

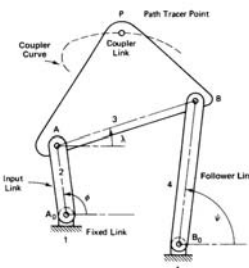
Mecanismos

Mecanismos com 1 GL

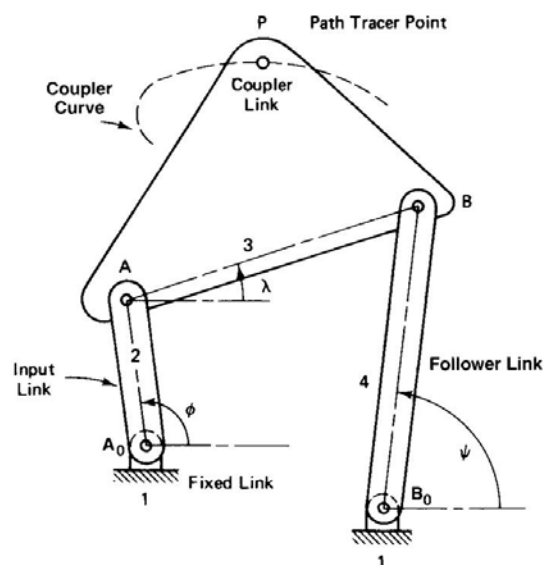
Mecanismo de 4 barras

- Análise Geral
- Análise de Pontos de Interesse

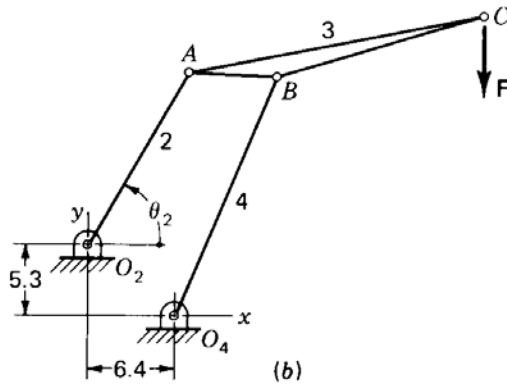
Prof. Jorge Luiz Erthal
jorgeerthal@gmail.com



Exemplos



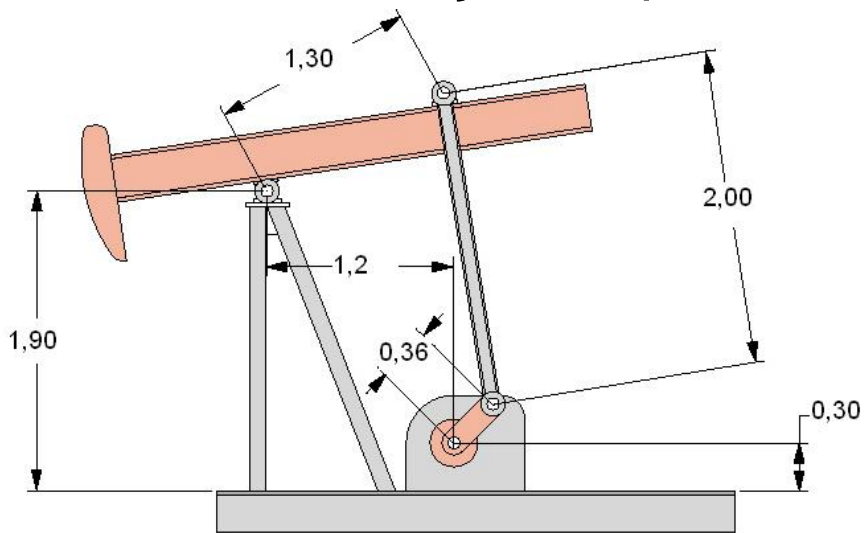
Guindaste



Mecanismo de 4 Barras

3

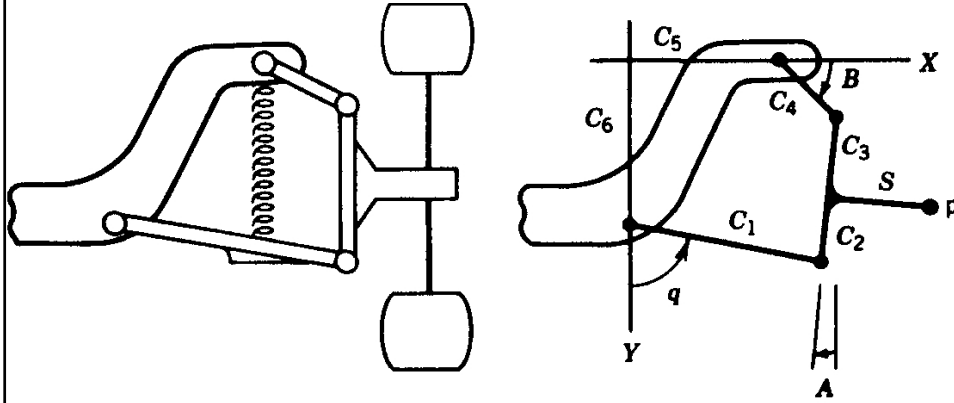
Bomba de extração de petróleo



Mecanismo de 4 Barras

4

Suspensão duplo A



Mecanismo de 4 Barras

5

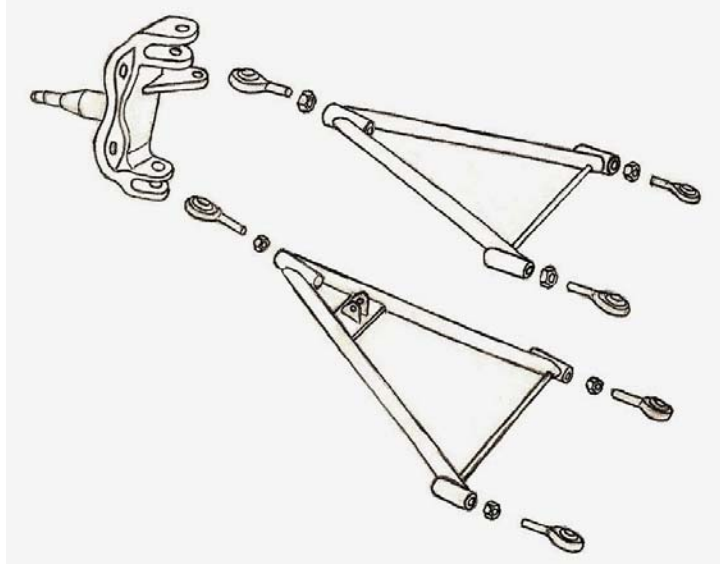
Suspensão duplo A



Mecanismo de 4 Barras

6

Suspensão duplo A



Mecanismo de 4 Barras

7

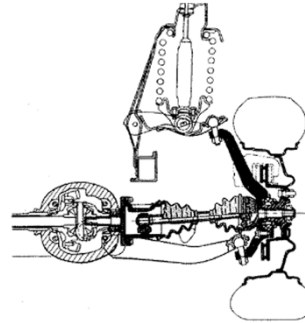
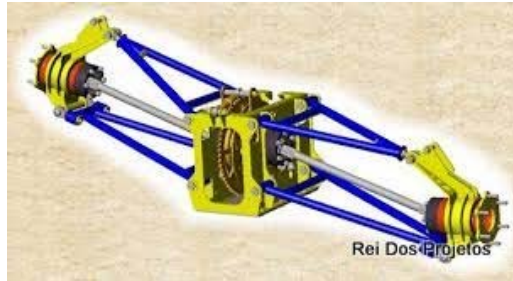
Suspensão duplo A



Mecanismo de 4 Barras

8

Suspensão duplo A



Mecanismo de 4 Barras

9

Sistemas de direção

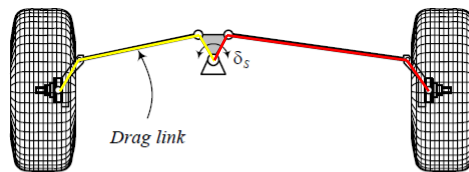


FIGURE 7.25. A lever arm steering system.

