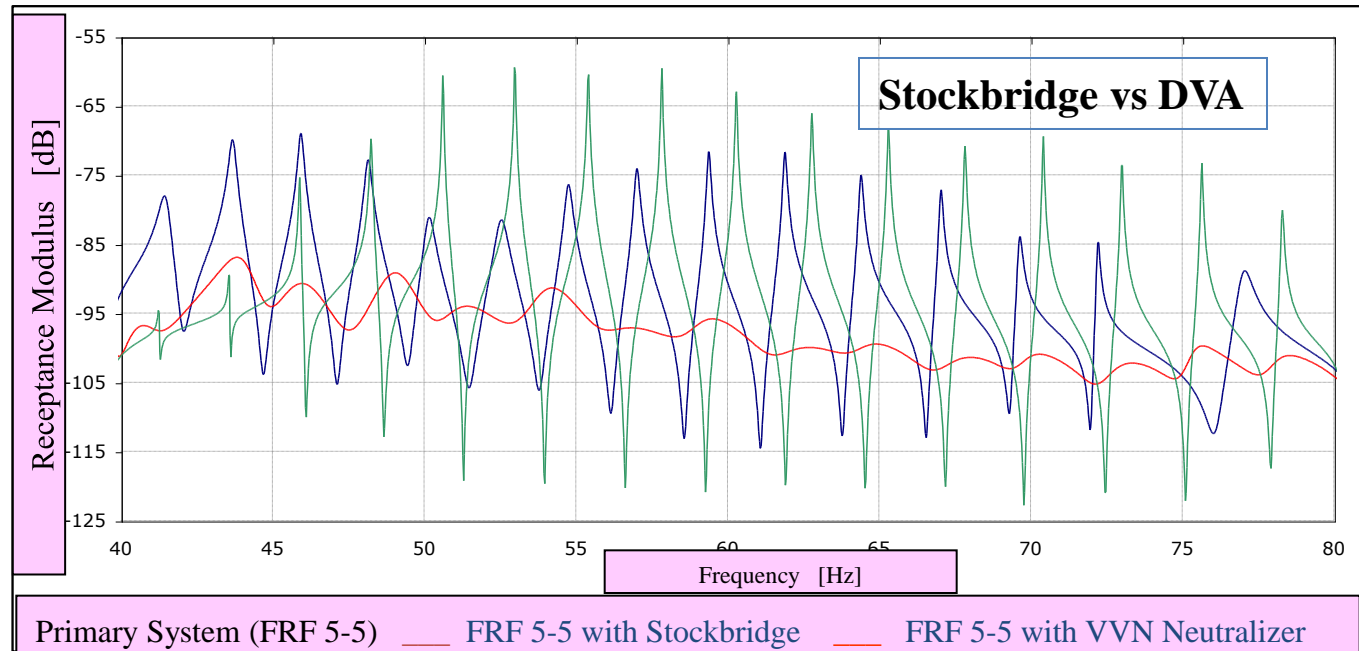
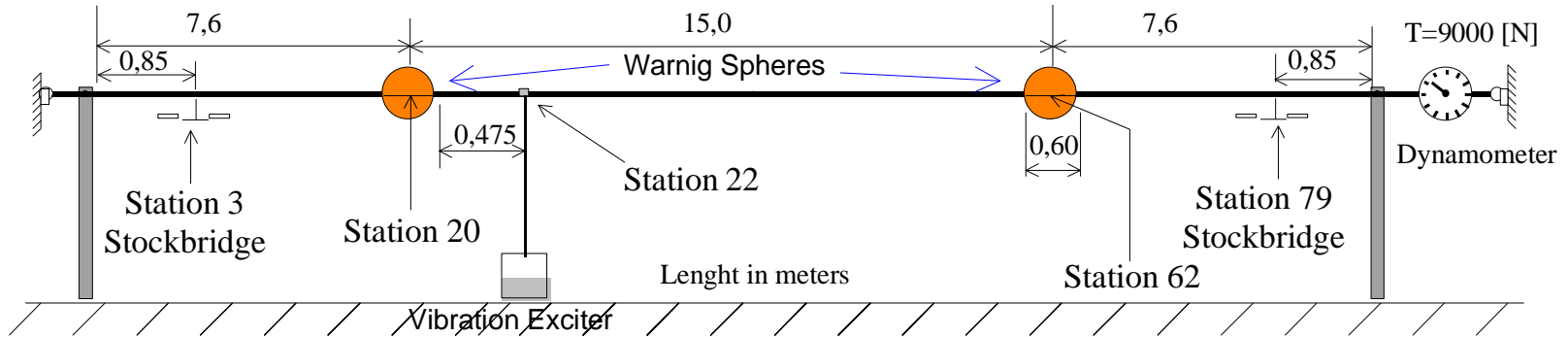


PISA/CNPq group

Frequency Response Function (FRF)

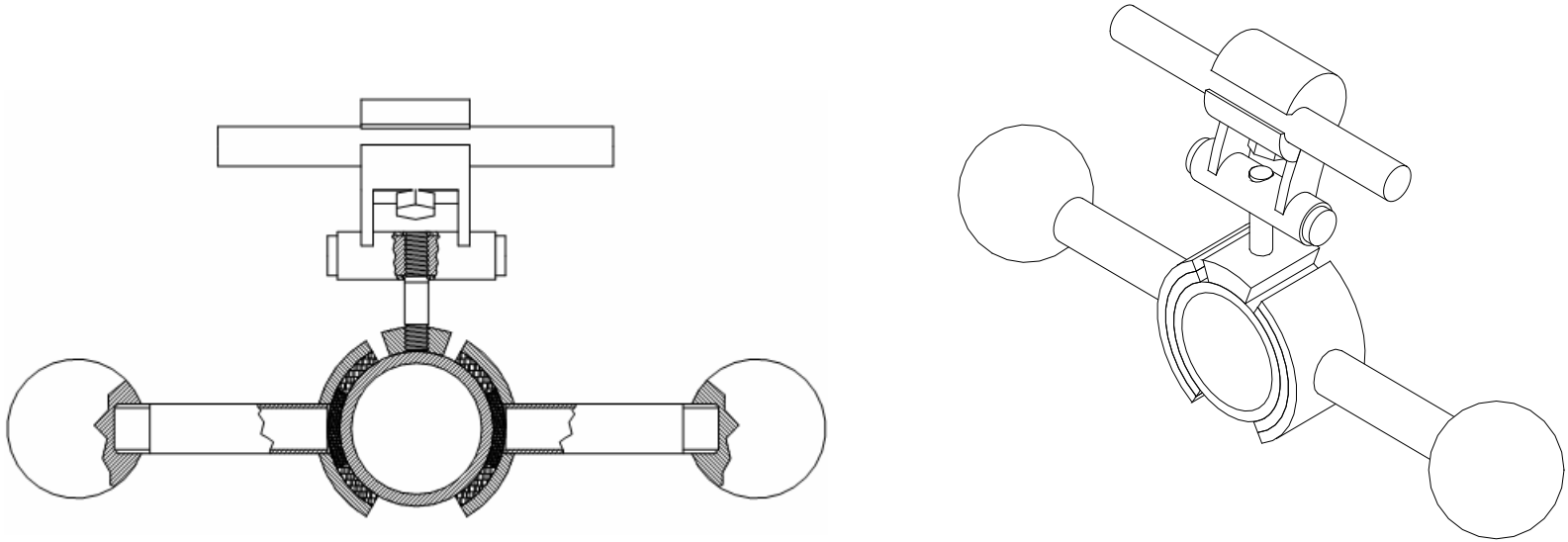


# ◆ A power transmission line





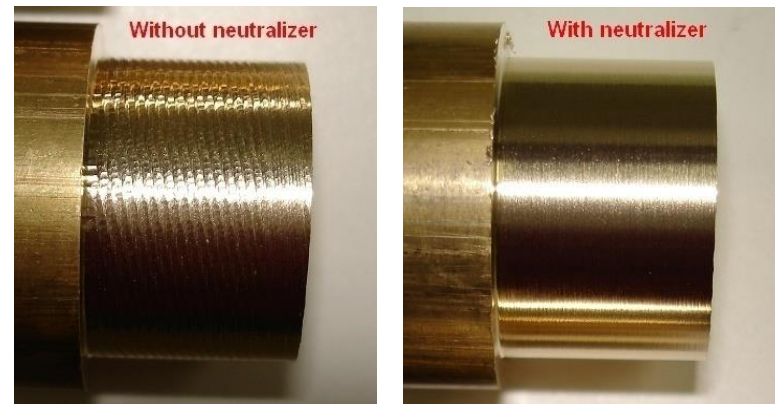
## ◆ A power transmission line



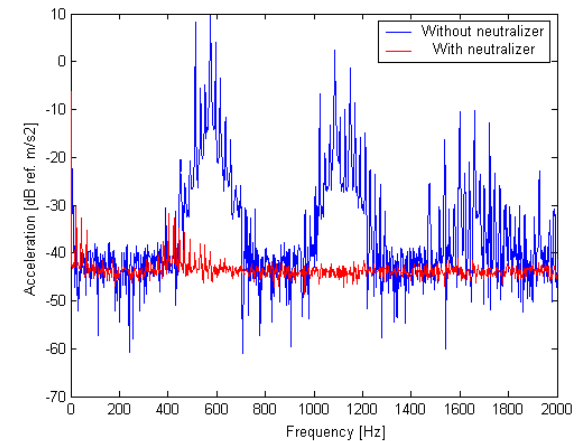
- Dynamic viscoelastic absorber
  - total additional mass ( 2,5% a 10%);
  - efficiently in a large frequency band;
  - allows more axial force on the transmission line;
  - then minor curve of the line;
  - Low towers.

# ◆ Instability-Chatter in Turning Processes

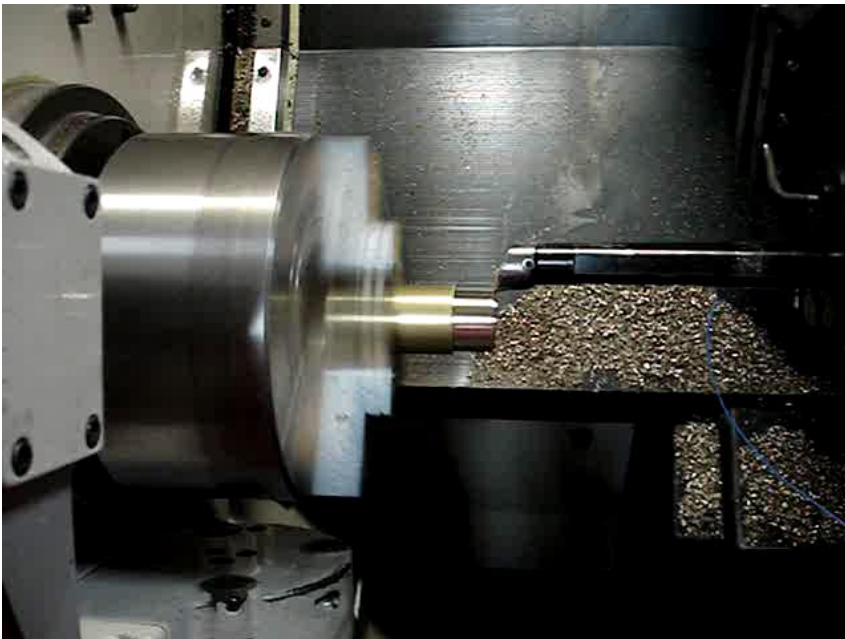
Set-up for the Tool Holder with Dynamic Absorber to eliminate Instability-Chatter in Turning Processes



Surface profile of the work piece machined



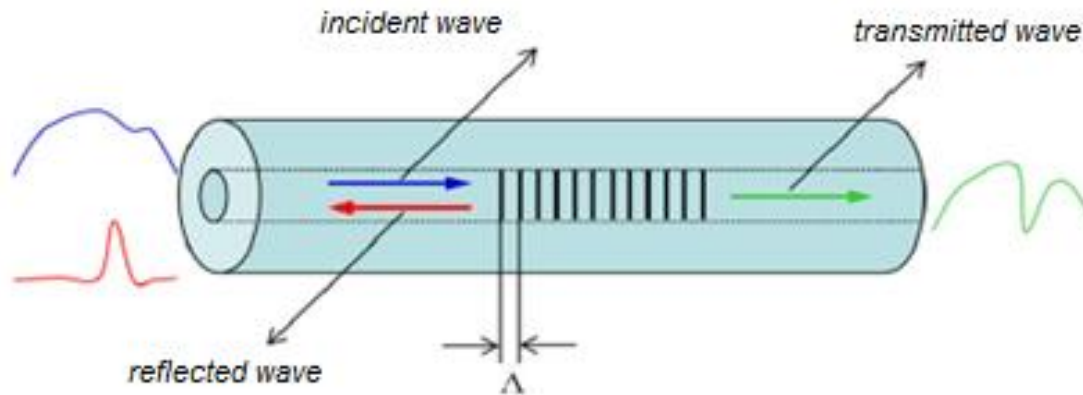
# ◆ Instability-Chatter in Turning Processes



Surface profile of the workpiece machined



◆ The Numerical example and experimental setup



A typical Fiber Bragg Grating (FBG) has a central wavelength reflected spectrum given by



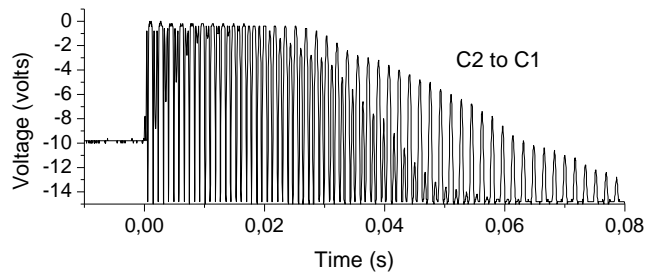
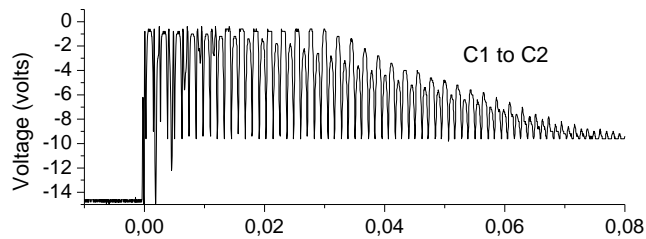
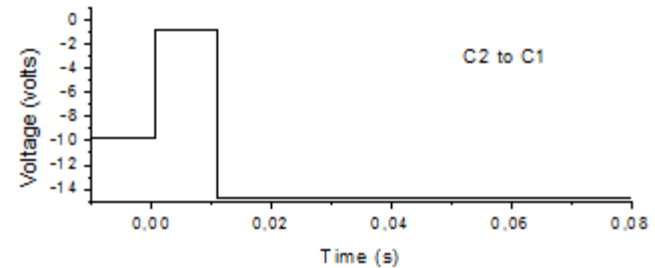
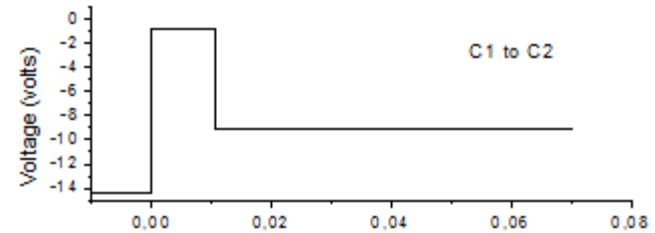
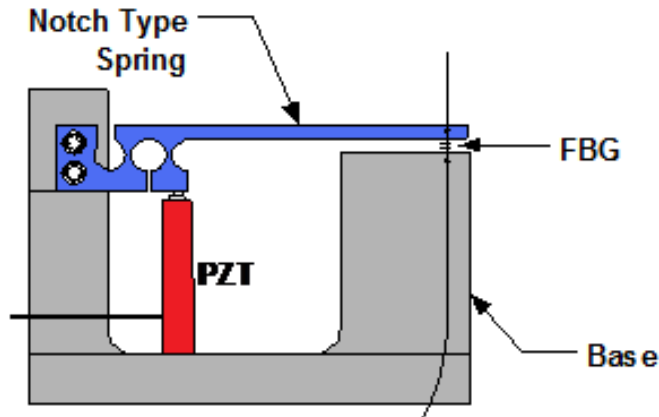
$$\lambda_B = 2n_{eff} \Lambda$$

Disregarding the variation of temperature  $\Delta T$ , which occurred in the present work, the change in Bragg wavelength due to application of a longitudinal strain is



$$\Delta\lambda_B = \lambda_B (1 - p_e) \epsilon_{xx}$$

◆ The Numerical example and experimental setup

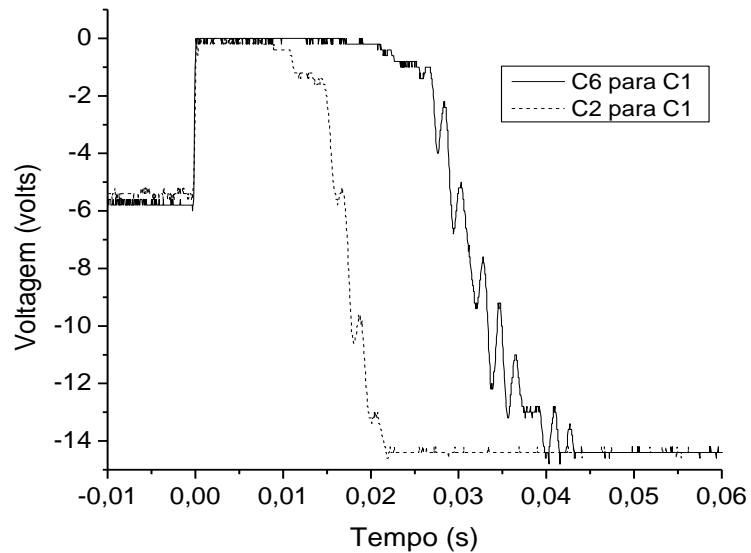
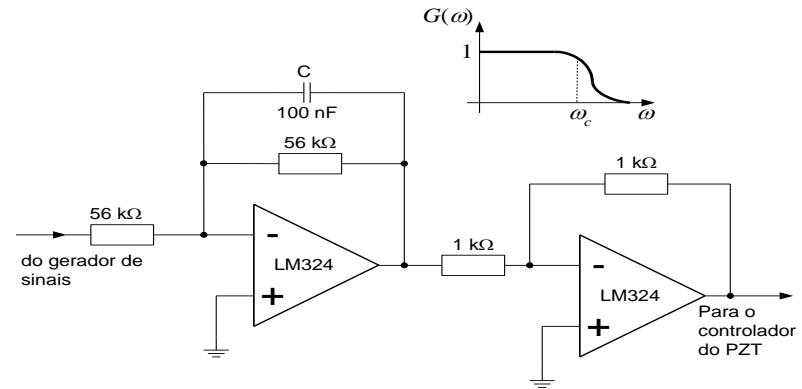


