(a) Find the components of the strain tensor.

(b) Find the unit elongation of an element initially in the direction of $\mathbf{e}_1 + \mathbf{e}_2$.

3.20. Consider the displacement field

$$u_1 = k(2X_1^2 + X_1 X_2), \quad u_2 = kX_2^2, \quad u_3 = 0,$$

where $k = 10^{-4}$.

(a) Find the unit elongations and the change of angle for two material elements $d\mathbf{X}^{(1)} = dX_1\mathbf{e}_1$ and $d\mathbf{X}^{(2)} = dX_2\mathbf{e}_2$ that emanate from a particle designated by $\mathbf{X} = \mathbf{e}_1 + \mathbf{e}_2$.

(b) Find the deformed shape of these two elements.

3.21. For the displacement field of Example 3.8.3, determine the increase in length for the diagonal element of the cube in the direction of $\mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3$ (a) by using the strain tensor and (b) by geometry.

3.22. With reference to a rectangular Cartesian coordinate system, the state of strain at a point is given by the matrix

$$[\mathbf{E}] = \begin{bmatrix} 5 & 3 & 0 \\ 3 & 4 & -1 \\ 0 & -1 & 2 \end{bmatrix} \times 10^{-4}$$

(a) What is the unit elongation in the direction $2\mathbf{e}_1 + 2\mathbf{e}_2 + \mathbf{e}_3$?

(b) What is the change of angle between two perpendicular lines (in the undeformed state) emanating from the point and in the directions of $2\mathbf{e}_1 + 2\mathbf{e}_2 + \mathbf{e}_3$ and $3\mathbf{e}_1 - 6\mathbf{e}_3$?

3.23. Do the previous problem for (a) the unit elongation in the direction $3\mathbf{e}_1 - 4\mathbf{e}_2$, (b) the change in angle between two elements in the direction $3\mathbf{e}_1 - 4\mathbf{e}_3$ and $4\mathbf{e}_1 + 3\mathbf{e}_3$.

3.24. (a)For Prob.3.22, determine the principal scalar invariants of the strain tensor.

(b) Show that the following matrix

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 2 \end{bmatrix} \times 10^{-4}$$

cannot represent the same state of strain of Prob.3.22.

3.25. For the displacement field

$$u_1 = kX_1^2, \quad u_2 = kX_2X_3, \quad u_3 = k(2X_1X_3 + X_1^2), \quad k = 10^{-6}$$

find the maximum unit elongation for an element that is initially at (1,0,0).

3.26. Given the matrix of an infinitesimal strain field

$$[\mathbf{E}] = \begin{bmatrix} k_1X_2 & 0 & 0 \\ 0 & -k_2X_2 & 0 \\ 0 & 0 & -k_2X_2 \end{bmatrix}$$

(a) Find the location of the particle that does not undergo any volume change.

(b) What should be the relation between k_1 and k_2 be such that no element changes volume?

3.27. The displacement components for a body are

$$u_1 = k(X_1^2 + X_2), \quad u_2 = k(4X_3^2 - X_1), \quad u_3 = 0, \qquad k = 10^{-4}$$

(a) Find the strain tensor.

(b) Find the change of length per unit length for an element which was at (1,2,1) and in the direction of $\mathbf{e}_1 + \mathbf{e}_2$.

(c) What is the maximum unit elongation at the same point (1,2,1)?

(d) What is the change of volume for the unit cube with a corner at the origin and with three of its edges along the positive coordinate axes.

3.28. For any motion the mass of a particle (material volume) remains constant. Consider the mass to be a product of its volume times its mass density and show that (a)for infinitesimal deformation $\rho(1 + E_{kk}) = \rho_o$, where ρ_o denotes the initial density and ρ the current density. (b) Use the smallness of E_{kk} to show that the current density is given by

$$\rho = \rho_o(1 - E_{kk})$$

3.29. True or false: At any point in a body, there always exist two mutually perpendicular material elements which do not suffer any change of angle in an arbitrary small deformation of the body. Give reasons.