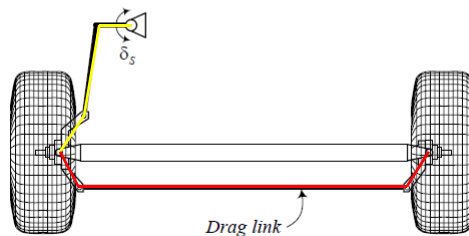
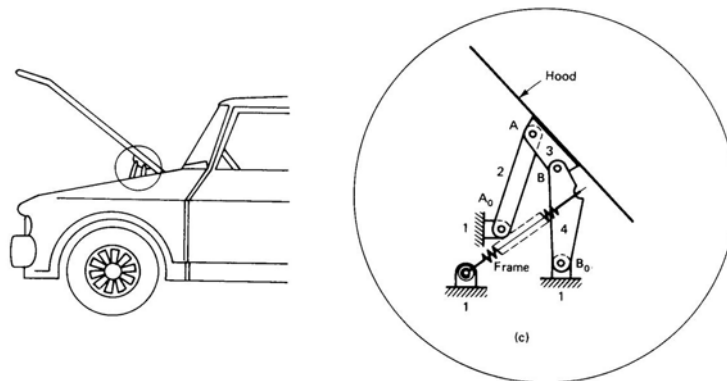


FIGURE 7.26. A drag link steering system.

Mecanismo de 4 Barras

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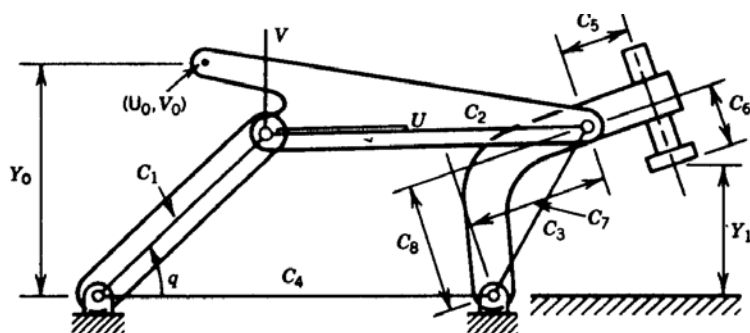
Articulação do capô do motor



Mecanismo de 4 Barras

11

Dispositivo de fixação



Mecanismo de 4 Barras

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1-Análise Geral

Mecanismo de 4 Barras

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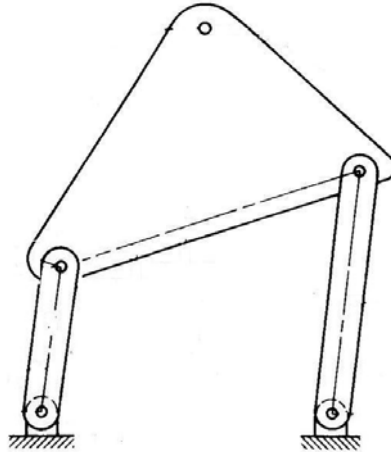
Conteúdo

- Análise Geral
- Critério de Grashof
- Variações no acionamento
 - Pontos singulares

Mecanismo de 4 Barras

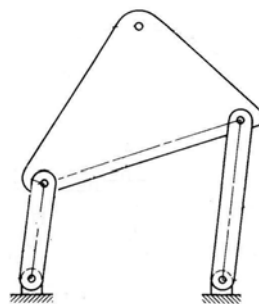
14

Mecanismo de Quatro Barras



15

1-Cálculo do número de graus de liberdade



$$N := 4 \qquad P_1 := 4 \qquad P_2 := 0$$

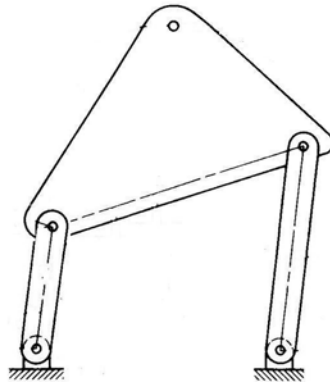
$$F := 3 \cdot (N - 1) - 2 \cdot P_1 - P_2 \qquad F = 1$$

Mecanismo de 4 Barras

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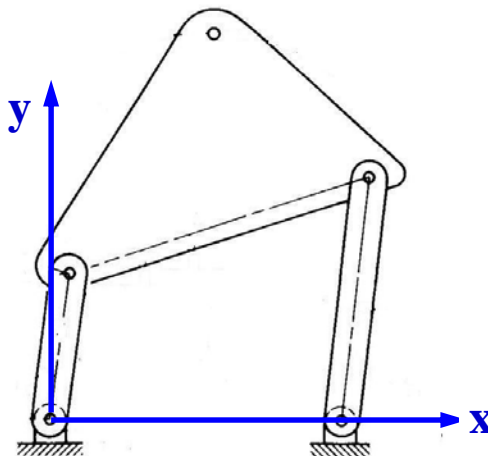
2-Decomposição dos pares superiores

Não há pares superiores



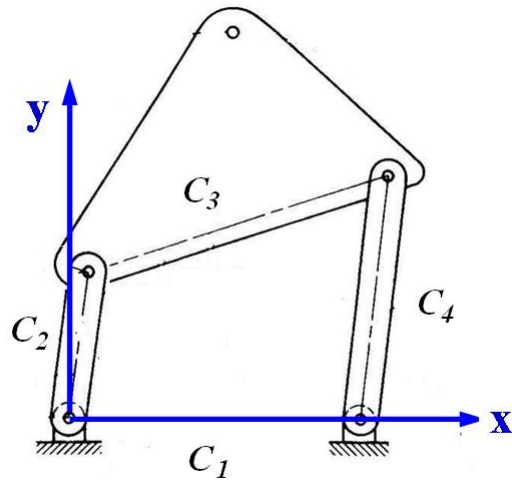
17

3-Sistema GLOBAL de coordenadas



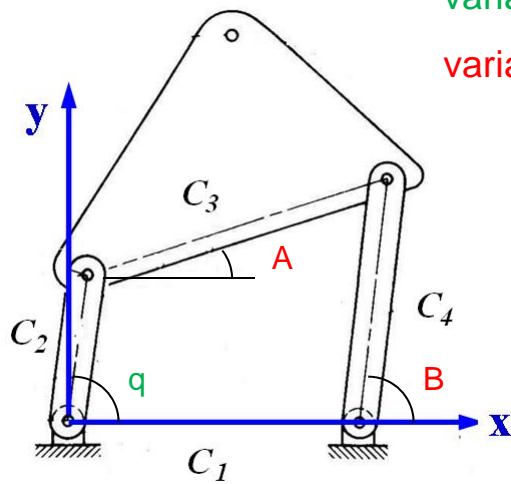
18

4-Identificação das medidas constantes



19

5-Definição das variáveis primárias e secundárias

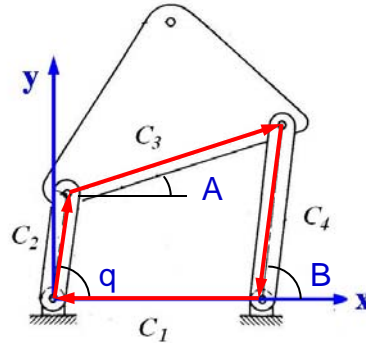


variável primária: q

variáveis secundárias: A, B

20

6-Equações cinemáticas de posição



$$C_2 \cdot \cos(q) + C_3 \cdot \cos(A) - C_4 \cdot \cos(B) - C_1 = 0$$

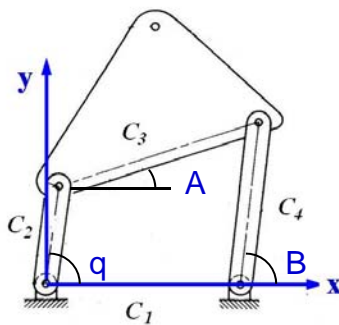
$$C_2 \cdot \sin(q) + C_3 \cdot \sin(A) - C_4 \cdot \sin(B) = 0$$