Tuning of channels 1 and 2.  
Ideal response and its  
experimental equivalent one



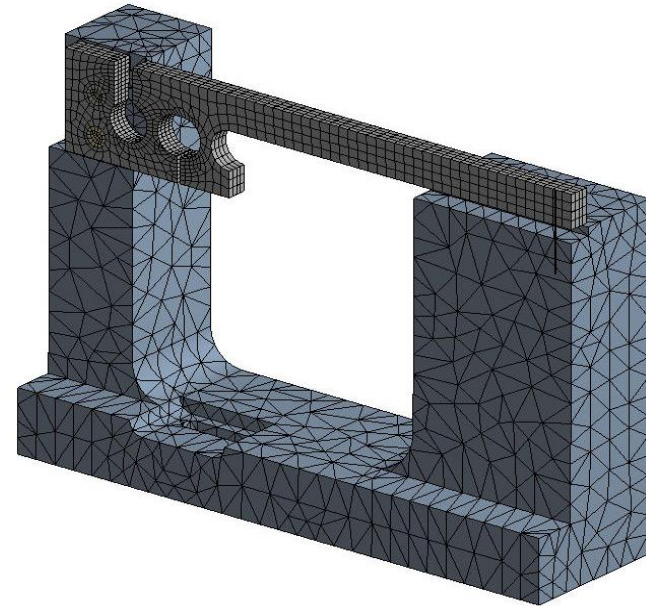
◆ The Numerical example and experimental setup

Electronic filter to reduce unwanted vibrations



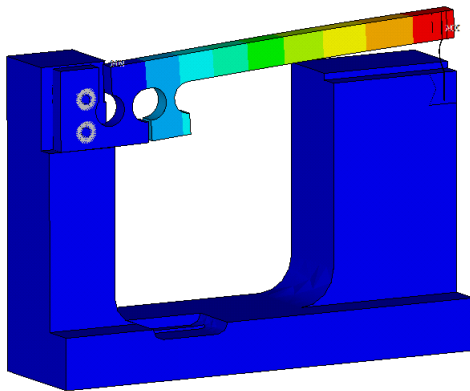
◆ The Numerical example and experimental setup

Finite Element Model

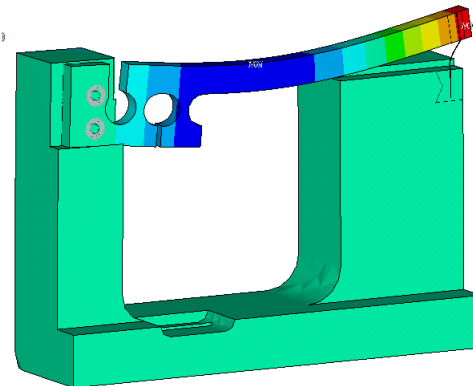


Modal Analysis of the dynamic system

```
NODAL SOLUTION
STEP=1
SUB =2
FREQ=480.627
U2 (AVG)
RSYS=0
DVK =6.262
SN1 =-1.45665
SN2 =6.191
```

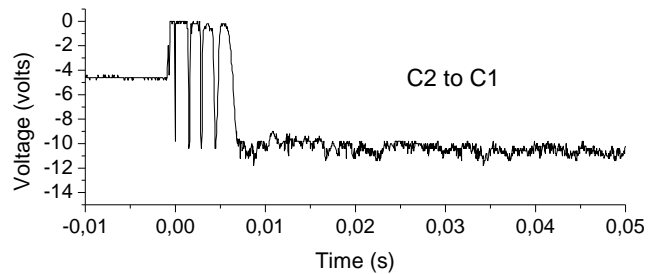
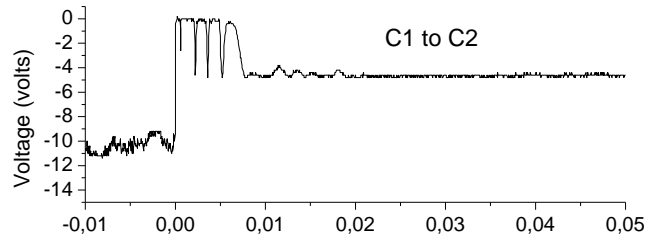
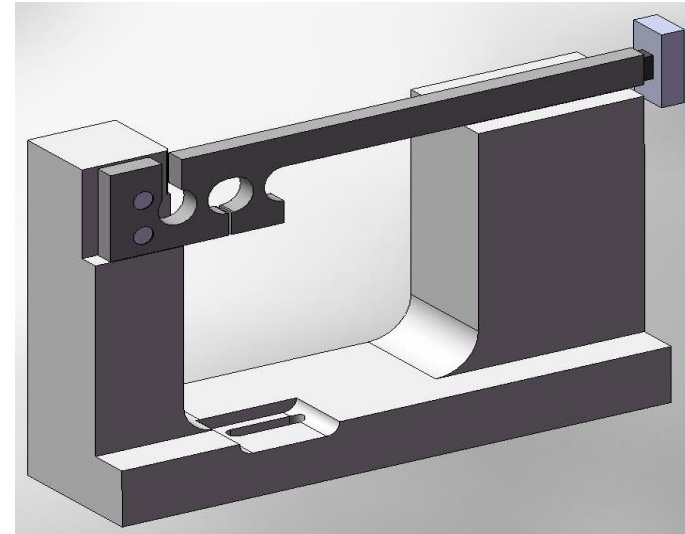


```
NODAL SOLUTION
STEP=1
SUB =7
FREQ=1652
U2 (AVG)
RSYS=0
DVK =4.519
SN1 =-2.209
SN2 =3.893
```

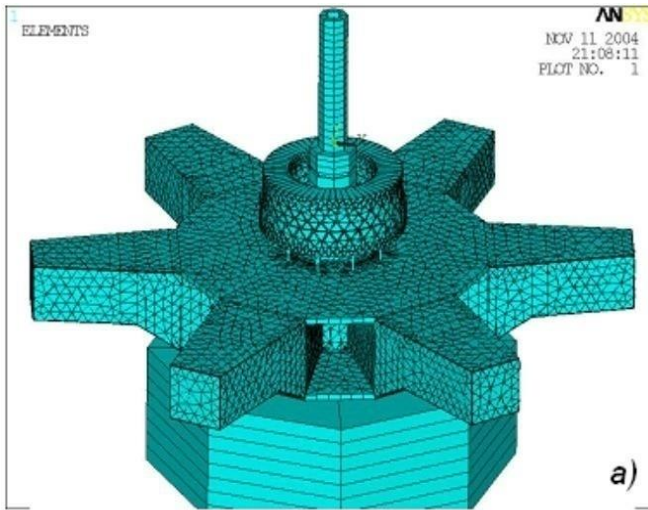


◆ The Numerical example and experimental setup

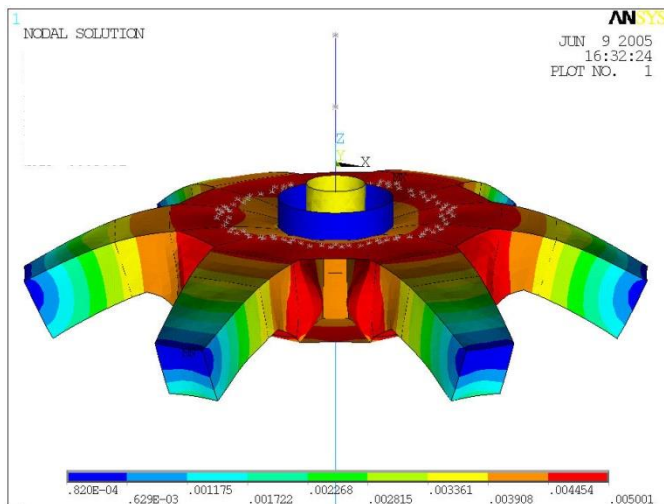
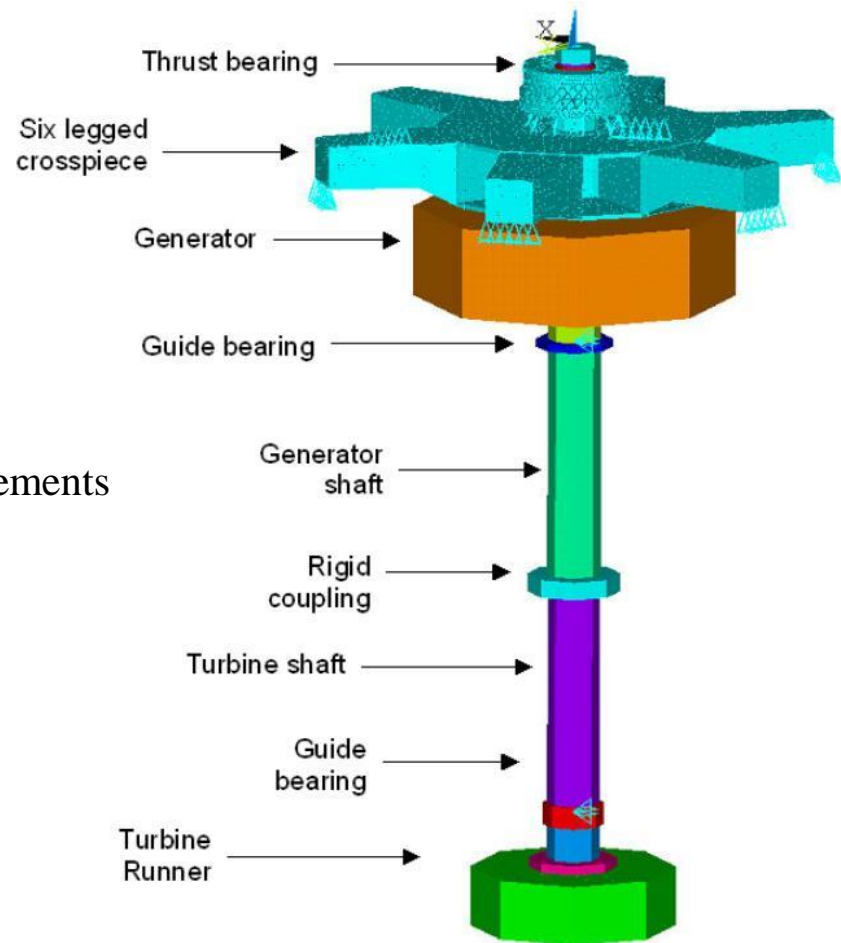
Optimal design of the dynamic viscoelastic neutralizer



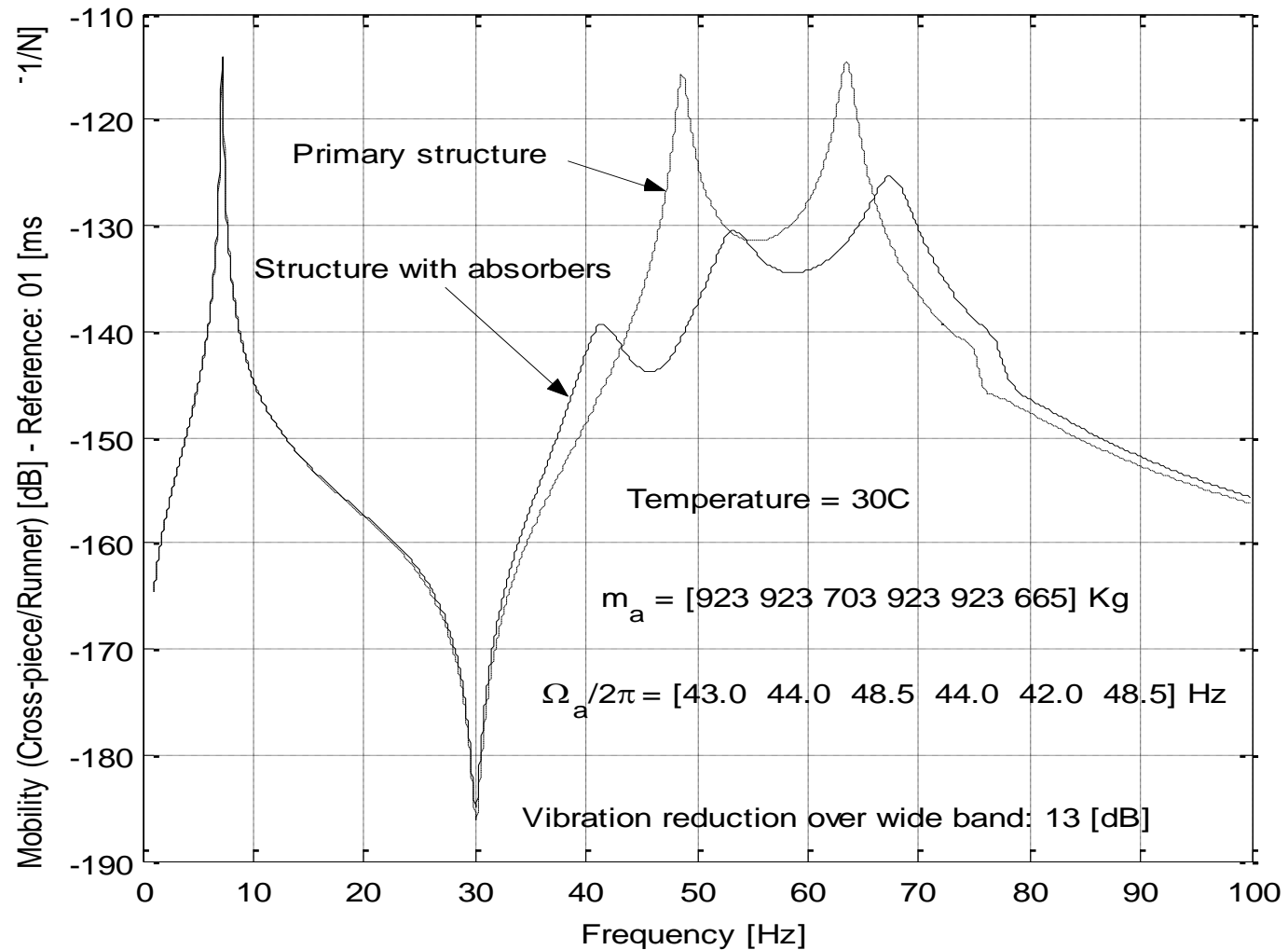
# ◆ FEM of Hydroelectric Group



**Level of axial vibration = 16 mm/s**



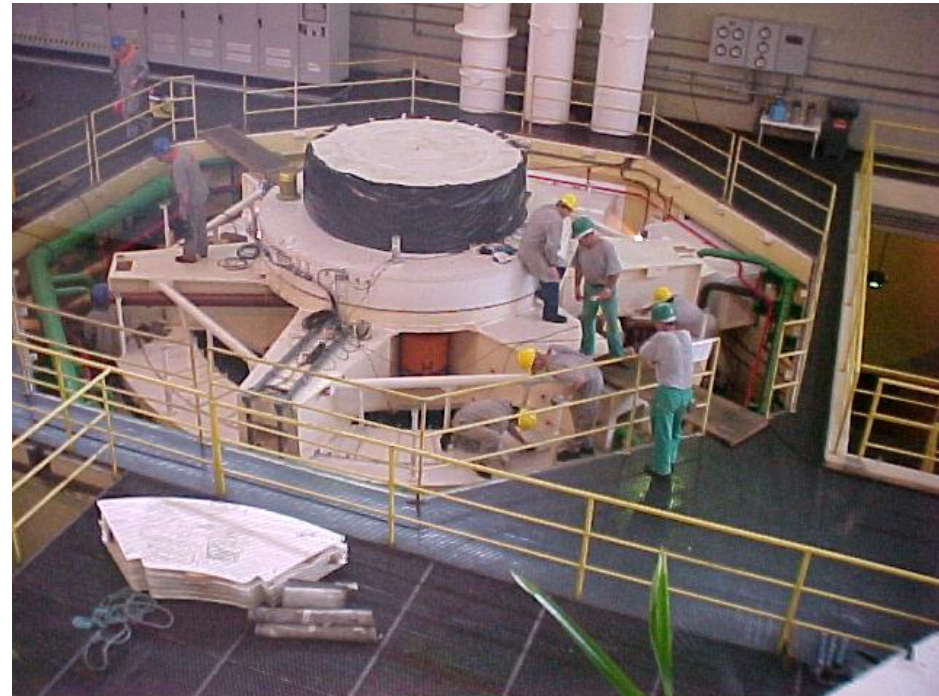
# ◆ FEM of Hydroelectric Group





## ◆ FEM of Hydroelectric Group

An Absorber in its Mounting Recess



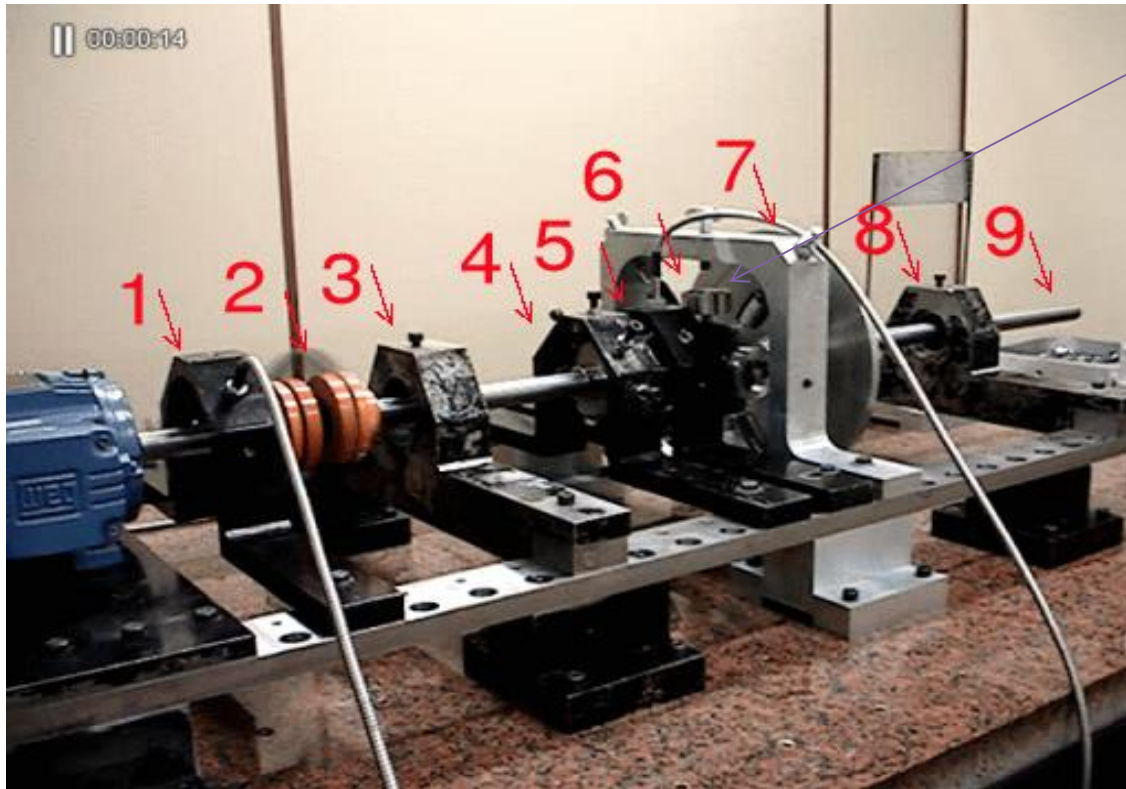
Panoramic View of the Six-Legged Crosspiece Showing an Dynamic Absorber Installed.

