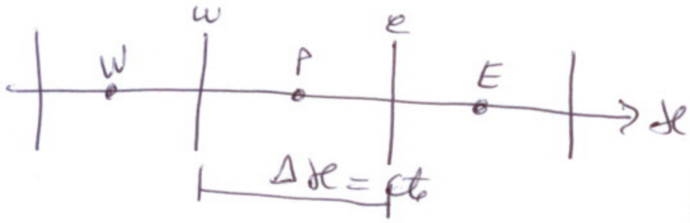
CDS

10 Jul 03

VOLUMES INTERNOS



$$Pe\frac{dT}{dx} = \frac{d^2T}{dx^2} \qquad T(0)=0 \quad T(1)=1$$

$$\int_{Vol} Pe\frac{dT}{dx}\,dV = \int_{Vol}\frac{d^2T}{dx^2}\,dV$$

$$\boxed{Pe(T_e - T_w)\cancel{A} = \left[\left(\frac{dT}{dx}\right)_e - \left(\frac{dT}{dx}\right)_w\right]\cancel{A}}$$

CDS: $T_e = \frac{(T_P + T_E)}{2} \qquad T_w = \frac{T_P + T_W}{2}$

$$\left(\frac{dT}{dx}\right)_e = \frac{(T_E - T_P)}{\Delta x} \qquad \left(\frac{dT}{dx}\right)_w = \frac{(T_P - T_W)}{\Delta x}$$

$$Pe\left[\frac{(\cancel{T_P}+T_E)}{2} - \frac{(\cancel{T_P}+T_W)}{2}\right] = \frac{(T_E - T_P)}{\Delta x} - \frac{(T_P - T_W)}{\Delta x}$$

$$\left(\frac{2}{\Delta x}\right)T_P = \left(\frac{1}{\Delta x} + \frac{Pe}{2}\right)T_W + \left(\frac{1}{\Delta x} - \frac{Pe}{2}\right)T_E$$

$$a_P T_P = a_W T_W + a_E T_E + b_P$$

$$a_W = \frac{1}{\Delta x} + \frac{Pe}{2} \qquad a_E = \frac{1}{\Delta x} - \frac{Pe}{2}$$

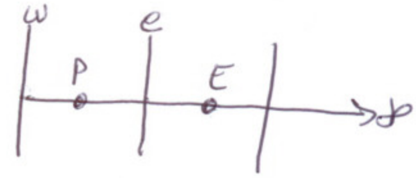
$$a_P = a_W + a_E = \frac{2}{\Delta x}$$

$$b_P = 0$$

Para $a_E \geq 0 \rightarrow \frac{1}{\Delta x} - \frac{Pe}{2} \geq 0$ (não oscilar)

$\frac{1}{\Delta x} \geq \frac{Pe}{2} \rightarrow 2 \geq Pe.\Delta x$ ou $Pe_{\Delta x} \leq 2$ volumes internos prevalece

1º VOLUME



$T_w = T(0) = 0$

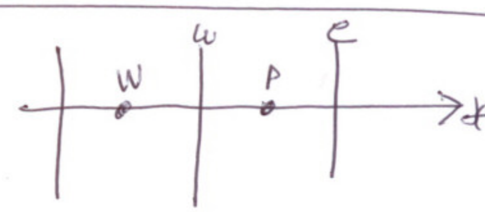
$$\left(\frac{dT}{dx}\right)_w = \frac{(T_P - \overset{=0}{T_w})}{\Delta x/2} = \frac{2T_P}{\Delta x}$$

$$Pe\left[\frac{(T_P + T_E)}{2} - 0\right] = \frac{(T_E - T_P)}{\Delta x} - \frac{2T_P}{\Delta x}$$

$a_W = 0, \qquad a_E = \frac{1}{\Delta x} - \frac{Pe}{2} \rightarrow Pe_{\Delta x} \leq 2$

$b_P = 0, \qquad a_P = \frac{3}{\Delta x} + \frac{Pe}{2}$

ÚLTIMO VOLUME



$T_e = T(1) = 1$

$$\left(\frac{dT}{dx}\right)_e = \frac{(\overset{=1}{T_e} - T_P)}{\Delta x/2} = 2\frac{(1 - T_P)}{\Delta x}$$

$$Pe\left[1 - \frac{(T_P + T_W)}{2}\right] = 2\frac{(1 - T_P)}{\Delta x} - \frac{(T_P - T_W)}{\Delta x}$$

$$a_W = \frac{1}{\Delta x} + \frac{Pe}{2} \qquad a_E = 0$$

$$a_P = \frac{3}{\Delta x} - \frac{Pe}{2}$$

$$b_P = \frac{2}{\Delta x} - Pe$$

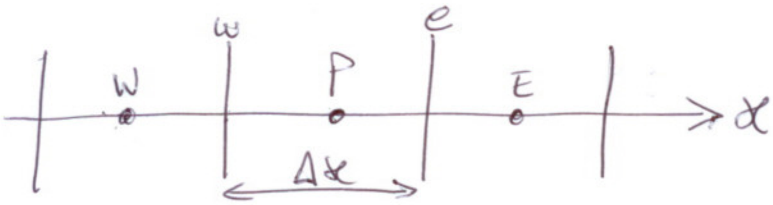
$a_P \geq 0 \rightarrow \frac{3}{\Delta x} - \frac{Pe}{2} \geq 0$