Mecanismo de 4 Barras

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7-Solução do sistema de equações

Solução numérica



Variável primária: $q := 60\text{deg}$

Valores iniciais: $A := 30\text{deg}$ $B := 90\text{deg}$

Solução numérica:

Given

$$C_2 \cdot \cos(q) + C_3 \cdot \cos(A) - C_4 \cdot \cos(B) - C_1 = 0$$

$$C_2 \cdot \sin(q) + C_3 \cdot \sin(A) - C_4 \cdot \sin(B) = 0$$

$$\begin{pmatrix} A \\ B \end{pmatrix} := \text{Find}(A, B)$$

$$A = 26.311 \text{ deg}$$

$$B = 75.229 \text{ deg}$$

Mecanismo de 4 Barras

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7-Solução para um intervalo de valores de q

Valores constantes: $C_1 := 60$ $C_2 := 20$ $C_3 := 70$ $C_4 := 50$

Variável primária: $q := 0 \text{ deg}, 10 \text{ deg} .. 360 \text{ deg}$

Valores iniciais: $A := 30 \text{ deg}$ $B := 90 \text{ deg}$

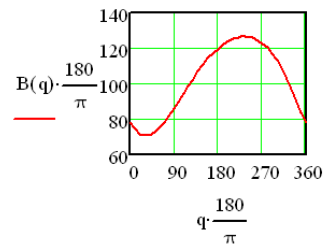
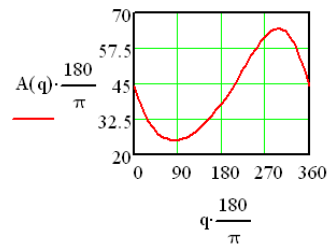
Solução numérica:

Given

$$C_2 \cdot \cos(q) + C_3 \cdot \cos(A) - C_4 \cdot \cos(B) - C_1 = 0$$

$$C_2 \cdot \sin(q) + C_3 \cdot \sin(A) - C_4 \cdot \sin(B) = 0$$

$$\begin{pmatrix} A(q) \\ B(q) \end{pmatrix} := \text{Find}(A, B)$$

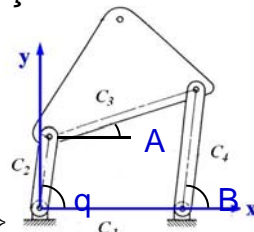


23

7-Solução do sistema de equações

vetor das equações cinemáticas :

$$f = \begin{Bmatrix} C_2 \cdot \cos(q) + C_3 \cdot \cos(a) - C_4 \cdot \cos(B) - C_1 \\ C_2 \cdot \sin(q) + C_3 \cdot \sin(a) - C_4 \cdot \sin(B) \end{Bmatrix}$$



vetor das variáveis secundárias :

$$S = \begin{Bmatrix} A \\ B \end{Bmatrix}$$

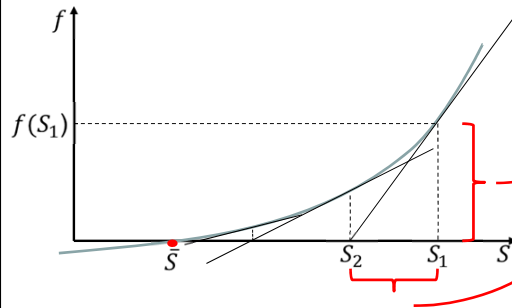
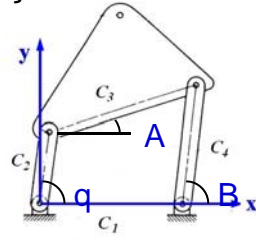
objetivo :

obter \bar{S} de modo que $f(\bar{S}) = 0$

Solução numérica
(as variáveis secundárias
são obtidas numericamente)

7-Solução do sistema de equações

Solução numérica
Método de Newton-Raphson



$$\frac{f(S_1)}{S_1 - S_2} = f'(S_1)$$

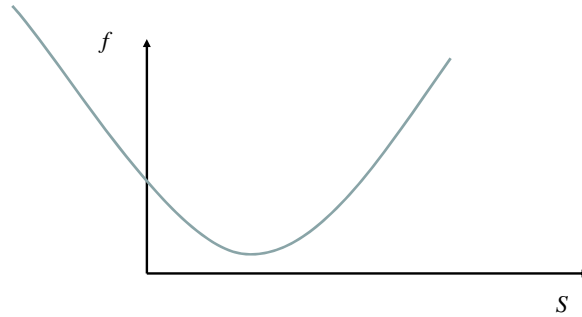
$$S_2 = -\frac{f(S_1)}{f'(S_1)} + S_1$$

$$f(S_1) \cong 0$$

7-Problemas com a solução numérica:

- **Existência**
- **Unicidade**
- **Intervalo de aplicação**

7-Existência da solução para um intervalo de q

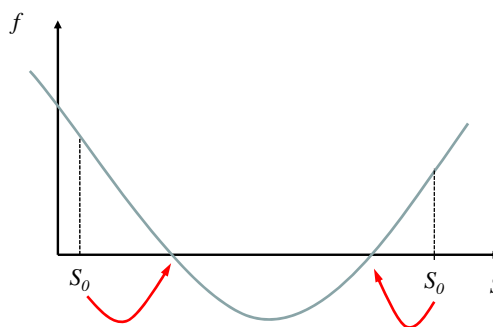


Relacionado com o “alcance” da variável

Mecanismo de 4 Barras

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7-Unicidade (número de soluções)

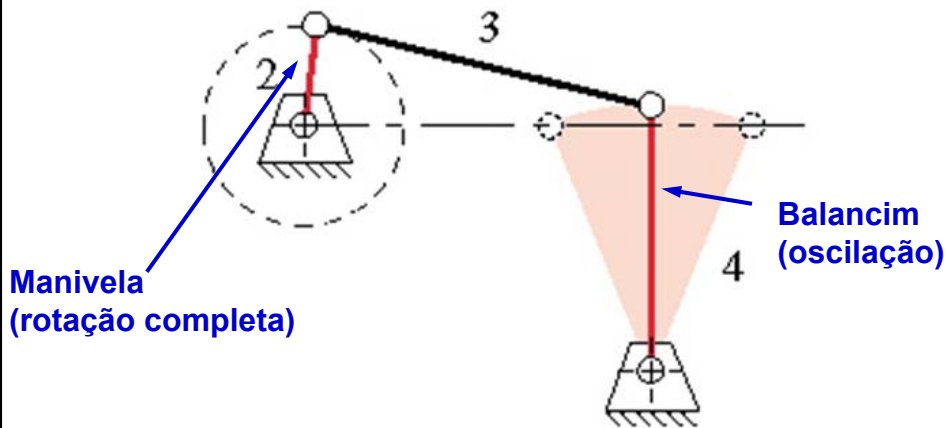


Valor inicial apropriado para atingir a solução desejada

Mecanismo de 4 Barras

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7-Intervalo de aplicação Movimentos dos elos

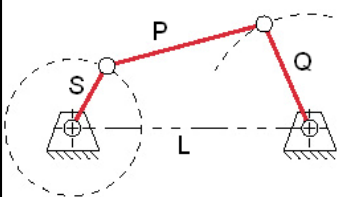


Mecanismo manivela-balancim

Mecanismo de 4 Barras

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Cr terio de **Grashof**:



S – comprimento do elo menor
L – comprimento do elo maior
P,Q – comprimentos dos elos restantes

Se $S+L < P+Q \Rightarrow$ Classe I (mecanismo de Grashof)

Se $S+L > P+Q \Rightarrow$ Classe II (mecanismo n o-Grashof)

Nenhum elo   capaz de girar completamente.