**After DVA, the level of axial vibration**

**= 2.5 mm/s**

◆ 8 - Rotating Systems

The rotor rig used in this work



Dynamic absorber

2, 4 e 5 – The steel disks

3 e 8 – The ball bearings

6 - The floating ball bearing

7 - The alloy disk

9 – The steel shaft

- Rotor Equations

In the frequency domain

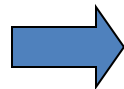
$$\left(-\Omega^2 M + i\Omega(C + G(\Omega_{rpm})) + K\right)Q(\Omega) = F(\Omega)$$

In term of the state variables

$$\left(i\Omega \begin{bmatrix} C_1 & M \\ M & 0 \end{bmatrix} + \begin{bmatrix} K & 0 \\ 0 & -M \end{bmatrix}\right)\{Y(\Omega)\} = \begin{Bmatrix} F(\Omega) \\ 0 \end{Bmatrix} \quad C_1 = C + G(\Omega_{rpm})$$

Considering the associated eigenvalue problem

$$B\theta = \lambda A\theta$$



$$B^T \psi = \lambda A^T \psi$$

Solving the whole system for all speed range, it is possible to obtain the Campbell diagram.

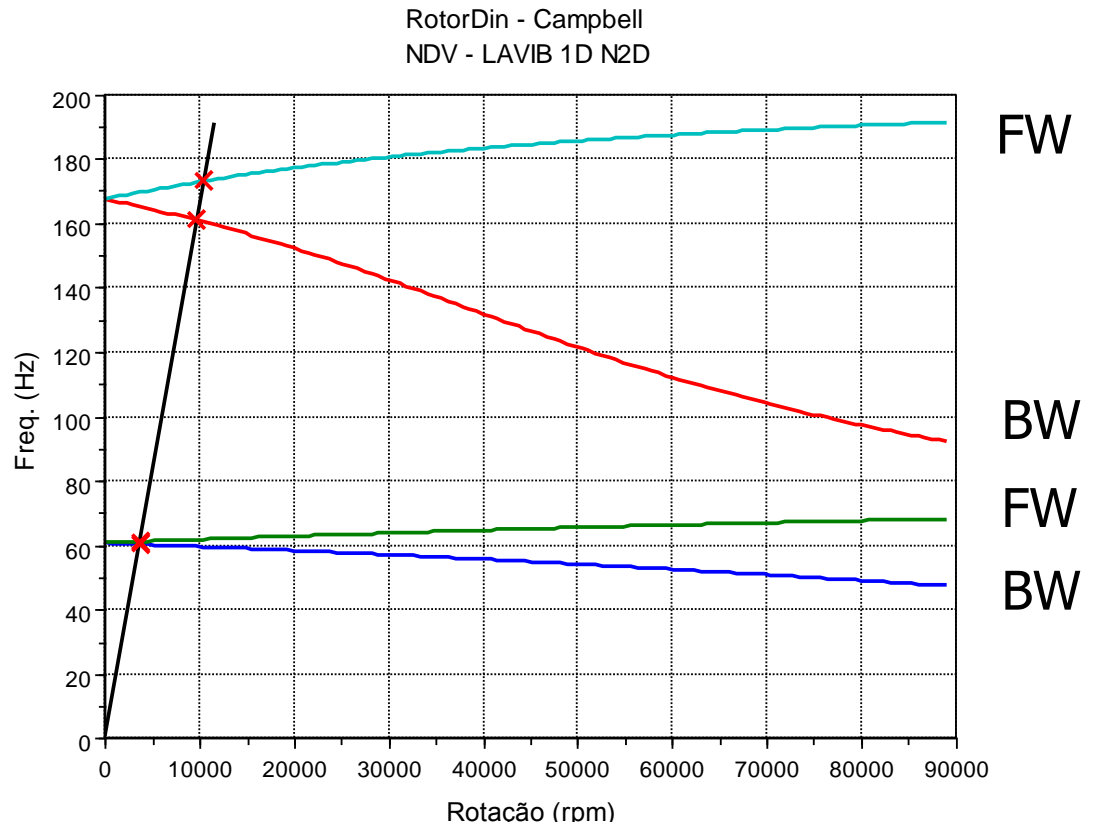


• Rotor Equations

Campbell diagram

$$C_1 = C + G(\Omega_{rpm})$$

$$\left[ \begin{array}{l} \Omega_{rpm} \\ B\theta = \lambda A\theta \\ B^T \psi = \lambda A^T \psi \end{array} \right.$$



- Rotating System with Dynamic Absorbers

Using the equivalent generalized concept

$$[\tilde{A}(\Omega)] = \begin{bmatrix} C_{eq}(\Omega) & M_{eq}(\Omega) \\ M_{eq}(\Omega) & 0 \end{bmatrix}$$

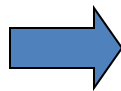
$$[\tilde{B}(\Omega)] = \begin{bmatrix} 0 & 0 \\ 0 & -M_{eq}(\Omega) \end{bmatrix}$$

$$M_{eq}(\Omega) = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ 0 & m_{eq_1}(\Omega) & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & m_{eq_p}(\Omega) & 0 \\ 0 & 0 & \dots & 0 & 0 \end{bmatrix}$$

$$C_{eq}(\Omega) = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ 0 & c_{eq_1}(\Omega) & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & c_{eq_p}(\Omega) & 0 \\ 0 & 0 & \dots & 0 & 0 \end{bmatrix}$$

$$\bar{A}(\Omega) = A + \tilde{A}(\Omega)$$

$$\bar{B}(\Omega) = B + \tilde{B}(\Omega)$$



■ For a given  $\Omega_{rpm}$

$$(i\Omega \bar{A}(\Omega) + \bar{B}(\Omega))Y(\Omega) = N(\Omega)$$

◆ The Response of the Compound System

Using the eigenvector of the matrix transformation

$$Y(\Omega) = \Theta P(\Omega)$$

It is possible to find the response in the modal space state

$$P(\Omega) = \left[ i\Omega(I + \Psi^T \tilde{A}\Theta) + (\Lambda + \Psi^T \tilde{B}\Theta) \right]^{-1} \Psi^T N(\Omega)$$

Then, the response  $q(t)$  can be obtained with a inverse Fourier transformation of the response in the space state using de transformation matrix = right eigenvector.

◆ Optimal Design of the Dynamic Absorbers

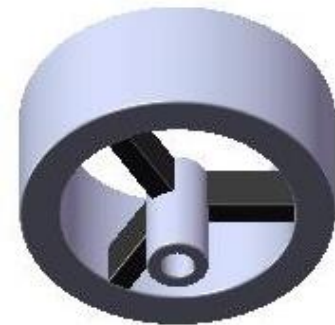
- The objective function is defined by

$$f_{cost}(x) = \left\| \max_{\Omega_1 < \Omega < \Omega_2} \left| \hat{P}(\Omega, x) \right| \right\|$$

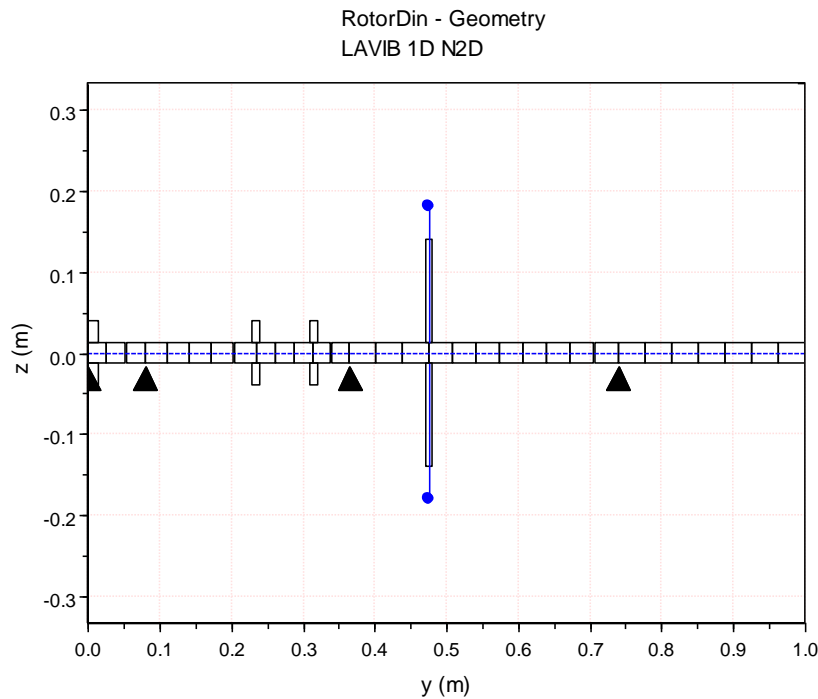
- where

$$x = (\Omega_{a1}, \Omega_{a2}, \dots, \Omega_{ap})$$

- After optimization procedure, the DVA's natural frequencies  $\Omega_n$  are known. Then, it is possible to do a physical realization.

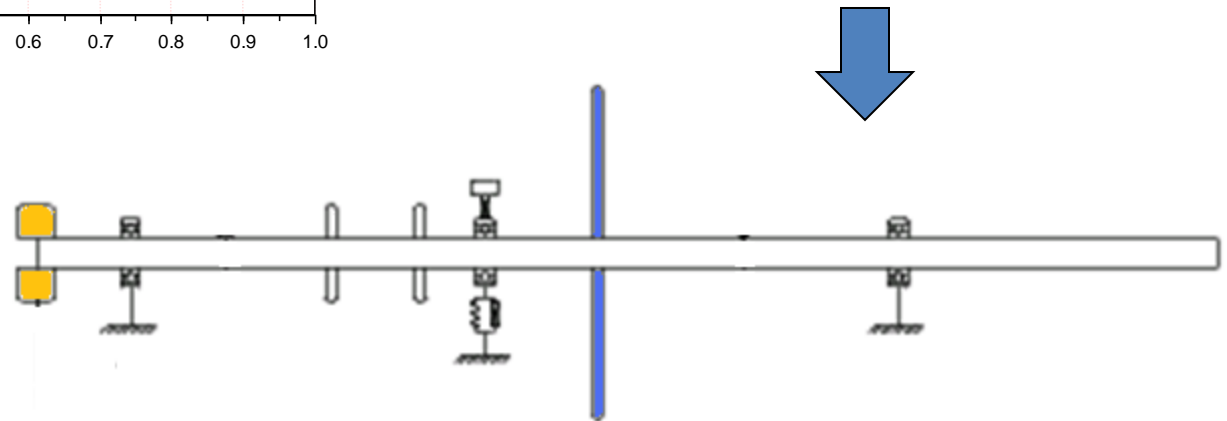


- Numerical Simulation

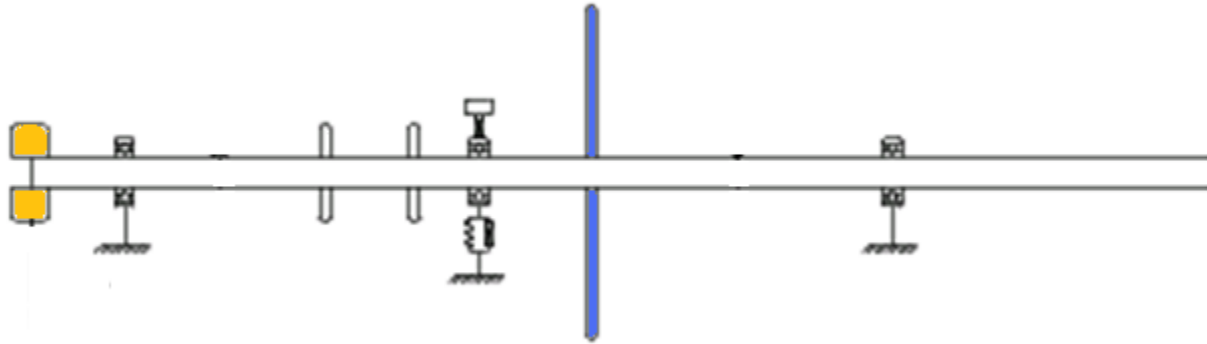


FEM model. Modal Analysis.  
Modal Parameters of the  
Rotor System

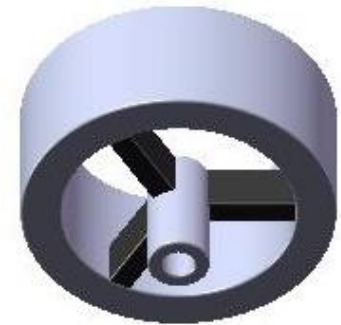
Compound System Model.  
Optimal Design of the  
Dynamic Absorber



- Numerical Simulation

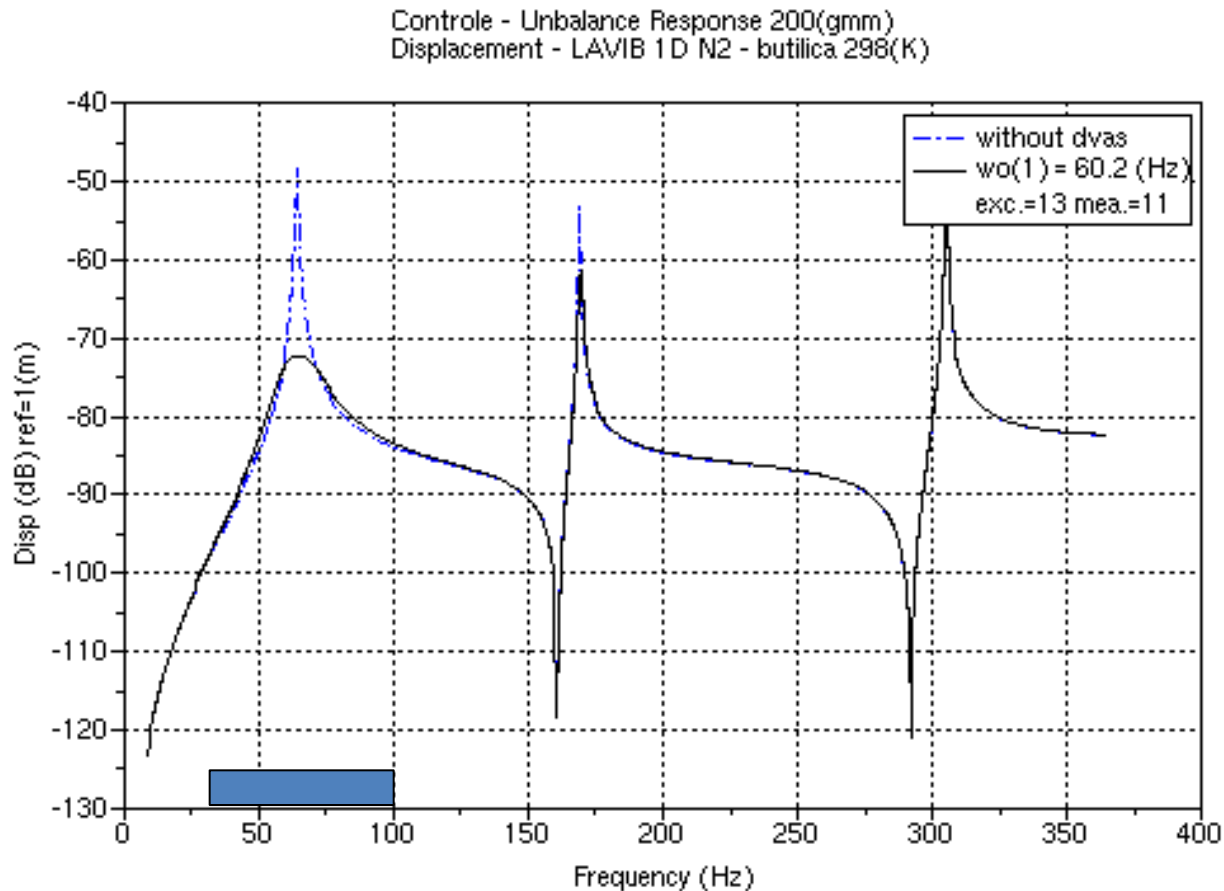


- After optimization procedure, the DVA's natural frequencies  $\Omega_n$  are known. Then, it is possible to do a physical realization.