3.30. Given the following strain components at a point in a continuum:

$$E_{11} = E_{12} = E_{22} = k, \quad E_{33} = 3k, \quad E_{13} = E_{23} = 0 \qquad k>0$$

Does there exist a material element at the point which decreases in length under the deformation? Explain your answer.

3.31. The unit elongations at a certain point on the surface of a body are measured experimentally by means of strain gages that are arranged 45° apart (called the 45° strain rosette) in the directions $\mathbf{e}_1$, $(\sqrt{2}/2)(\mathbf{e}_1 + \mathbf{e}_2)$ and $\mathbf{e}_2$. If these unit elongations are designated by a,b,c respectively, what are the strain components E_{11}, E_{22}, E_{12}.

3.32. (a) Do Problem 3.31 if the measured strains are 200×10^{-6}, 50×10^{-6}, 100×10^{-6}, respectively.

(b) If $E_{33} = E_{32} = E_{31} = 0$, find the principal strains and directions of part (a).

(c) How will the result of part (b) be altered if $E_{33} \neq 0$?

3.33. Repeat Problem 3.32 except that $a = b = c = 1000\times10^{-6}$.

3.34. The unit elongations at a certain point on the surface of a body are measured experimentally by means of strain gages that are arranged 60^o apart (called the 60^o strain rosette) in the directions $\mathbf{e}_1$, $\frac{1}{2}(\mathbf{e}_1 + \sqrt{3}\mathbf{e}_2)$, and $\frac{1}{2}(-\mathbf{e}_1 + \sqrt{3}\mathbf{e}_2)$. If these elongations are designated by a,b,c respectively, what are the strain components E_{11}, E_{22}, E_{12}?

3.35. Do Problem 3.34 if the strain rosette measurements give $a = 2\times10^{-6}$, $b = 1\times10^{-6}$, $c = 1.5\times10^{-6}$.

3.36. Do Problem 3.35 except that $a = b = c = 2000\times10^{-6}$.

3.37. For the velocity field, $\mathbf{v} = (kx_2^2)\mathbf{e}_1$

(a) Find the rate of deformation and spin tensors.

(b) Find the rate of extensions of a material element $d\mathbf{x} = (ds)\mathbf{n}$ where

$$\mathbf{n} = (\sqrt{2}/2)(\mathbf{e}_1 + \mathbf{e}_2) \quad \text{at} \quad \mathbf{x} = 5\mathbf{e}_1 + 3\mathbf{e}_2.$$

3.38. For the velocity field

$$\mathbf{v} = \left(\frac{t+k}{1+x_1}\right)\mathbf{e}_1$$

find the rates of extension for the following material elements: $d\mathbf{x}^{(1)} = ds_1\mathbf{e}_1$ and $d\mathbf{x}^{(2)} = (ds_2/\sqrt{2})(\mathbf{e}_1 + \mathbf{e}_2)$ at the origin at time $t = 1$.

3.39. (a) Find the rate of deformation and spin tensors for the velocity field $\mathbf{v} = (\cos t)(\sin \pi x_1)\mathbf{e}_2$.

(b) For the velocity field of part (a), find the rates of extension of the elements $d\mathbf{x}^{(1)} = (ds_1)\mathbf{e}_1$, $d\mathbf{x}^{(2)} = (ds_2)\mathbf{e}_2$, $d\mathbf{x}^{(3)} = ds_3/\sqrt{2}(\mathbf{e}_1 + \mathbf{e}_2)$ at the origin at t = 0.

3.40. Show that the following velocity components correspond to a rigid body motion:

$$v_1 = x_2 - x_3, \quad v_2 = -x_1 + x_3, \quad v_3 = x_1 - x_2$$

3.41. For the velocity field of Prob.3.15

(a) Find the rate of deformation and spin tensors.

(b) Find the rate of extension of a radial material line element.

3.42. Given the two-dimensional velocity field in cylindrical coordinates

$$v_r = 0, \quad v_\theta = 2r + \frac{4}{r}$$

(a) Find the acceleration at $r = 2$.

(b) Find the rate of deformation tensor at $r = 2$.