Se $S+L = P+Q \Rightarrow$ Classe III

Mecanismo de 4 Barras

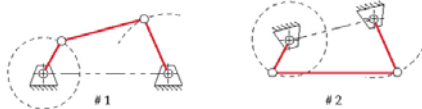
30

Critério de *Grashof*:

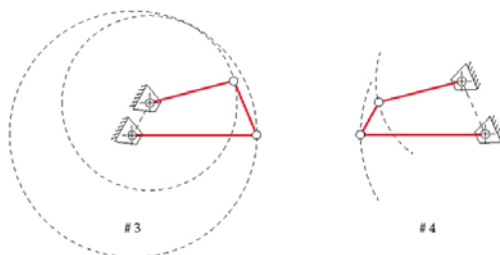
Classe I ($S+L \leq P+Q$)

exemplo: quatro Barras_CI.dv

1. Dupla manivela se S for fixo
2. Duplo balancim se S for oposto ao fixo
3. Manivela-balancim se S for ligado ao fixo



Mecanismo manivela-balancim



Mecanismo
dupla manivela

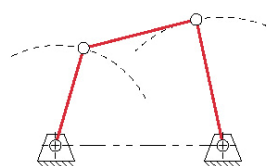
Mecanismo
duplo balancim

31

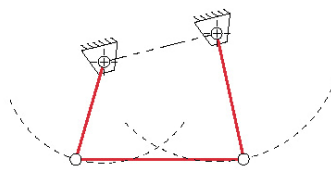
Critério de *Grashof*:

Classe II ($S+L > P+Q$)

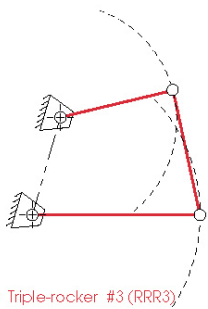
Duplo balancim



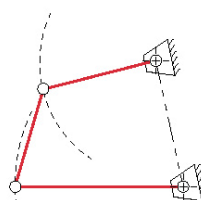
(a) Triple-rocker #1 (RRR1)



(b) Triple-rocker #2 (RRR2)



(c) Triple-rocker #3 (RRR3)



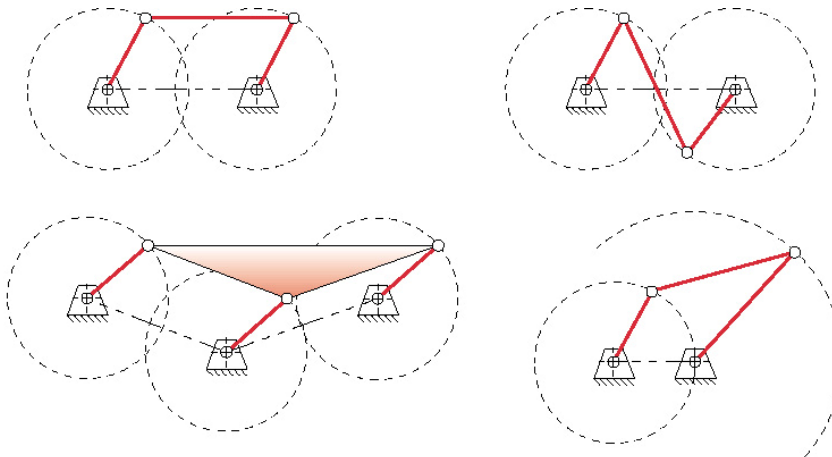
(d) Triple-rocker #4 (RRR4)

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Critério de *Grashof*:

Classe III ($S+L = P+Q$)

Configuração incerta

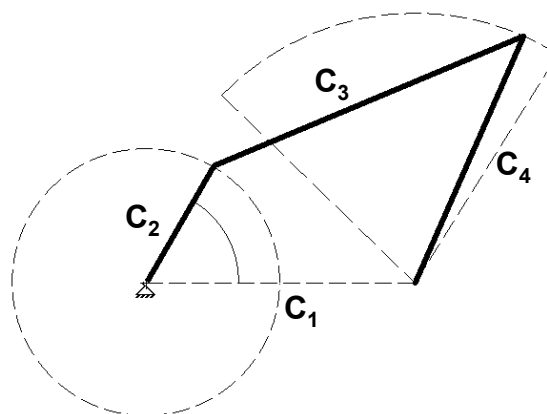


Mecanismo de 4 Barras

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Limites do mecanismo manivela-balancim

posições limites do elo de saída (não caracteriza travamento)



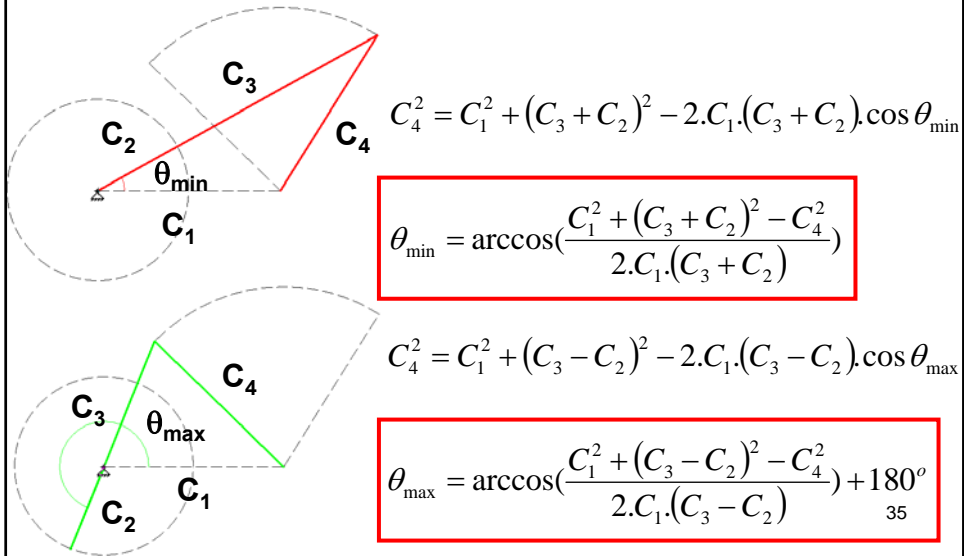
exemplo: [limites_4bMB.dv](#)

Mecanismo de 4 Barras

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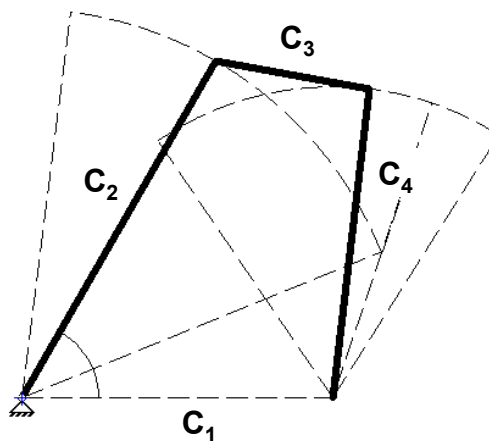
Limites do mecanismo manivela-balancim

Alinhamento do elo de entrada (C_2) com o intermediário (C_3).