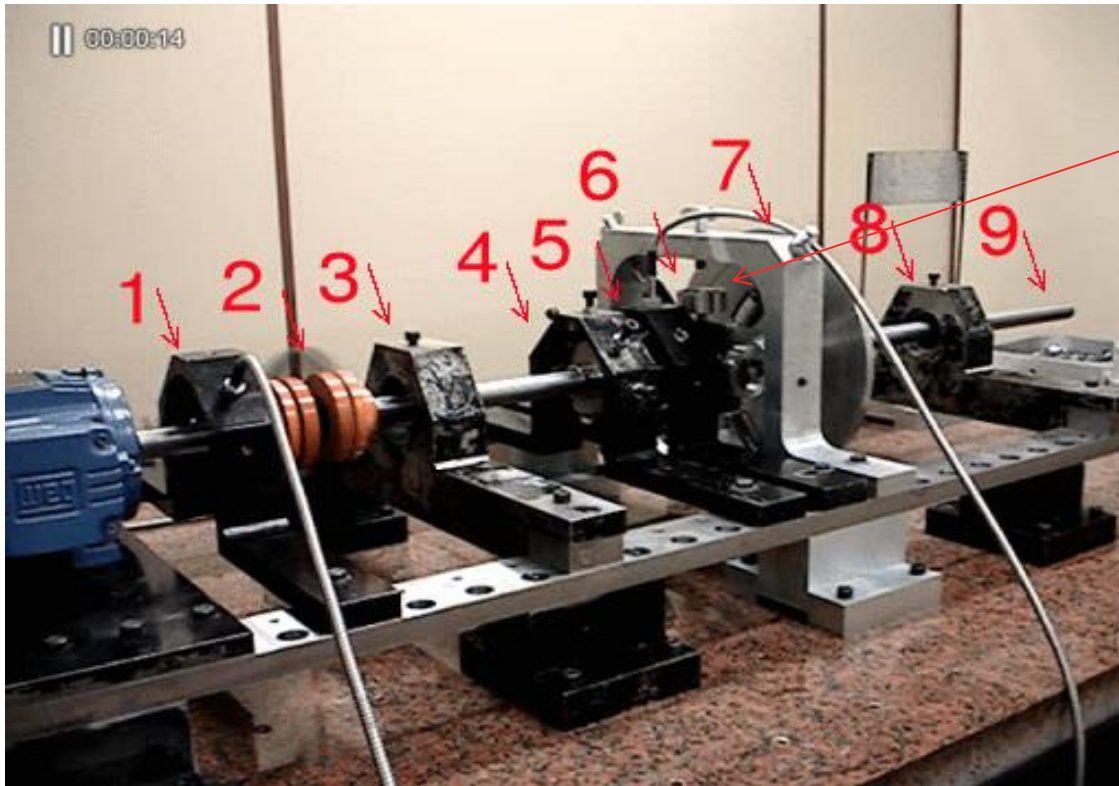
## ◆ Numerical Solution

Then the first 16 eigenvalues have been used. The barrier frequencies were 33 and 100 Hz and the DVA's initial natural frequency was adopted to be 59 Hz.



The optimal natural frequency of the four absorbers is  $\Omega_a = 60.2$  Hz.

## ◆ Experimental Setup



Dynamic absorber

2, 4 e 5 – The steel disks

3 e 8 – The ball bearings

6 - The floating ball bearing

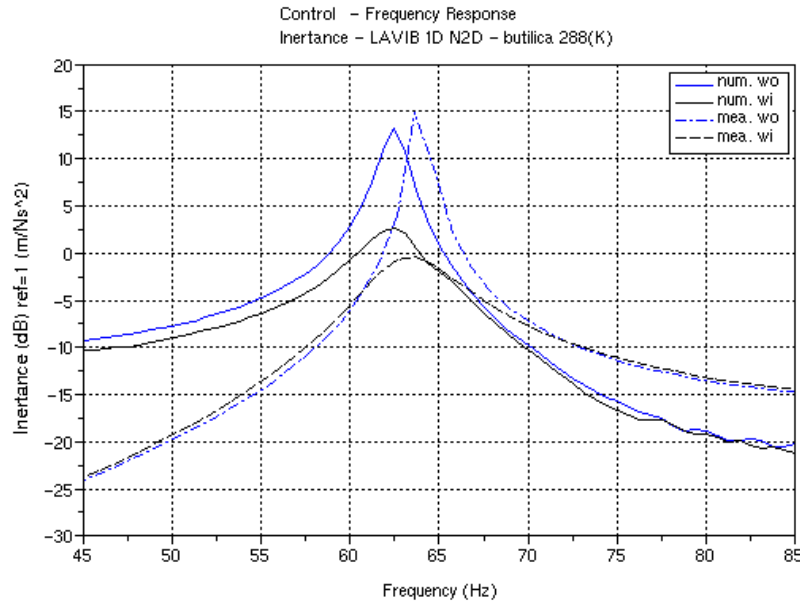
7 - The alloy disk

9 – The steel shaft

The rotor rig

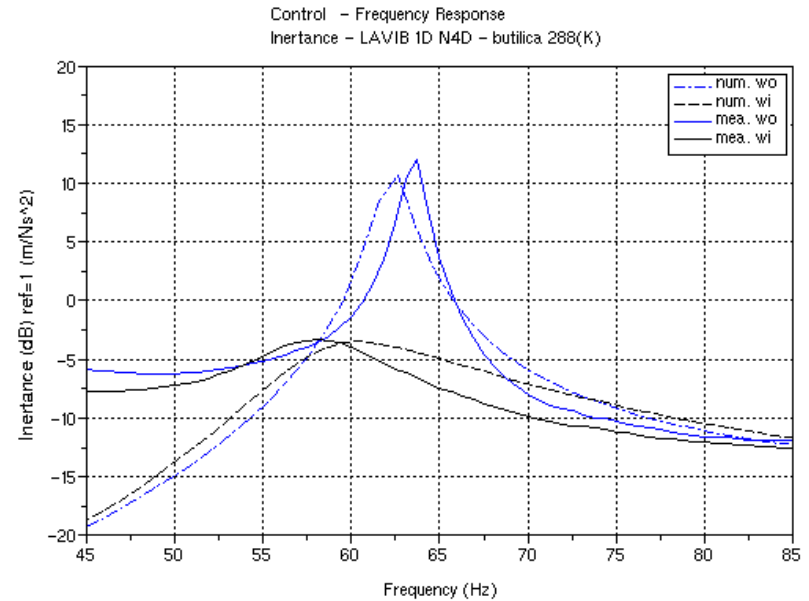


- Frequency Response Function



with 2 dynamic absorbers

mea = measured curves (mea)  
wi = with absorbers  
wo = without absorbers

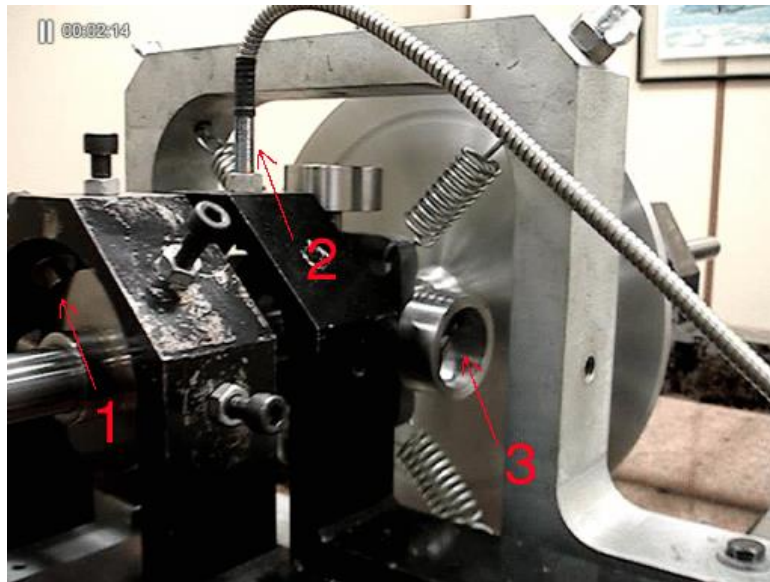


with 4 dynamic absorbers

num = numerical curves  
exp = experimental curves

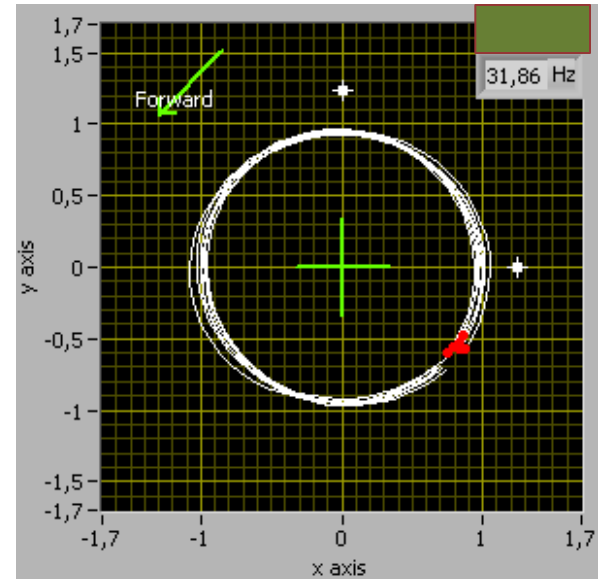
## ◆ Experimental Setup

2 - The dynamic orbit measuring has been done using a 90 (degrees) proximeters set

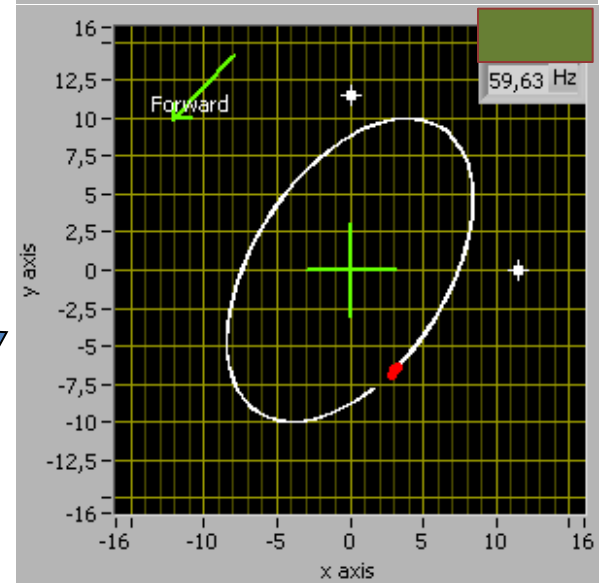


- 1 - Kind of excursion limiters
- 2 - Proximeter set
- 3 - Dynamic Viscoelastic Absorbers

Reference

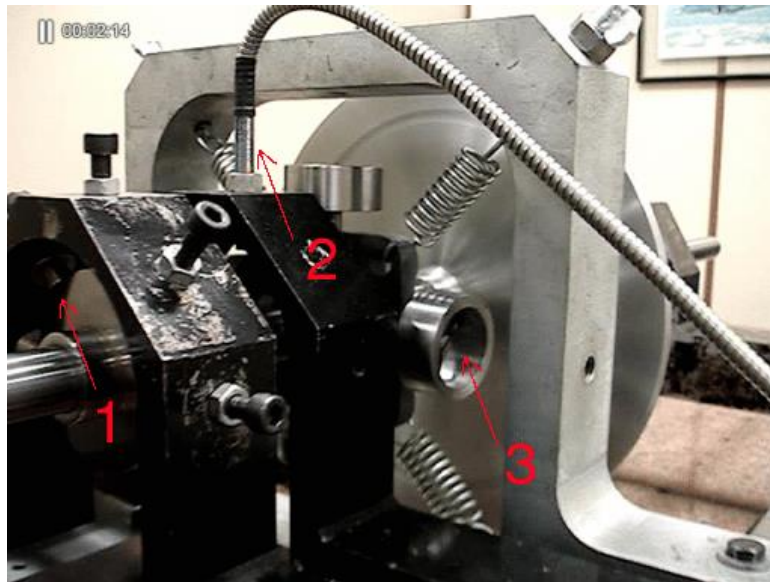


Without Absorbers



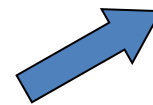
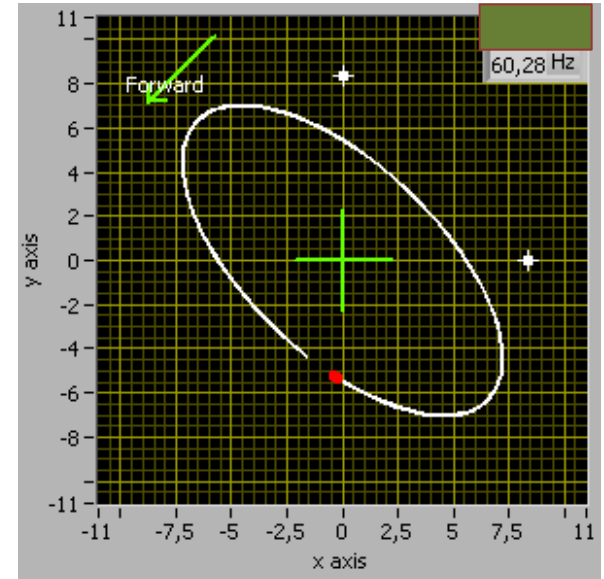
## ◆ Experimental Setup

2 - The dynamic orbit measuring has been done using a 90 (degrees) proximeters set

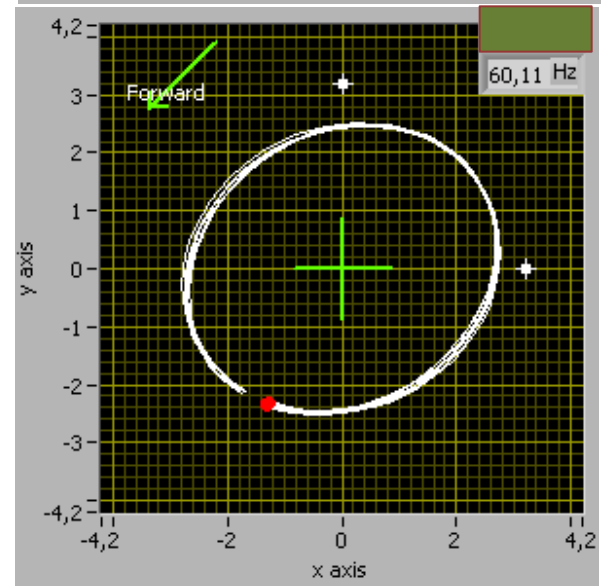


- 1 - Kind of excursion limiters
- 2 - Proximeter set
- 3 - Dynamic Viscoelastic Absorbers

2 Absorbers



With 4 Absorbers



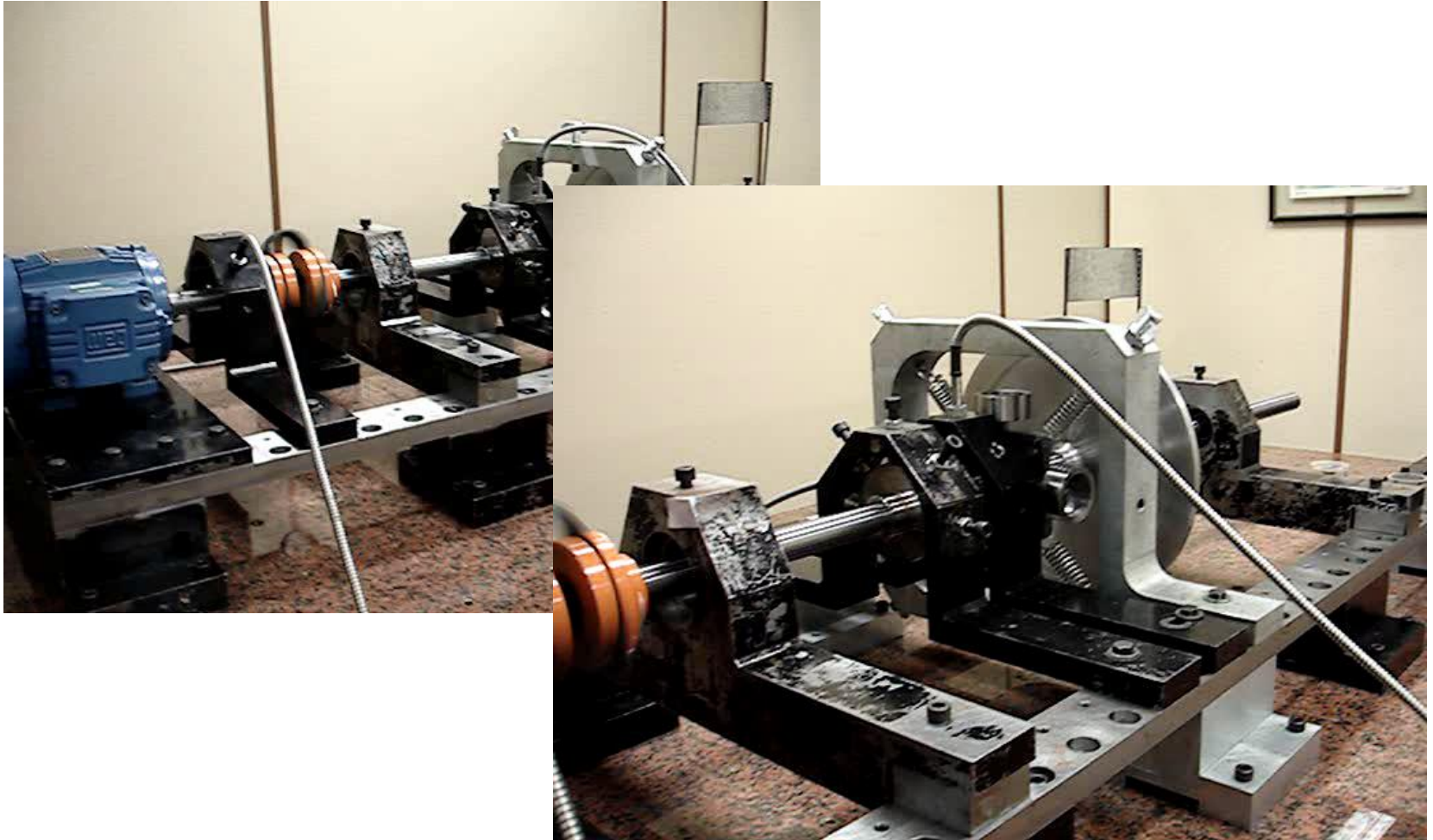
◆ Experimental Setup



the excursion limiters



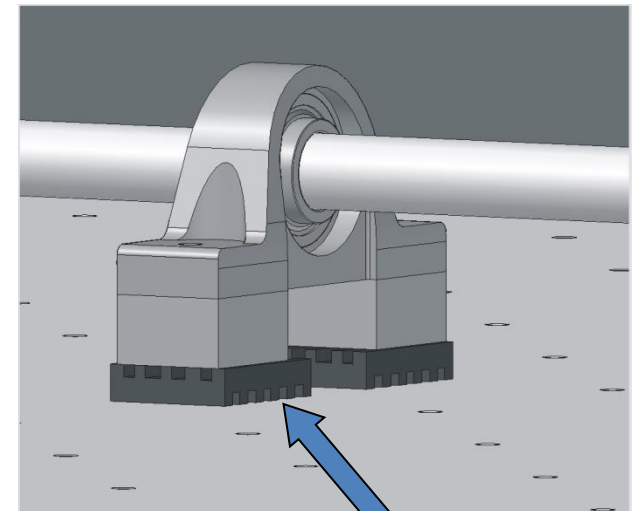
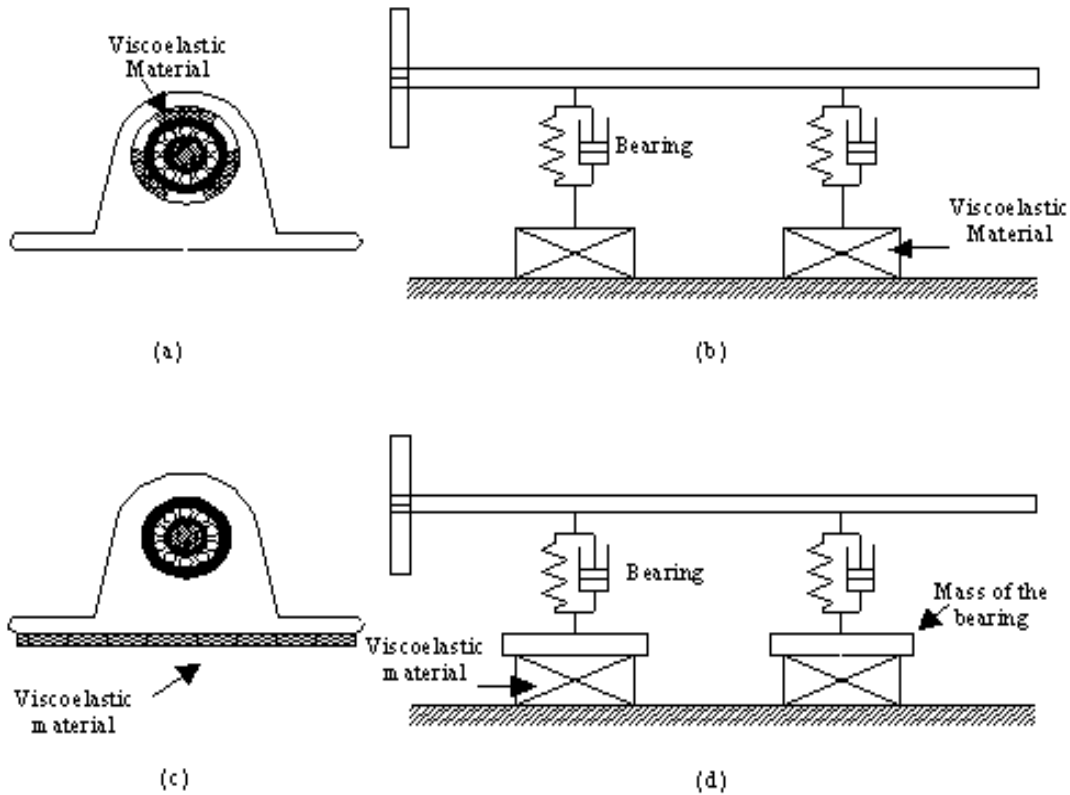
◆ Experimental Setup