ou $\frac{3}{\Delta x} \geq \frac{Pe}{2} \rightarrow 6 \geq Pe.\Delta x$

$Pe_{\Delta x} \leq 6$

UDS

10 Jul 03



$\Delta x = cte$

$$Pe\,\frac{dT}{dx} = \frac{d^2T}{dx^2} \qquad T(0)=0 \quad T(1)=1$$

$$\int_{Vol} Pe\,\frac{dT}{dx}\,dx\,dy\,dz = \int_{Vol} \frac{d^2T}{dx^2}\,dx\,dy\,dz$$

$$\boxed{Pe\,(T_e - T_w)\cancel{A} = \left[\left(\frac{dT}{dx}\right)_e - \left(\frac{dT}{dx}\right)_w\right]\cancel{A}}$$

UDS: $\left.\begin{array}{l} T_e = T_P \\ T_w = T_W \end{array}\right\}$ p/ $Pe>0$

CDS: $\left(\frac{dT}{dx}\right)_e = \frac{(T_E - T_P)}{\Delta x}$

$$\left(\frac{dT}{dx}\right)_w = \frac{(T_P - T_W)}{\Delta x}$$

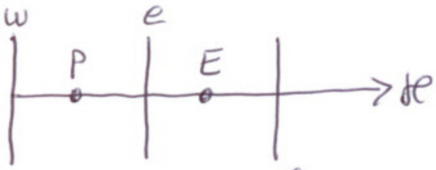
$$Pe\,(T_P - T_W) = \frac{(T_E - T_P)}{\Delta x} - \frac{(T_P - T_W)}{\Delta x}$$

$$\left(\frac{2}{\Delta x} + Pe\right)T_P = \left(\frac{1}{\Delta x} + Pe\right)T_W + \left(\frac{1}{\Delta x}\right)T_E$$

$$a_P T_P = a_w T_W + a_e T_E + b_P$$

$$\left.\begin{array}{l} a_P = \frac{2}{\Delta x} + Pe = a_w + a_e \\ a_w = \frac{1}{\Delta x} + Pe \\ a_e = \frac{1}{\Delta x} \\ b_P = 0 \end{array}\right\} \text{VOLUMES INTERNOS}$$

1º VOLUME



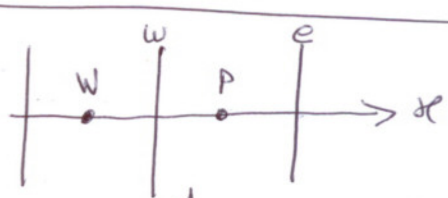
$T_w = T(0) = 0 \qquad \left(\frac{dT}{dx}\right)_w = \frac{(T_P - \overset{=0}{T_w})}{\Delta x/2} = \frac{2T_P}{\Delta x}$

$$Pe\,(T_P - 0) = \frac{(T_E - T_P)}{\Delta x} - \frac{2T_P}{\Delta x}$$

$a_w = 0, \quad a_e = \frac{1}{\Delta x}, \quad b_P = 0$

$a_P = \frac{3}{\Delta x} + Pe$

ÚLTIMO VOLUME:



$T_e = T(1) = 1 \qquad \left(\frac{dT}{dx}\right)_e = \frac{(\overset{=1}{T_e} - T_P)}{\Delta x/2} = \frac{2(1-T_P)}{\Delta x}$

$$Pe\,(T_P - T_W) = \frac{2(1-T_P)}{\Delta x} - \frac{(T_P - T_W)}{\Delta x}$$

$a_w = \frac{1}{\Delta x} + Pe \qquad a_e = 0$

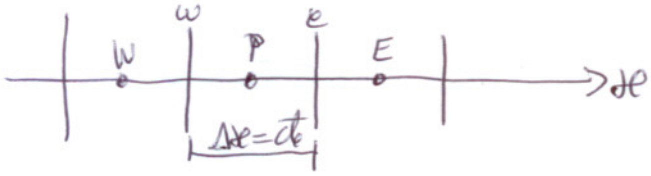
$a_P = \frac{3}{\Delta x} + Pe$

$b_P = \frac{2}{\Delta x}$

EXATO

10 Jul 03

VOLUMES INTERNOS



$$Pe \frac{dT}{dx} = \frac{d^2T}{dx^2} \qquad T(0)=0 \quad T(1)=1$$

$$\int_{Vol} Pe \frac{dT}{dx} dV = \int_{Vol} \frac{d^2T}{dx^2} dV$$

$$\boxed{Pe\,(T_e - T_w)\,\cancel{A} = \left[\left(\frac{dT}{dx}\right)_e - \left(\frac{dT}{dx}\right)_w\right]\cancel{A}}$$