Limites do mecanismo duplo-balancim

posições limites do elo de entrada (caracteriza travamento)

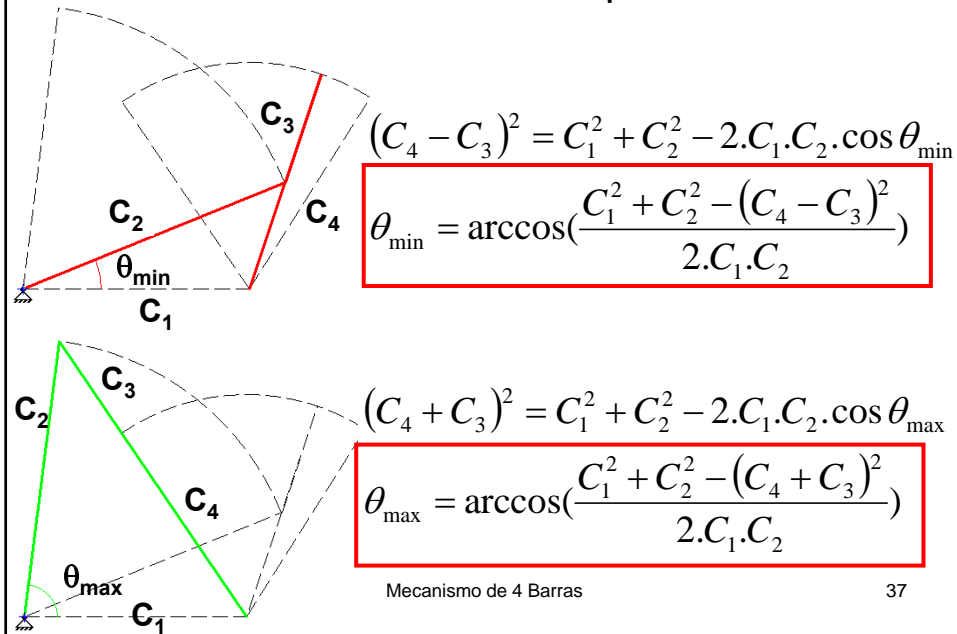


exemplo: [limites_4bBB.dv](#)

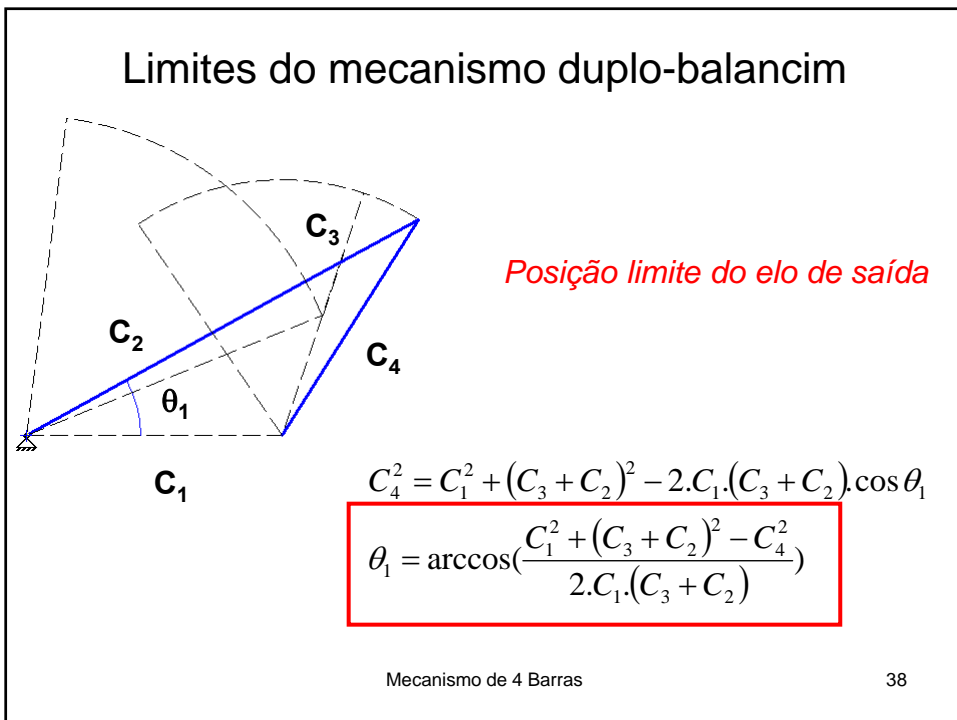
Mecanismo de 4 Barras

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Limites do mecanismo duplo-balancim

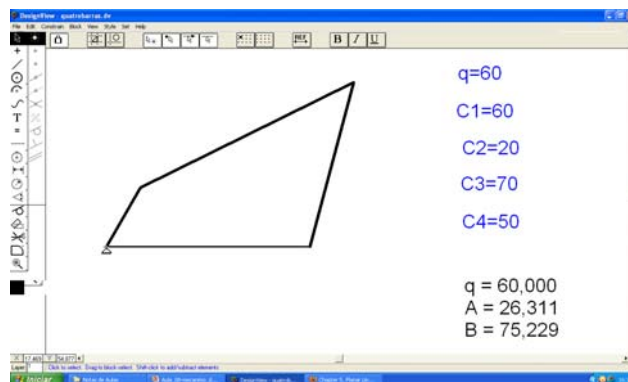


Limites do mecanismo duplo-balancim



Exemplo

Arq.: quatro Barras.dv (Design View)



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Equações das velocidades

Derivadas das equações de posição em relação ao tempo

$$-C_2 \cdot \sin(q) \cdot \dot{q} - C_3 \cdot \sin(A) \cdot \dot{A} + C_4 \cdot \sin(B) \cdot \dot{B} = 0$$

$$C_2 \cdot \cos(q) \cdot \dot{q} + C_3 \cdot \cos(A) \cdot \dot{A} - C_4 \cdot \cos(B) \cdot \dot{B} = 0$$

$$\begin{pmatrix} -C_3 \cdot \sin(A) & C_4 \cdot \sin(B) \\ C_3 \cdot \cos(A) & -C_4 \cdot \cos(B) \end{pmatrix} \cdot \begin{pmatrix} \dot{A} \\ \dot{B} \end{pmatrix} + \begin{pmatrix} -C_2 \cdot \sin(q) \\ C_2 \cdot \cos(q) \end{pmatrix} \cdot \dot{q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Matriz
jacobiana

Vetor das velocidades
secundárias

Vetor dos
coeficientes
constantes

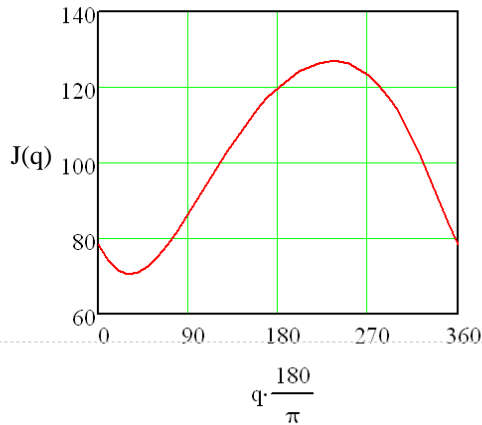
Velocidade
primária

$$J \cdot \dot{S} + Q \cdot \dot{q} = 0$$

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Determinante da matriz Jacobiana

$$J(q) := \begin{pmatrix} -C_3 \cdot \sin(A(q)) & C_4 \cdot \sin(B(q)) \\ C_3 \cdot \cos(A(q)) & -C_4 \cdot \cos(B(q)) \end{pmatrix} \quad \det J(q) := C_3 \cdot C_4 \cdot \sin(A(q) - B(q))$$



Mecanismo de 4 Barras

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Solução para as velocidades secundárias

$$\begin{pmatrix} \dot{A} \\ \dot{B} \end{pmatrix} = - \begin{pmatrix} -C_3 \cdot \sin(A) & C_4 \cdot \sin(B) \\ C_3 \cdot \cos(A) & -C_4 \cdot \cos(B) \end{pmatrix}^{-1} \cdot \begin{pmatrix} -C_2 \cdot \sin(q) \\ C_2 \cdot \cos(q) \end{pmatrix} \cdot \dot{q} \quad \dot{S} = -J^{-1} \cdot Q \cdot \dot{q}$$

$$\begin{pmatrix} \dot{A} \\ \dot{B} \end{pmatrix} = \begin{pmatrix} \frac{C_2 \cdot \sin(B - q)}{C_3 \cdot \sin(A - B)} \\ \frac{C_2 \cdot \sin(A - q)}{C_4 \cdot \sin(A - B)} \end{pmatrix} \cdot \dot{q}$$