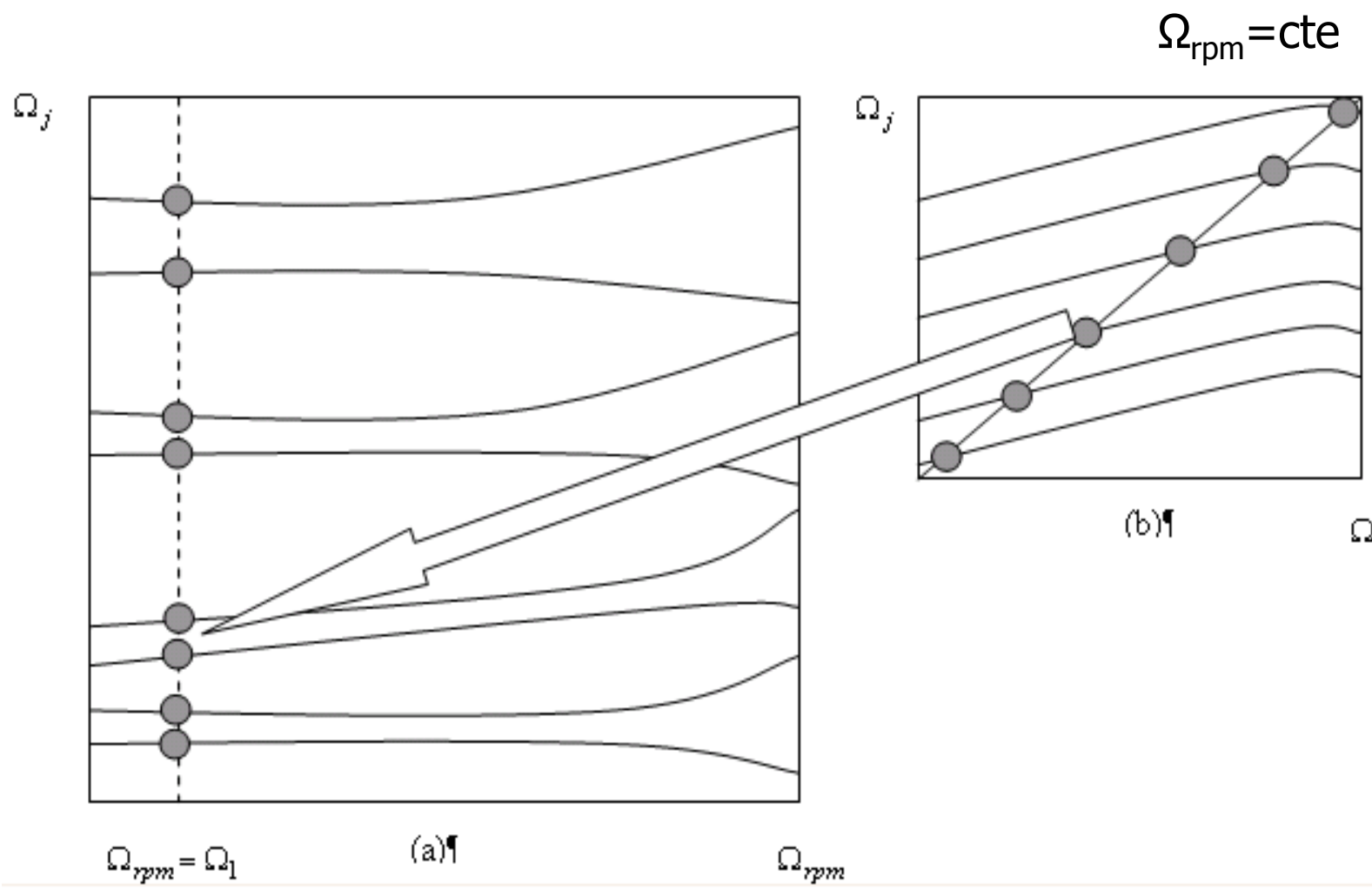
## ◆ Rotors with Flexible Bearings

The instability problems, when working at high rotations, can be solved by including damping in the bearings.



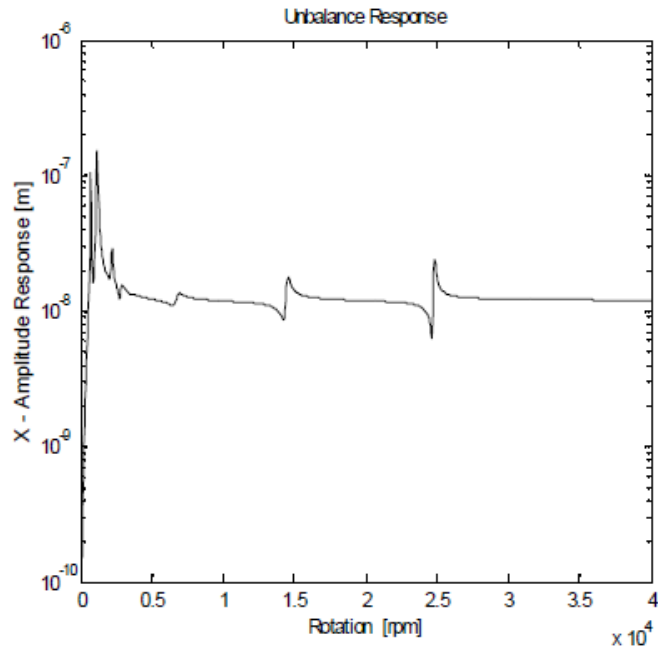
Viscoelastic Material

◆ Rotors with Flexible Bearings

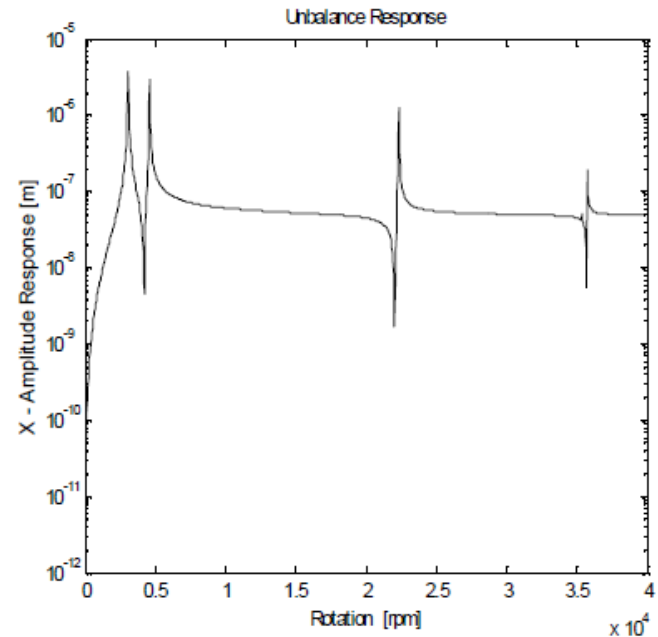


## ◆ Rotors with Flexible Bearings

The instability problems, when working at high rotations, can be solved by including damping in the bearings.



with flexible bearings



rigid ball bearings

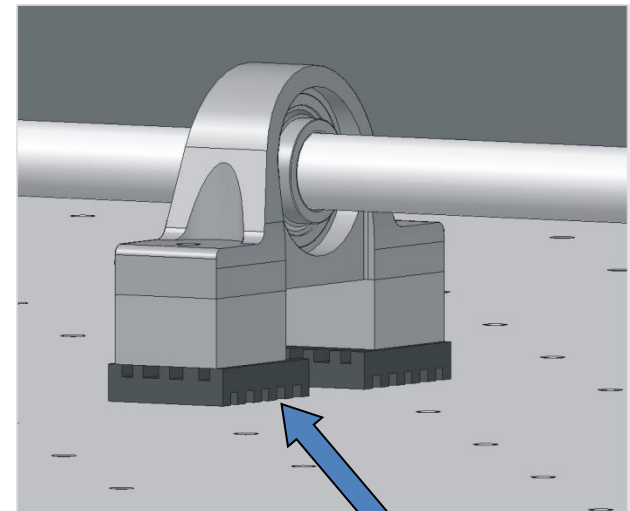
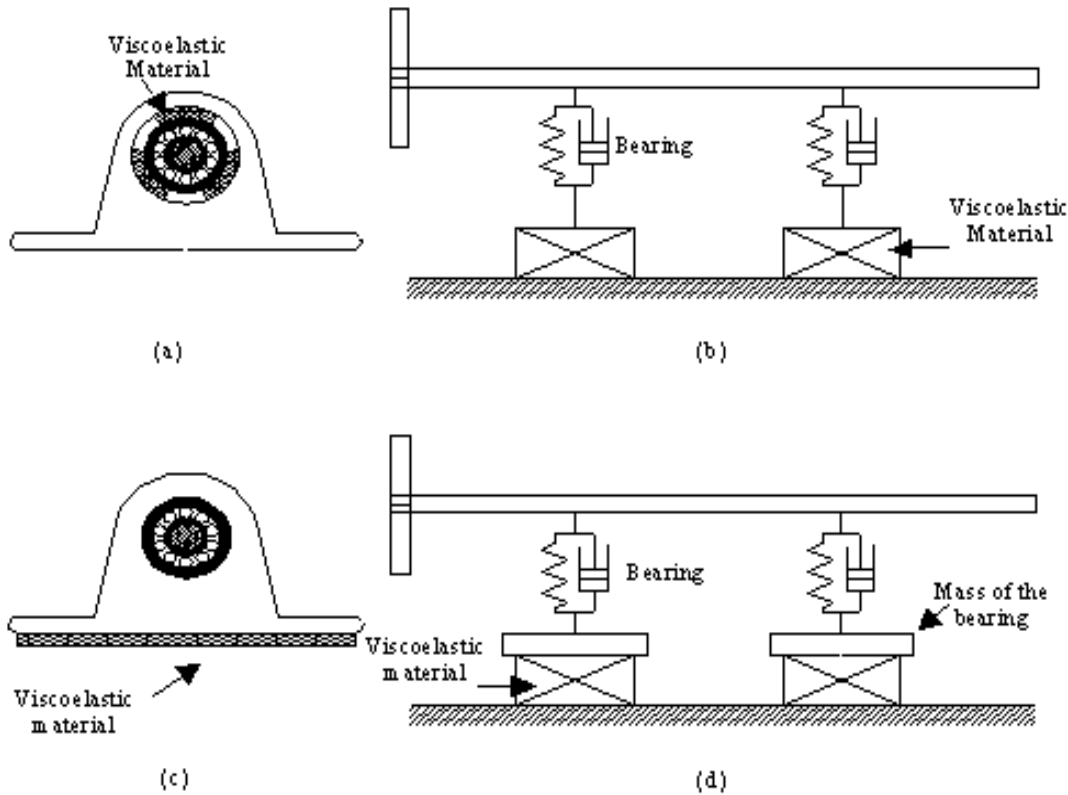


◆ Works in Progress – Laboratory of Sound and Vibration



◆ Rotors with Flexible Bearings

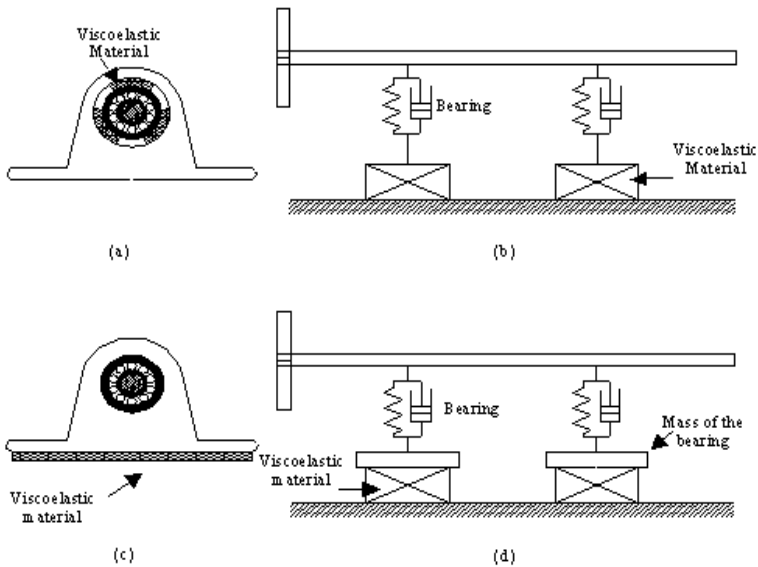
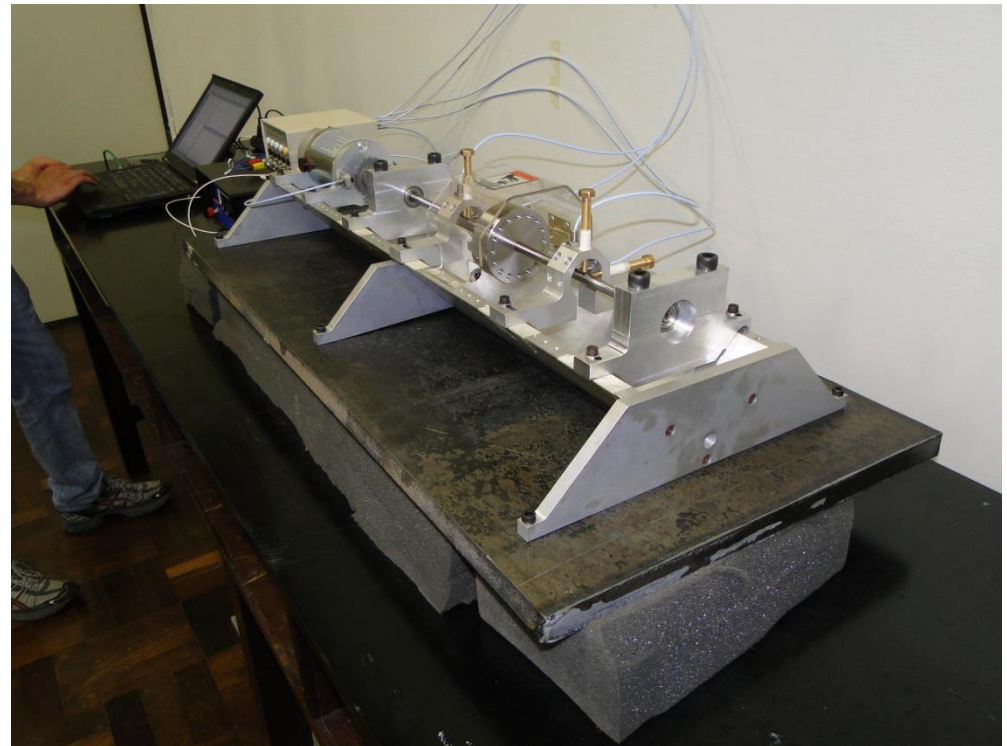
The instability problems, when working at high rotations, can be solved by including damping in the bearings.



Viscoelastic Material

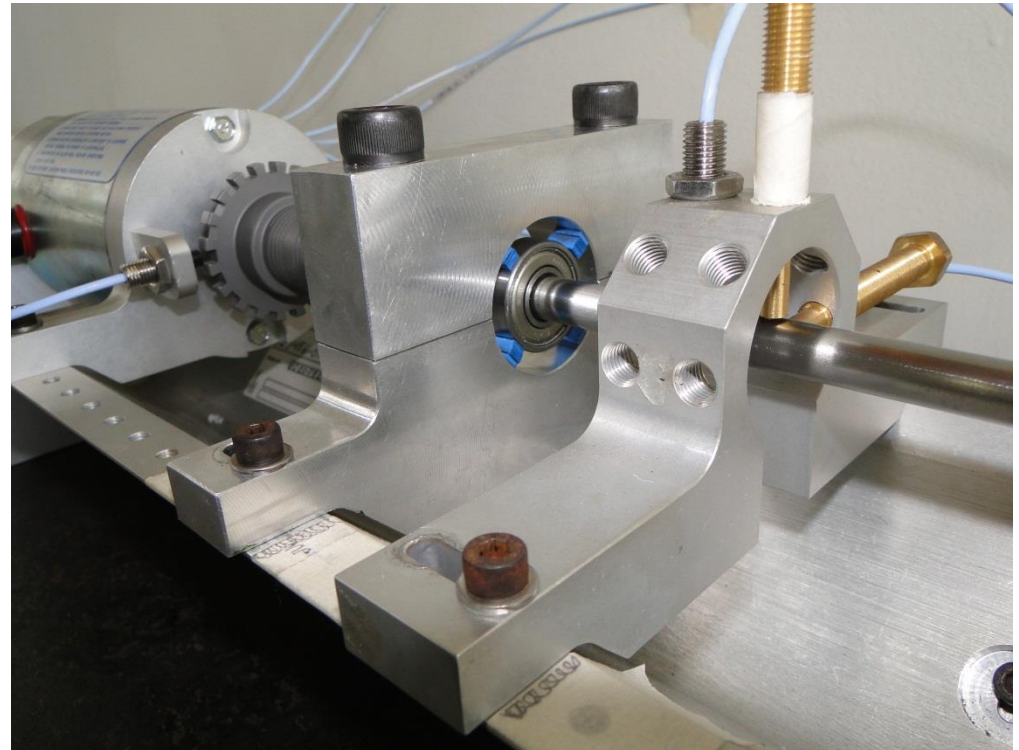
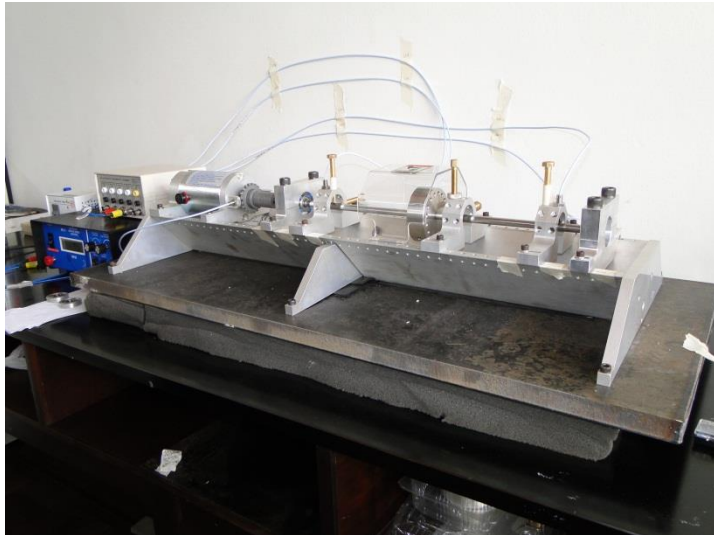
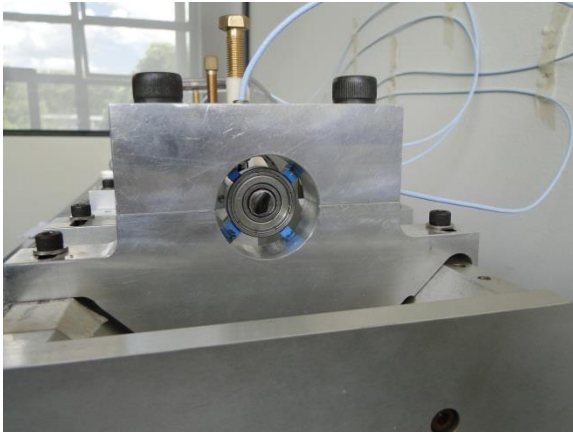
# ◆ Rotors with Flexible Bearings

The instability problems, when working at high rotations, can be solved by including damping in the bearings.

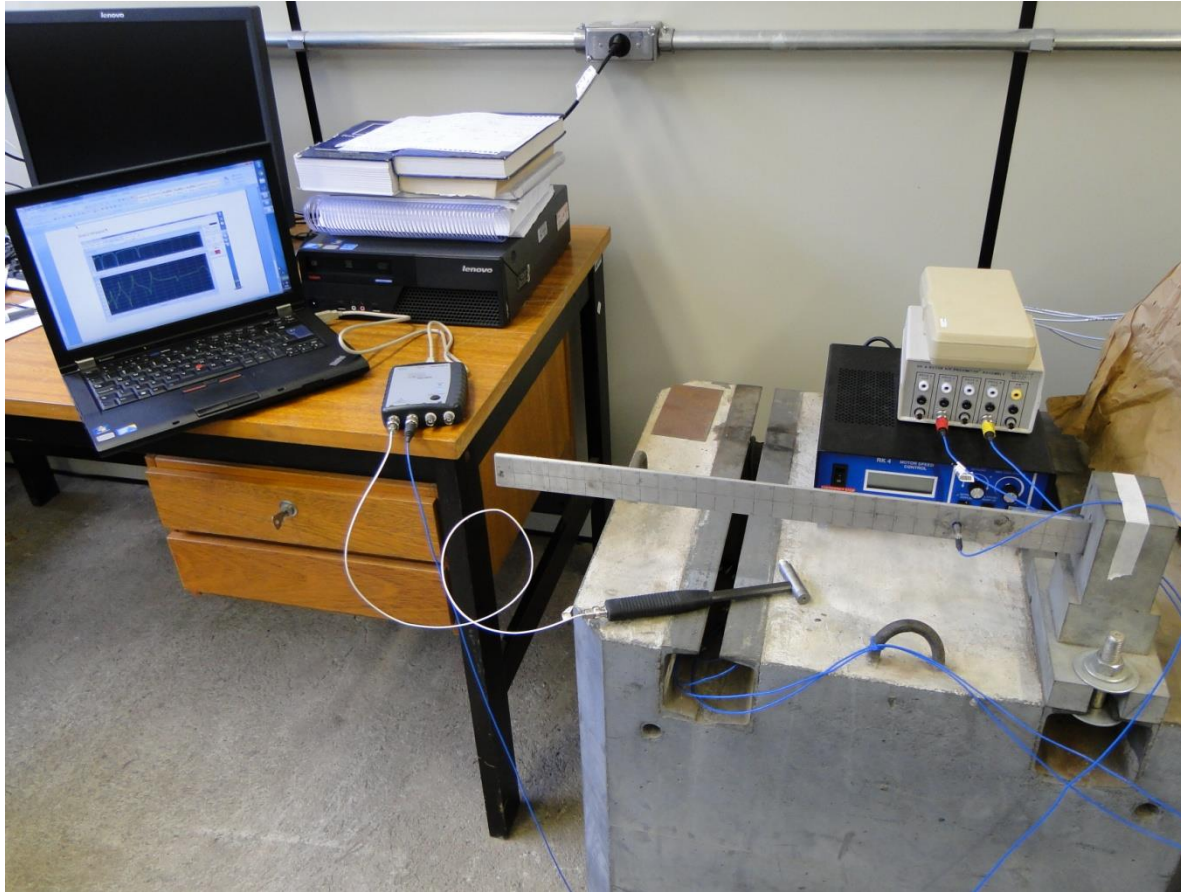




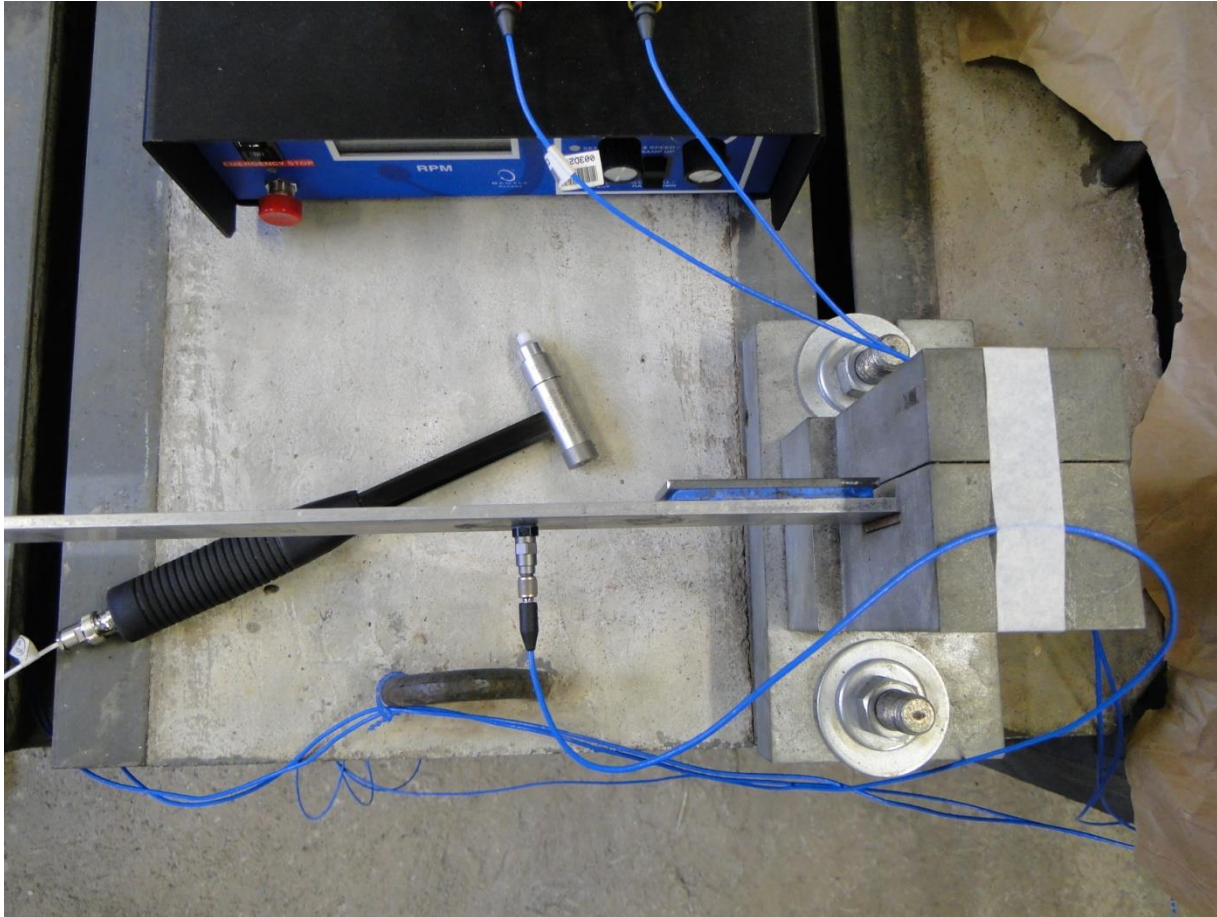
◆ Rotors with Flexible Bearings



◆ The Numerical example and experimental setup of Constrained Layers and Sandwich beams

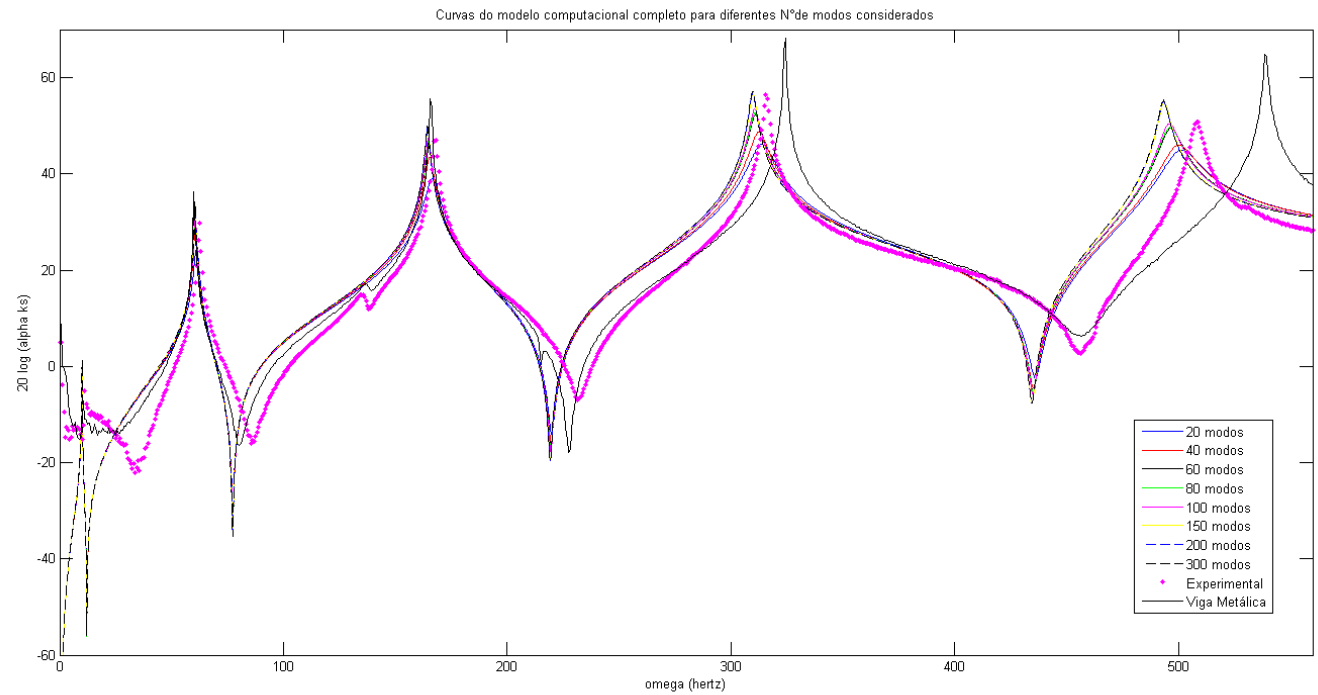
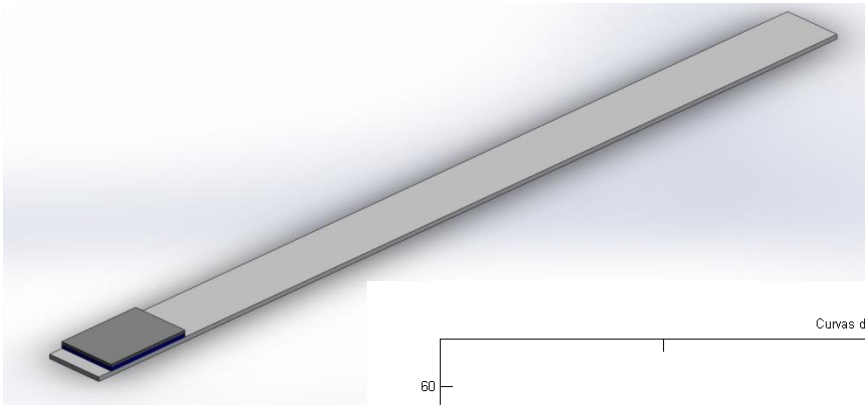


◆ The Numerical example and experimental setup of Constrained Layers



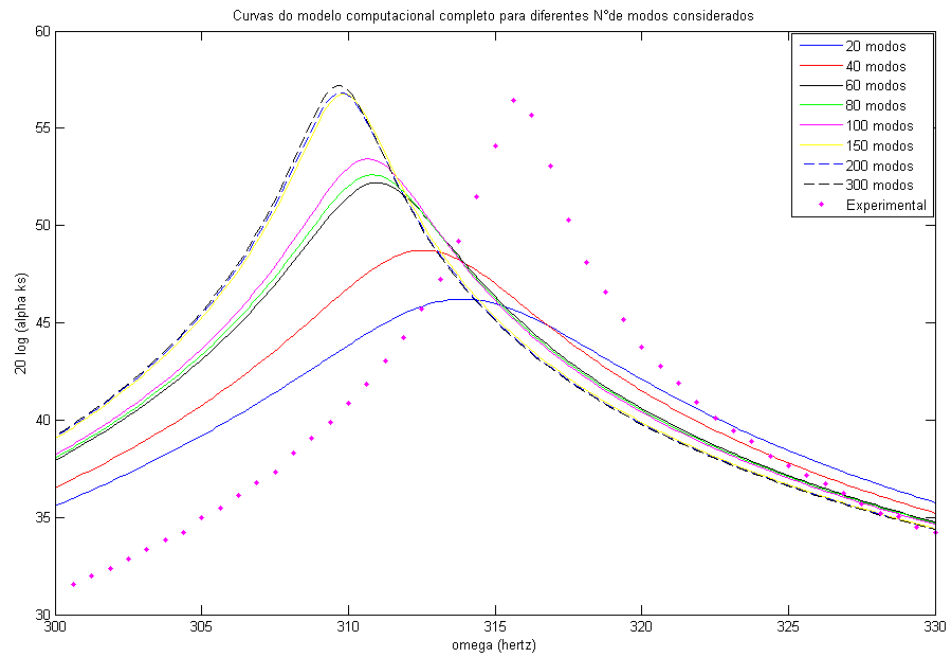


## ◆ The Numerical example and experimental setup of Constrained Layers



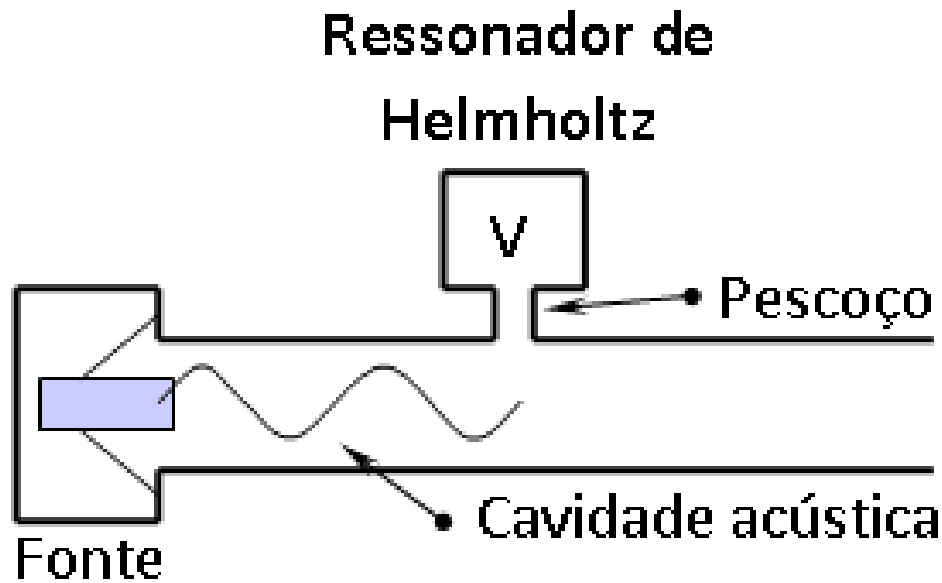
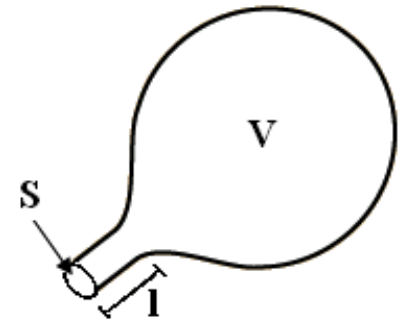
◆ The Numerical example and experimental setup of Constrained Layers