$$K_a(q) = \frac{C_2 \cdot \sin(B(q) - q)}{C_3 \cdot \sin(A(q) - B(q))}$$

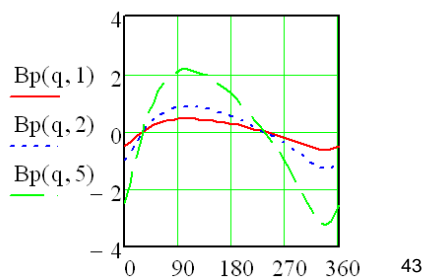
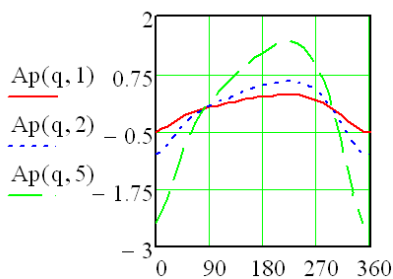
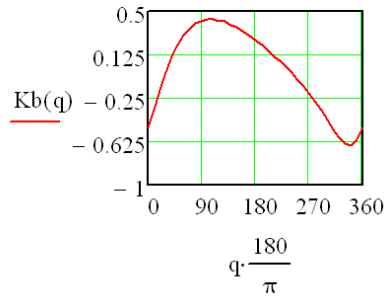
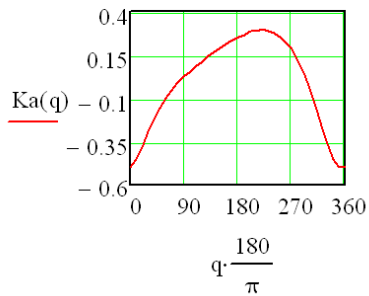
$$K_b(q) = \frac{C_2 \cdot \sin(A(q) - q)}{C_4 \cdot \sin(A(q) - B(q))}$$

$$\dot{S} = K \cdot \dot{q}$$

$$\dot{A}(q) = K_a(q) \cdot \dot{q}$$

$$\dot{B}(q) = K_b(q) \cdot \dot{q}$$

Velocidades secundárias



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Acelerações secundárias

$$\ddot{A}(q) = Ka(q) \cdot \ddot{q} + La(q) \cdot \dot{q}^2$$

$$\ddot{B}(q) = Kb(q) \cdot \ddot{q} + Lb(q) \cdot \dot{q}^2$$

$$La(q) = \frac{d}{dq} Ka(q) \quad Lb(q) = \frac{d}{dq} Kb(q)$$

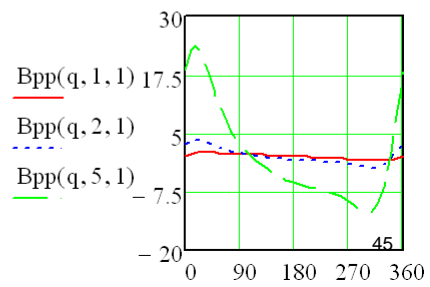
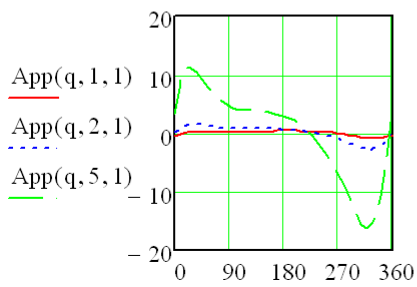
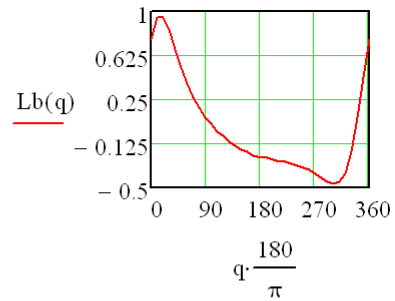
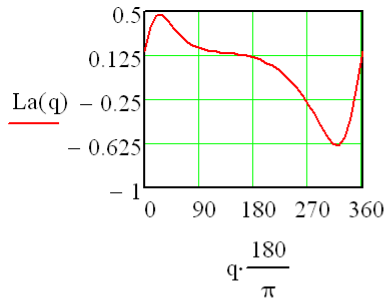
$$La(q) := \frac{C_2 \cdot \cos(B(q) - q) \cdot (Kb(q) - 1)}{C_3 \cdot \sin(A(q) - B(q))} - \frac{C_2 \cdot \sin(B(q) - q) \cdot \cos(A(q) - B(q)) \cdot (Ka(q) - Kb(q))}{C_3 \cdot \sin(A(q) - B(q))^2}$$

$$Lb(q) := \frac{C_2 \cdot \cos(A(q) - q) \cdot (Ka(q) - 1)}{C_4 \cdot \sin(A(q) - B(q))} - \frac{C_2 \cdot \sin(A(q) - q) \cdot \cos(A(q) - B(q)) \cdot (Ka(q) - Kb(q))}{C_4 \cdot \sin(A(q) - B(q))^2}$$

Mecanismo de 4 Barras

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Acelerações secundárias



2-Análise de Pontos de Interesse

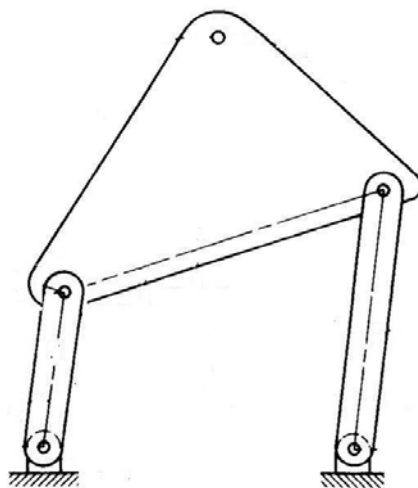
Conteúdo

- Escolha do ponto de interesse
- Cálculo da posição, velocidade e aceleração do ponto
- Solução no MathCAD
- Desenho de deslocamentos
- Exemplo: guindaste

Mecanismo de 4 Barras

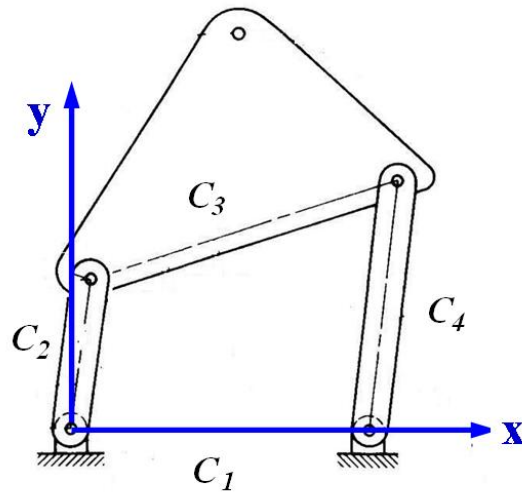
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Exemplo



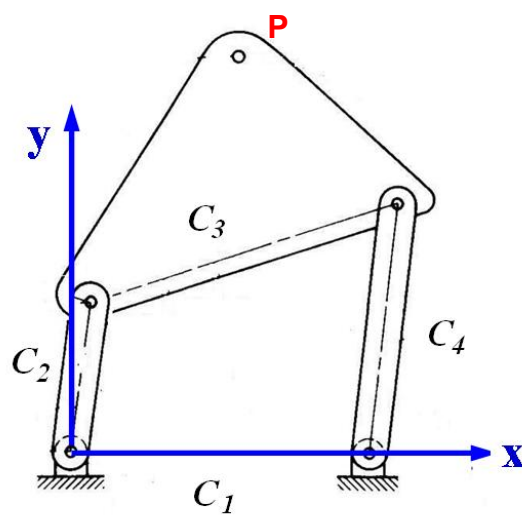
48

1- Sistema global (x,y)