Zoom da FRF medida e obtida numericamente  
ao redor do quarto modo





◆ Helmholtz resonators



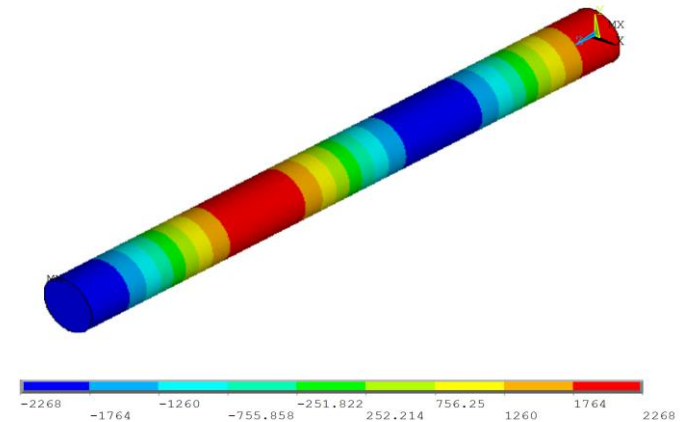
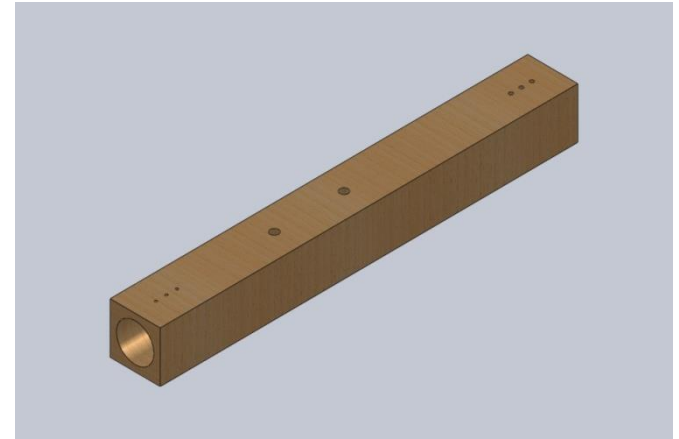
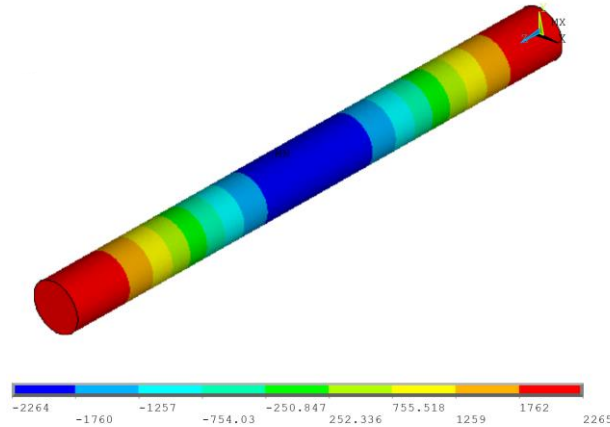
# ◆ Helmholtz resonators



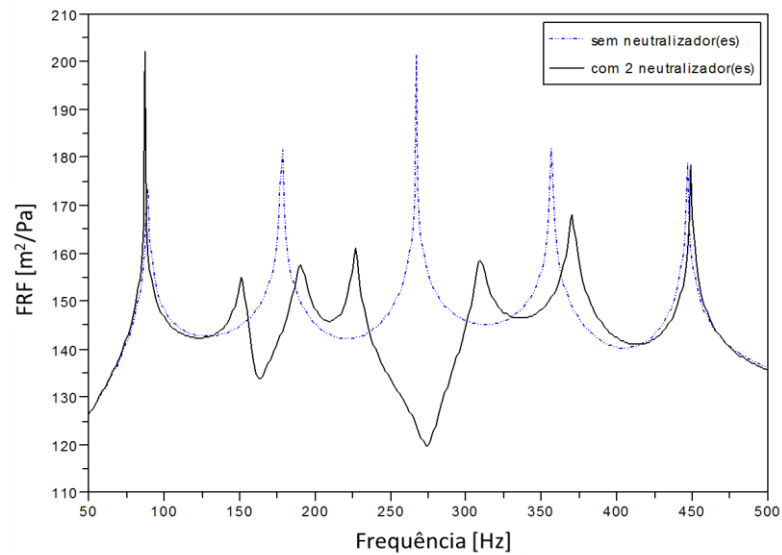
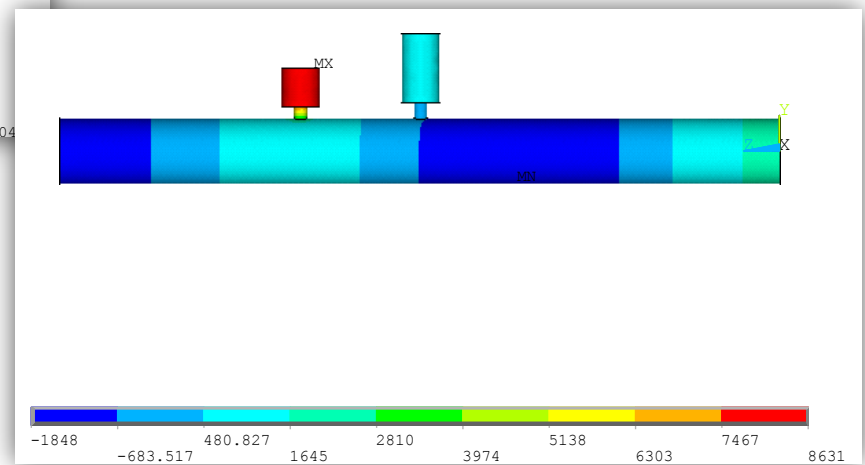
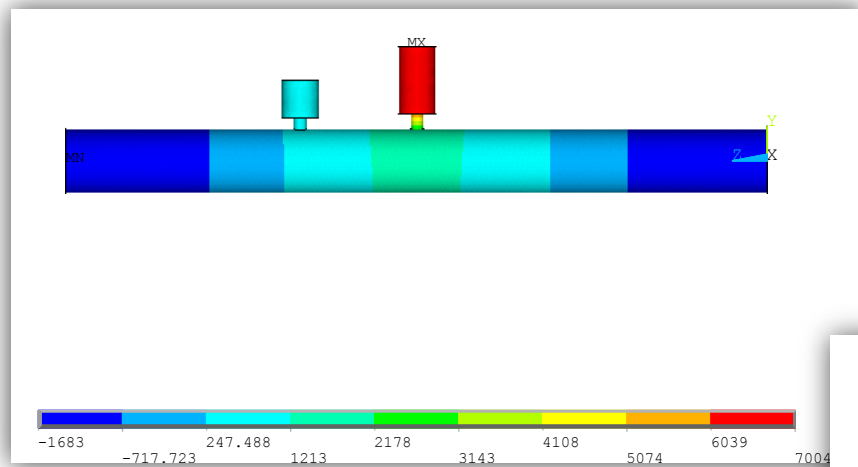
# ◆ Modal Analysis

## ○ Modal Parameters

Modo	Frequência [Hz]
1	88,8974
2	177,92
3	267,163
4	356,792
5	446,811
6	537,518
7	628,859
8	721,273
9	814,658
10	909,115

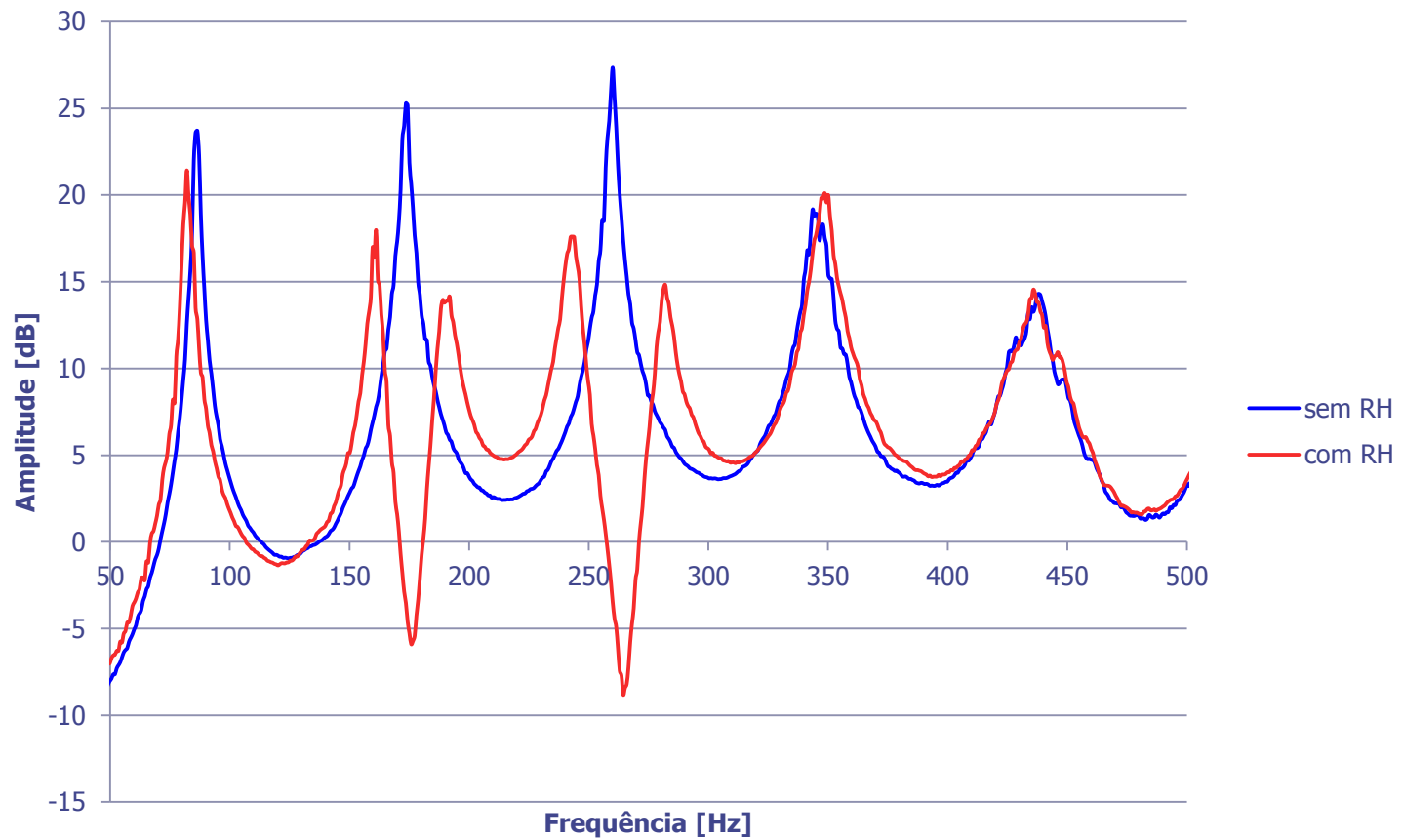


# Numerical simulations

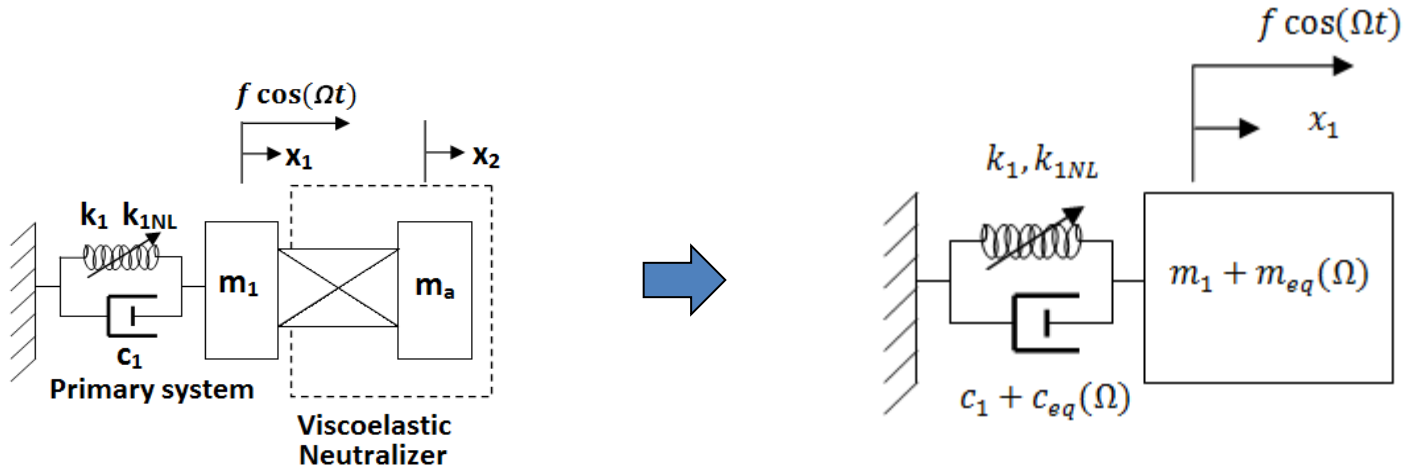


# ◆ Experimental Results

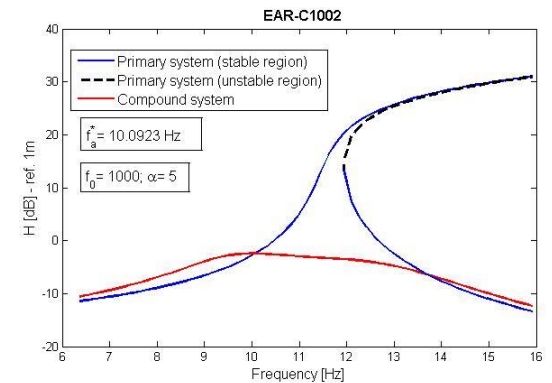
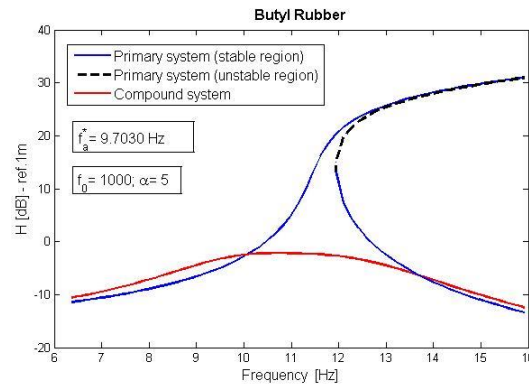
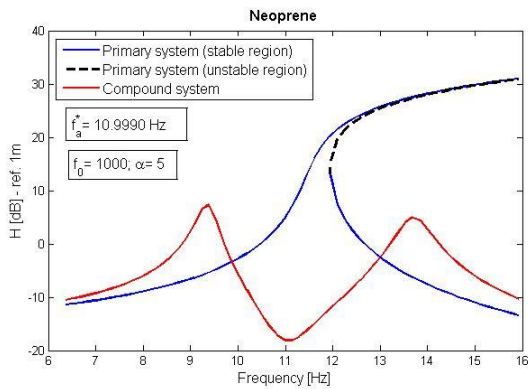
## Ruído Branco



# Optimal design of the viscoelastic neutralizer applied a non linear systems

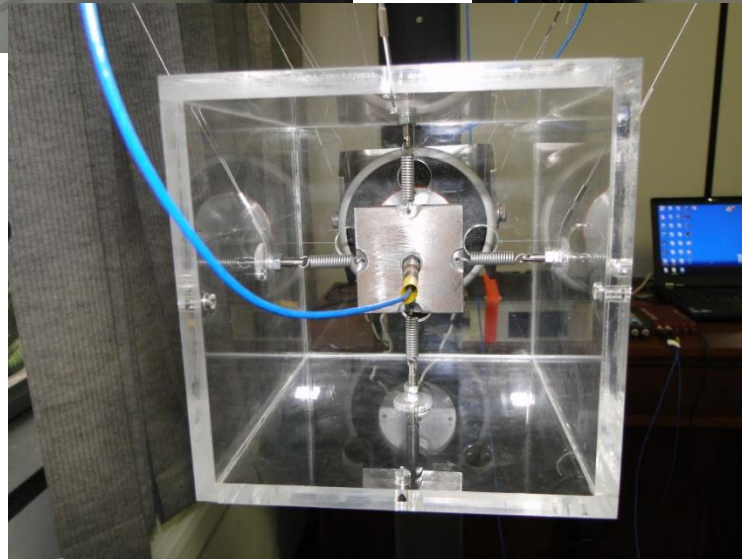
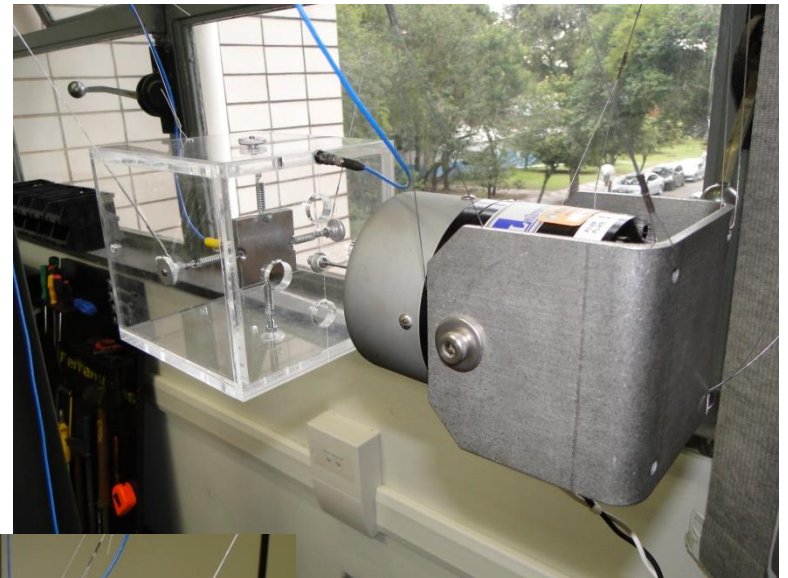
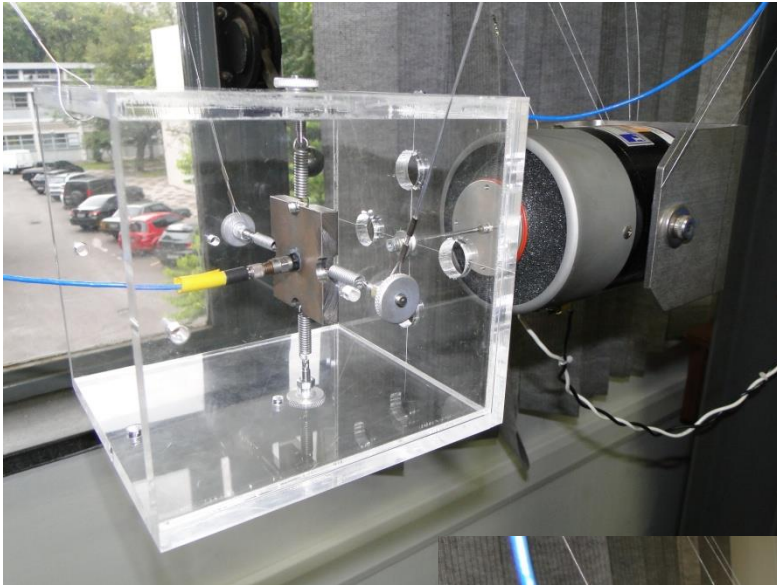


$$(m_1 + m_e(\Omega))\ddot{x}_1 + k_1 x_1 + k_{1NL} x_1^3 + (c_1 + c_e(\Omega))\dot{x}_1 = f \cos(\Omega t)$$