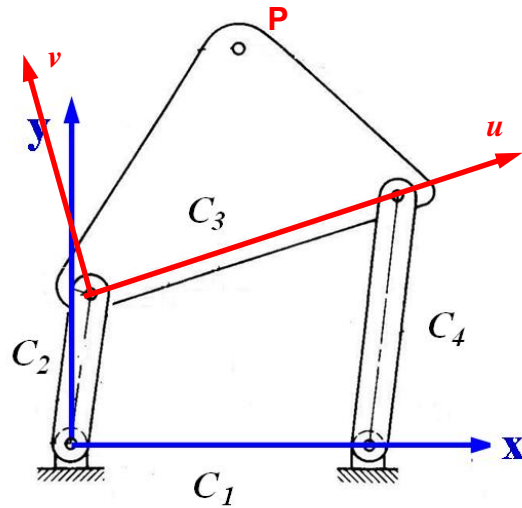
49

2- Seleção do ponto de interesse (P)



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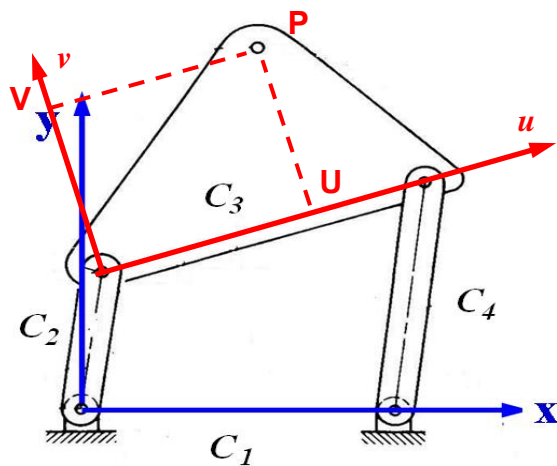
3-Sistema LOCAL (u,v)



51

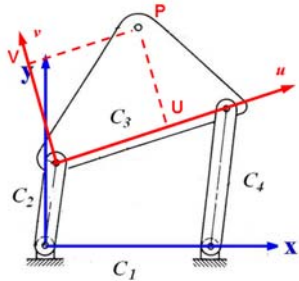
4-Coordenadas locais (U,V)

$$U_P := 50 \quad V_P := 50$$



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5-Coordenadas globais (X,Y)



COORDENADAS LOCAIS

$$\begin{pmatrix} X_P \\ Y_P \end{pmatrix} = \begin{pmatrix} O_X \\ O_Y \end{pmatrix} + \begin{pmatrix} u_X & v_X \\ u_Y & v_Y \end{pmatrix} \cdot \begin{pmatrix} U_P \\ V_P \end{pmatrix}$$

COORDENADAS GLOBAIS

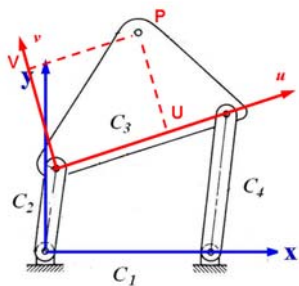
ORIGEM DO SISTEMA LOCAL

MATRIZ DE ROTAÇÃO

Barras

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5-Coordenadas globais (X,Y) (posição)



$$\begin{pmatrix} X_P \\ Y_P \end{pmatrix} = \begin{pmatrix} O_X \\ O_Y \end{pmatrix} + \begin{pmatrix} u_X & v_X \\ u_Y & v_Y \end{pmatrix} \cdot \begin{pmatrix} U_P \\ V_P \end{pmatrix}$$

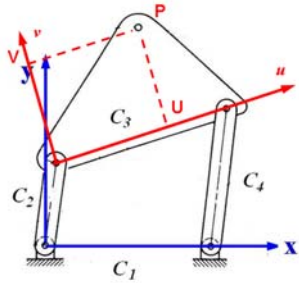
$$\begin{pmatrix} X(q) \\ Y(q) \end{pmatrix} = \begin{pmatrix} C_2 \cdot \cos(q) \\ C_2 \cdot \sin(q) \end{pmatrix} + \begin{pmatrix} \cos(A(q)) & -\sin(A(q)) \\ \sin(A(q)) & \cos(A(q)) \end{pmatrix} \cdot \begin{pmatrix} U_P \\ V_P \end{pmatrix}$$

$$X(q) := C_2 \cdot \cos(q) + \cos(A(q)) \cdot U_P - \sin(A(q)) \cdot V_P$$

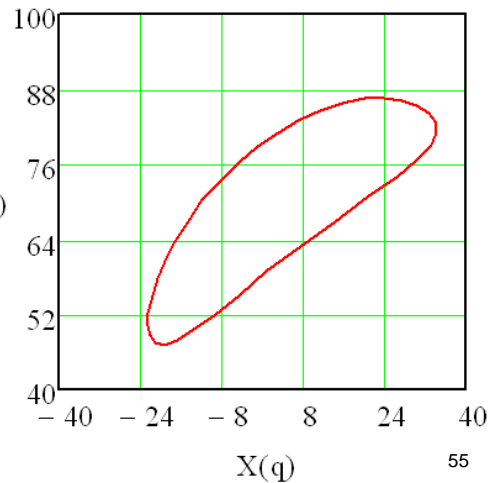
$$Y(q) := C_2 \cdot \sin(q) + \sin(A(q)) \cdot U_P + \cos(A(q)) \cdot V_P$$

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5-Coordenadas globais (X,Y) (posição)



$Y(q)$



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Equações das velocidades

$$X(q) := C_2 \cdot \cos(q) + \cos(A(q)) \cdot U_P - \sin(A(q)) \cdot V_P$$

$$Y(q) := C_2 \cdot \sin(q) + \sin(A(q)) \cdot U_P + \cos(A(q)) \cdot V_P$$

Derivadas das equações de posição em relação ao tempo

$$K_{px}(q) := -C_2 \cdot \sin(q) - \sin(A(q)) \cdot K_a(q) \cdot U_P - \cos(A(q)) \cdot K_a(q) \cdot V_P$$

$$K_{py}(q) := C_2 \cdot \cos(q) + \cos(A(q)) \cdot K_a(q) \cdot U_P - \sin(A(q)) \cdot K_a(q) \cdot V_P$$

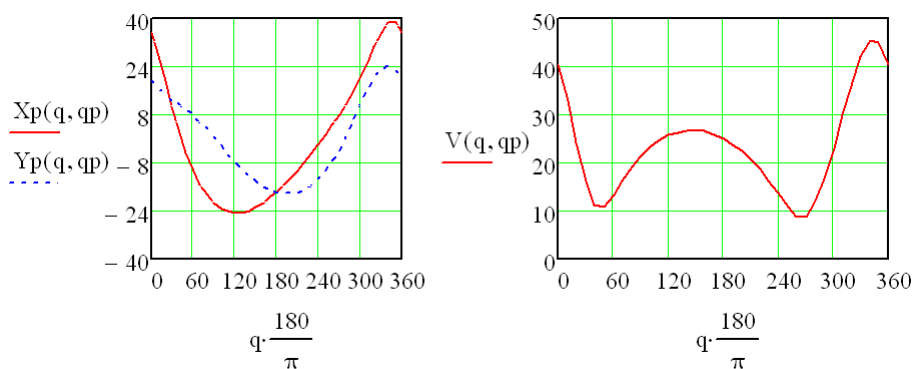
$$\dot{X}(q, qp) := K_{px}(q) \cdot qp$$

$$\dot{Y}(q, qp) := K_{py}(q) \cdot qp$$

$$V(q, qp) := \sqrt{\dot{X}(q, qp)^2 + \dot{Y}(q, qp)^2}$$

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Velocidade do ponto P



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Equações das acelerações

$$L_{Px}(q) := -Ka(q)^2 \cdot (U_P \cdot \cos(A(q)) - V_P \cdot \sin(A(q))) - C_2 \cdot \cos(q) - La(q) \cdot (U_P \cdot \sin(A(q)) + V_P \cdot \cos(A(q)))$$

$$L_{Py}(q) := (-U_P \cdot \sin(A(q)) - V_P \cdot \cos(A(q))) \cdot Ka(q)^2 - C_2 \cdot \sin(q) + (U_P \cdot \cos(A(q)) - V_P \cdot \sin(A(q))) \cdot La(q)$$