◆ Optimal design of the viscoelastic neutralizer applied a non linear systems

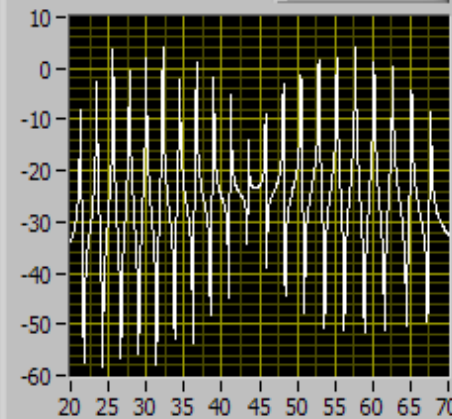


# Projeto ótimo dos neutralizadores:

Arquivos | Dados da estrutura | Parâmetros de cálculo | Dados dos neutral. | Mat. Viscoelástico | Calcular | Ajuda | Opções

Encerrar o programa **STOP**

FRF do sistema primário



Selecione o caminho para os arquivos de Dados de Entrada - Fortran  
C:\usu\bavastri\bavastri\_ant\bavastri\cefet\PPGEM\herbert\Herbert6\executaveis\Lavib\_Interface\exemplos

Selecione o caminho para os arquivos de Parâmetros modais  
20a70.eig

Selecione o caminho para o arquivo de Desenho da estrutura

Informar posição

Parâmetro de execução  
Otimização e Gráfico de FRF (PX0,FF0)

Iniciar novo projeto

Carregar dados anteriores

Graficar: Inertância

Gerar FRF

Automático

Ponto de excitação	Ponto de resposta
2	4

Automático

Limite inferior	Limite Superior
20	70



# Projeto ótimo dos neutralizadores:

Arquivos Dados da estrutura Parâmetros de cálculo Dados dos neutral. Mat. Viscoelástico Calcular Ajuda Opções

Origem dos parâmetros modais

ANSYS  
 ICATS

Número de graus de liberdade: 80  
Número de modos identificados: 21

Tipo de entrada dos modos:  
 Normal

Analisar arquivo de parâmetros modais

Tipo de Função Objetivo (TIPOSAI)  
Delta dirac em um ponto

Ponto j de H (i,j): 4  
Ponto i de H (i,j): 0

Tipo do gráfico de resposta (TIPOSAII)  
resposta no ponto px0 a um delta de dirac no ponto pf0

Ponto de excitação para graficar: 3  
Ponto de resposta para graficar: 2

Graficar: Inertância

Gerar FRF

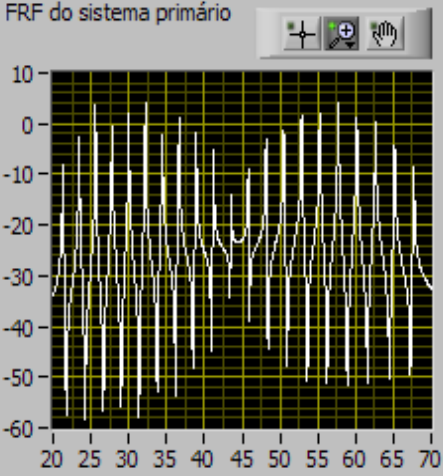
Automático  Inertância

Ponto de excitação: 2  
Ponto de resposta: 4

Automático  Limite inferior: 20  
Limite Superior: 70

Encerrar o programa **STOP**

FRF do sistema primário



The plot shows a highly oscillatory signal with a primary peak around 30 Hz and a secondary peak around 45 Hz. The y-axis ranges from -60 to 10, and the x-axis ranges from 20 to 70.

# Projeto ótimo dos neutralizadores:

Arquivos | Dados da estrutura | Parâmetros de cálculo | Dados dos neutral. | Mat. Viscoelástico | Calcular | Ajuda | Opções

Encerrar o programa **STOP**

Selecione as TONL a serem usadas:  
Algoritmo Genético + Quase Newton

Parâmetros do algoritmo genético

Número de indivíduos	Porcentagem de Crossover
100	50%
Número de gerações	Porcentagem de mutação
5	1%

Parâmetros do Quase Newton

Critério de parada	Fator de ponderação
1E-7	0,002

Faixa a analisar:      Faixa a graficar:

Frequência Inferior	Frequência Inferior
22	10
Frequência Superior	Frequência Superior
66	80
Discretização	Discretização
400	2000

Selecione os modos a controlar:

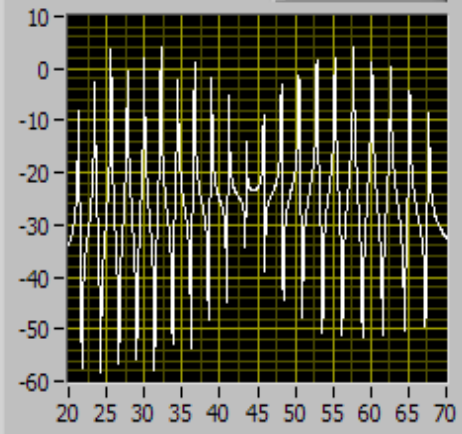
Modo	Relação de massas
1º Modo	0
2º Modo	0,1
3º Modo	0,1
4º Modo	0,1
5º Modo	0,1
6º Modo	0,1
7º Modo	0,1
8º Modo	0,1
9º Modo	0,1
10º Modo	0,1
11º Modo	0,1
12º Modo	0,1
13º Modo	0,1
14º Modo	0,1
15º Modo	0,1
16º Modo	0,1
17º Modo	0,1
18º Modo	0,1

Relação de massas

Selecione faixa  
Selecione todos  
Selecione nenhum

Tipo de relação  
Diferentes  
Valor da relação: 0,1

FRF do sistema primário



10  
0  
-10  
-20  
-30  
-40  
-50  
-60

20 25 30 35 40 45 50 55 60 65 70

Graficar:      Gerar FRF

Inertância

Automático

Ponto de excitação	Ponto de resposta
2	4

Automático

Limite inferior	Limite Superior
20	70

# Projeto ótimo dos neutralizadores:

Arquivos Dados da estrutura Parâmetros de cálculo Dados dos neutral. Mat. Viscoelástico Calcular Ajuda Opções

Encerrar o programa **STOP**

Modelo de neutralizador Massa do Núcleo

Modelo de Neutralizador Com Centro de Percussão 0,1

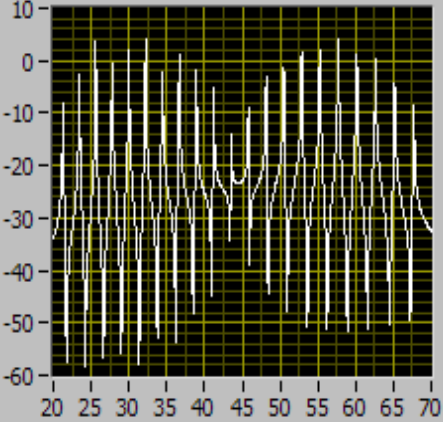
Número de neutralizadores

2

Posicionar graficamente

Posição Nodal	Frequência Natural (Hz)	Restrição Inferior (Hz)	Restrição Superior (Hz)	Faixa de localizações válidas	
2	30	10	80	0	0
78	30	10	80	0	0

FRF do sistema primário



10  
0  
-10  
-20  
-30  
-40  
-50  
-60

20 25 30 35 40 45 50 55 60 65 70

Graficar: Gerar FRF

Inertância

Automático  Ponto de excitação 2 Ponto de resposta 4

Automático  Limite inferior 20 Limite Superior 70

# Projeto ótimo dos neutralizadores:

Arquivos Dados da estrutura Parâmetros de cálculo Dados dos neutral. Mat. Viscoelástico Calcular Ajuda Opções

Encerrar o programa **STOP**

Escolha a borracha:  
Parâmetros levantados PISA - Neoprene 55 Shore A - 19/06/2005

Editar arquivo de dados dos materiais VE

Nomograma

Temperatura de trabalho (K): 303  
Temperatura de referência (K): 277,7  
G0 - Assíntota inferior: 5,32E+6  
Goo - Assíntota Superior: 1,48E+8  
Parâmetro de derivada fracionária: 0,359  
FIOMOD: 0,0054  
Teta 1: 10,1  
Teta 2: 137

Temperatura (K): 353, 313, 293, 273, 253

Módulo de Cisalhamento (MPa) e Fator de Perda

Frequência (Hz)

Frequência Reduzida (Hz)

FRF do sistema primário

Graficar: Inertância

Automático

Ponto de excitação: 2

Ponto de resposta: 4

Automático

Limite inferior: 20

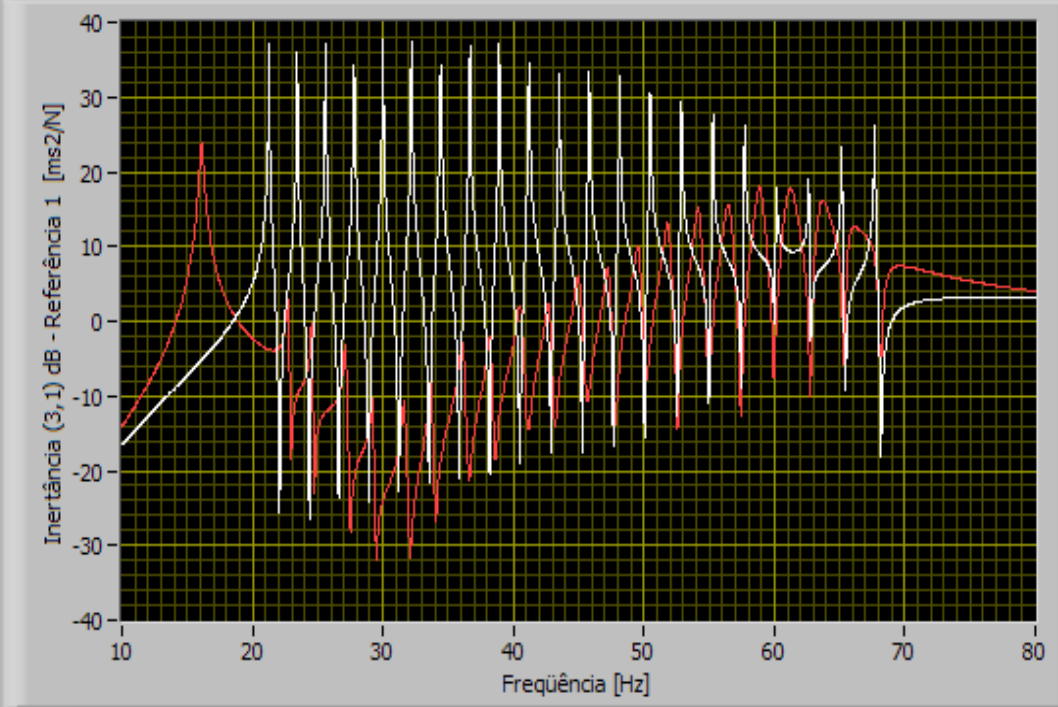
Limite Superior: 70

Gerar FRF

# Projeto ótimo dos neutralizadores:

Arquivos Dados da estrutura Parâmetros de cálculo Dados dos neutral. Mat. Viscoelástico Calcular Ajuda Opções

Gráfico





Inertância (3,1) dB - Referência 1 [ms<sup>2</sup>/N]

30  
20  
10  
0  
-10  
-20  
-30  
-40

10 20 30 40 50 60 70 80

Freqüência [Hz]

S.Prim.   
S.Contr. 






  

Gráfico de alfa 0  
Carregar

Gráfico de alfa 1  
Carregar

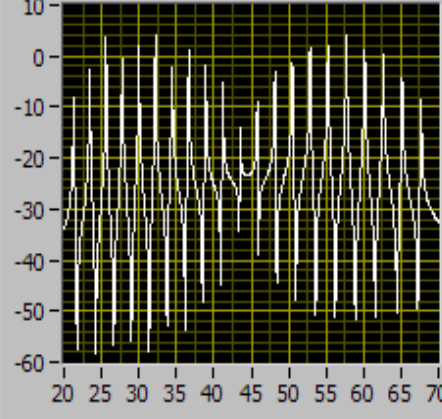
Parâmetros otimizados  
Carregar

Salvar alterações 

Iniciar cálculo 



Encerrar o programa **STOP**


FRF do sistema primário




10  
0  
-10  
-20  
-30  
-40  
-50  
-60

20 25 30 35 40 45 50 55 60 65 70

Graficar: Inertância  Gerar FRF 

Automático  Ponto de excitação 2 Ponto de resposta 4

Automático  Limite inferior 20 Limite Superior 70

Resultados da otimização

11/03/2007 - key [413301]  
\*\*\* Resultados da otimização \*\*\*  
30.707113864559 <XBEST(1)> Freqüência natural ótima do neutralizador  
19.275795770135 <XBEST(2)> Freqüência natural ótima do neutralizador  
0.649621933318 <MA(1)> Massa do neutralizador (g)  
0.649621933318 <MA(2)> Massa do neutralizador (g)



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TECHNOLOGY SECTOR

WEBSITE: <http://www.pgmec.ufpr.br>

## PROGRAMA DE PÓS-GRADUAÇÃO EM ENGENHARIA MECÂNICA

**Mestrado e Doutorado - *stricto sensu* - NOTA CAPES: 5**

Áreas de Concentração:	Linhas de Pesquisa:
Manufatura	1 - Engenharia de Superfícies 2 - Engenharia de Materiais e Fabricação
Fenômenos de Transporte e Mecânica dos Sólidos	3 - Fenômenos de Transporte e Engenharia Térmica 4 - Mecânica dos Sólidos e Projeto Mecânico



Mechanics of Solids and Vibrations




FEDERAL UNIVERSITY OF PARANA  
TECHNOLOGY SECTOR

COORDENAÇÃO	
Coordenador	<u>Prof. Dr. Paulo Victor Prestes Marcondes</u> telefone: ☎ (41) 3361-3701 gabinete:Sala 7-02 B, Depto. Eng. Mecânica/UFPR
Vice-Coordenador	<u>Prof. Dr. Carlos Alberto Bavastri</u> telefone: ☎ (41) 3361-3309 gabinete:Sala 7-07 Depto. Eng. Mecânica/UFPR
Secretário	Sr. Marcio Brandani Tenório telefone: ☎ (41) 3361-3701



## FEDERAL UNIVERSITY OF PARANA TECHNOLOGY SECTOR

<b>SECRETARIA</b>	
Local	Departamento de Engenharia Mecânica Setor de Tecnologia Centro Politécnico da UFPR Bloco IV Bairro Jardim das Américas Curitiba - PR
Endereço postal	Programa de Pós-Graduação em Engenharia Mecânica (PG-MEC) Depto. Engenharia Mecânica - UFPR Rua Francisco H. dos Santos, S/Nº Caixa postal 19011 81531-980, Curitiba, PR
Atendimento	Horário de expediente Interno: 2ª a 6ª das 07:30 às 11:30 horas Horário de atendimento Externo: 2ª a 6ª das 13:00 às 17:00 horas
e-mail	<a href="mailto:pgmec@ufpr.br">pgmec@ufpr.br</a>
website	<a href="http://www.pgmec.ufpr.br">www.pgmec.ufpr.br</a>
Telefone/Fax	 (41) 3361-3701





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TECHNOLOGY SECTOR

<b>COLEGIADO DO CURSO</b>		
<b>Titular</b>	<b>Suplente</b>	<b>Representante</b>
Prof. Ana Sofia C. M. D'Oliveira	Prof. Carlos José de M. Siqueira	Linha de Pesquisa: Engenharia de Superfícies
Prof. Silvio Francisco Brunatto	Profª. Thaís Helena S. Flores- Sahagun	Linha de Pesquisa: Engenharia de Materiais e Fabricação
Prof. Christian J. Losso Hermes	Profª. Maria José J. de S. Ponte	Linha de Pesquisa: Fenômenos de Transporte e Engenharia Térmica
Prof. Eduardo M. de O. Lopes	Prof. Heraldo Nélio Cambraia	Linha de Pesquisa: Mecânica dos Sólidos e Projeto Mecânico
Cristiano José Scheuer	João do Carmo Lopes Gonçalves	Alunos
Prof. Paulo Victor P. Marcondes	Coordenador do PG-Mec	
Prof. Carlos Aberto Bavastri	Vice-Coordenador do PG-Mec	



# FEDERAL UNIVERSITY OF PARANA TECHNOLOGY SECTOR

<b>CURSOS</b>	
Tipo	Mestrado acadêmico e Doutorado (stricto sensu)
Custo	Gratuito
Data de criação do Mestrado	Mestrado: 15 de março de 2000 Doutorado: 13 de março de 2006
Nota CAPES/MEC	Mestrado: 5 Doutorado: 5
Alunos regulares 2012/1	29 = Mestrado 36 = Doutorado
Alunos Disciplinas Isoladas 2012/1	62
Cota de Bolsas de estudo 2012/1	Mestrado: CAPES/PROAP= 12 CAPES/REUNI = 05 Doutorado: CAPES/PROAP= 05 CAPES/REUNI = 05 Fundação Araucária = 01
Defesas de Dissertação realizadas	131
Defesas de Tese realizadas	14
Períodos das aulas	diurno (manhã e tarde)
Períodos letivos	trimestrais
Professores Credenciados	22 doutores
Laboratórios	18
Grupos de pesquisa CNPq	10

Thank you for your  
attention

MUITO OBRIGADO