$$L_{Px} = \frac{d}{dq} K_{Px} \quad L_{Py} = \frac{d}{dq} K_{Py}$$

$$\ddot{X}_P = K_{Px} \cdot \ddot{q} + L_{Px} \cdot \dot{q}^2$$

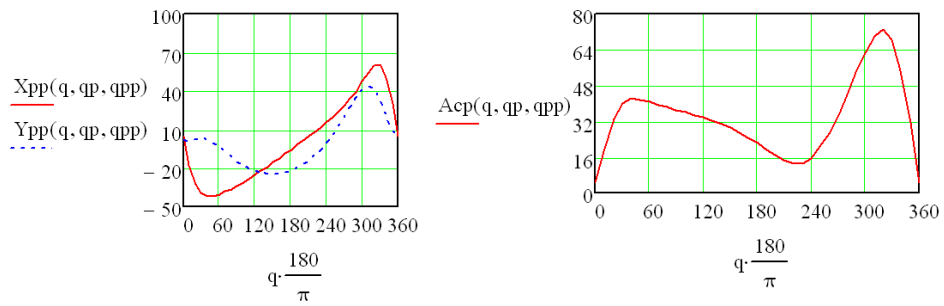
$$\ddot{Y}_P = K_{Py} \cdot \ddot{q} + L_{Py} \cdot \dot{q}^2$$

$$A_P = \sqrt{\ddot{X}_P^2 + \ddot{Y}_P^2}$$

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Aceleração do ponto P



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Solução no Matlab

[Arquivo: quatrobarras.m](#)

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Desenhos de deslocamentos

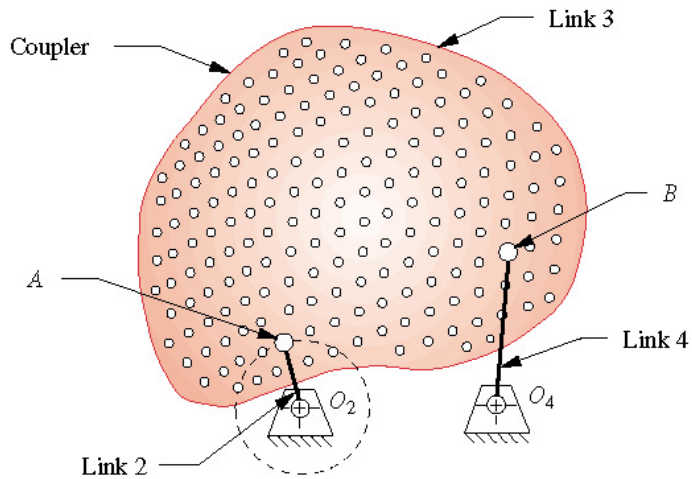


FIGURE 3-15

The fourbar coupler extended to include a large number of coupler points

Desenhos de deslocamentos

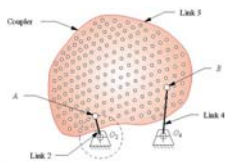


FIGURE 3-15
The fourbar coupler extended to include a large number of coupler points

- | | |
|---------------------|------------------|
| (a) Pseudo ellipse | (g) Teardrop |
| (b) Kidney bean | (h) Scimitar |
| (c) Banana | (i) Umbrella |
| (d) Crescent | (j) Triple cusp |
| (e) Single straight | (k) Figure eight |
| (f) Double straight | (l) Triple loop |

Mecanismo FIGURE 3-16 Part 1
A "Cursory Catalog" of coupler curve shapes

FIGURE 3-16 Part 2
A "Cursory Catalog" of coupler curve shapes

Desenhos de deslocamentos

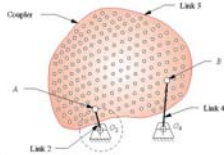
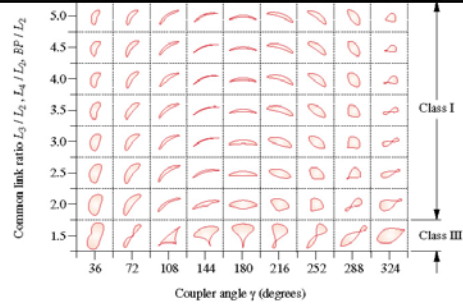
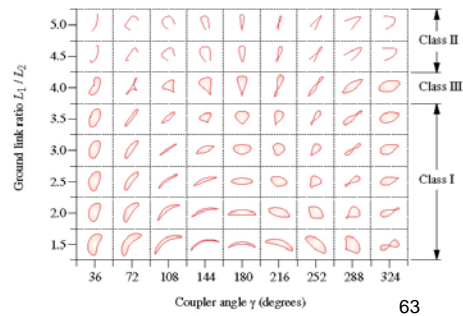


FIGURE 3-16 The four-bar coupler extended to include a large number of coupler points



(a) Variation of coupler curve shape with common link ratio and coupler angle for a ground link ratio $L_1/L_2 = 2.0$

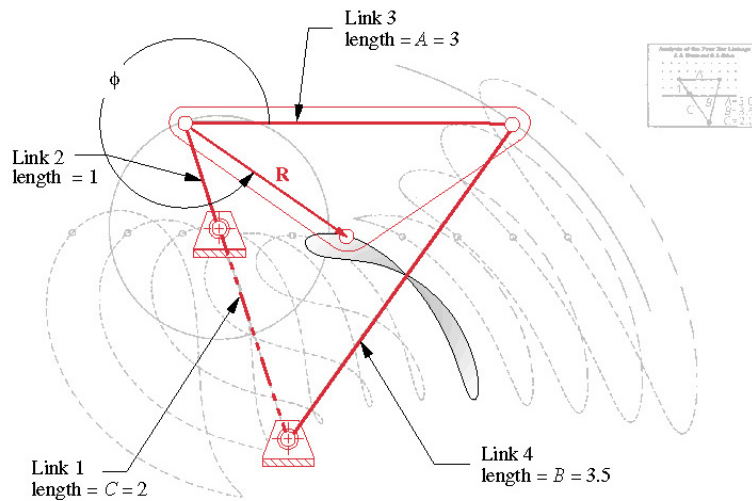


(b) Variation of coupler curve shape with ground link ratio and coupler angle for a common link ratio $L_3/L_2 = L_4/L_2 = BP/L_2 = 2.5$

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Desenhos de deslocamentos



(b) Creating the linkage from the information in the atlas

FIGURE 3-17

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Selecting a coupler curve and constructing the linkage from the Hrones and Nelson atlas

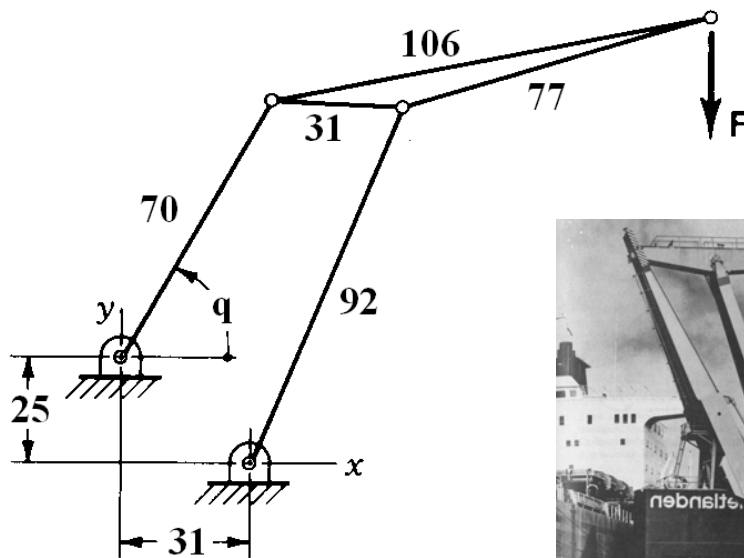
Desenhos de deslocamentos

[Arquivo: quatrobarras.dv](#)

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Exercício



[Arquivo: guindaste.dv](#)

Mecanismo de 4 Barras

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