EXATO: $T_e = (1/2+\alpha)T_P + (1/2-\alpha)T_E$

$T_w = (1/2+\alpha)T_W + (1/2-\alpha)T_P$

$$\left(\frac{dT}{dx}\right)_e = \beta\,\frac{(T_E - T_P)}{\Delta x} \qquad \left(\frac{dT}{dx}\right)_w = \beta\,\frac{(T_P - T_W)}{\Delta x}$$

$$\alpha = \frac{1}{2}\ \cancel{4}\ \frac{(e^{Pe_{\Delta x}/2} - 1)}{(e^{Pe_{\Delta x}} - 1)}$$

$$\beta = Pe_{\Delta x}\frac{e^{Pe_{\Delta x}/2}}{(e^{Pe_{\Delta x}} - 1)}$$

Solução analítica da adv.-difusão [ver anexo]

p/ $Pe > 0$

$$Pe_{\Delta x} = \frac{Pe}{N}$$

$$Pe\left[(1/2+\alpha)T_P + (1/2-\alpha)T_E - (1/2+\alpha)T_W - (1/2-\alpha)T_P\right] = \frac{\beta(T_E - T_P)}{\Delta x} - \frac{\beta(T_P - T_W)}{\Delta x}$$

$$a_P T_P = a_W T_W + a_E T_E + b_P$$

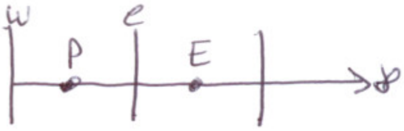
$$a_W = \frac{\beta}{\Delta x} + (1/2+\alpha)Pe$$

$$a_E = \frac{\beta}{\Delta x} - (1/2-\alpha)Pe$$

$$a_P = \frac{2\beta}{\Delta x} + 2\alpha Pe = a_W + a_E$$

$$b_P = 0$$

1º VOLUME



$T_w = T(0) = 0$

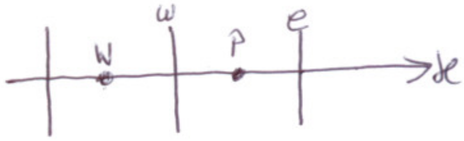
$$\left(\frac{dT}{dx}\right)_w = \frac{Pe}{(e^{Pe}-1)}$$

$$Pe\left[(1/2+\alpha)T_P + (1/2-\alpha)T_E - 0\right] = \beta\frac{(T_E - T_P)}{\Delta x} - \frac{Pe}{(e^{Pe}-1)}$$

$a_W = 0,\quad a_E = \frac{\beta}{\Delta x} - (1/2-\alpha)Pe$

~~$a_P = a_E$~~ $\quad a_P = \frac{\beta}{\Delta x} + (1/2+\alpha)Pe, \quad b_P = -\frac{Pe}{(e^{Pe}-1)}$

ÚLTIMO VOLUME



$T_e = T(1) = 1$

$$\left(\frac{dT}{dx}\right)_e = \frac{Pe\,e^{Pe}}{(e^{Pe}-1)} \rightarrow \text{OU} \rightarrow \cancel{\left(\frac{dT}{dx}\right)_e = \beta\frac{(T_e - T_P)}{\Delta x/2} = \frac{2(1-T_P)\beta}{\Delta x}}$$

$$Pe\left[1 - (1/2+\alpha)T_W - (1/2-\alpha)T_P\right] = \frac{Pe\,e^{Pe}}{(e^{Pe}-1)} - \frac{(T_P - T_W)\beta}{\Delta x}$$

$a_W = \frac{\beta}{\Delta x} + (1/2+\alpha)Pe, \quad a_E = 0$

$a_P = \frac{\beta}{\Delta x} - (1/2-\alpha)Pe, \quad \cancel{b_P = Pe\frac{(e^{Pe}-1)}{(e^{Pe}-1)} - Pe}$

OU

$$\cancel{Pe\left[1 - (1/2+\alpha)T_W - (1/2-\alpha)T_P\right] = \beta(T_e - T_P)}$$

$$b_P = Pe\left[\frac{e^{Pe}}{(e^{Pe}-1)} - 1\right] = Pe\frac{(e^{Pe} - e^{Pe} + 1)}{(e^{Pe}-1)}$$

$$b_P = \frac{Pe}{(e^{Pe}-1)}$$

$$T = \frac{(e^{xPe} - 1)}{(e^{Pe} - 1)} \qquad x=0 \rightarrow T(0)=0 \quad x=1 \rightarrow T(1)=1$$

$$\frac{dT}{dx} = Pe\frac{(e^{xPe})}{(e^{Pe}-1)} \qquad x=0 \rightarrow \left(\frac{dT}{dx}\right)_0 = \frac{Pe}{(e^{Pe}-1)} \quad x=1 \rightarrow \left(\frac{dT}{dx}\right)_1 = Pe\frac{e^{Pe}}{(e^{Pe}-1)}$$

ADUP / MODELOS E PADRÕES / 1

~~[illegible]~~ EXPONENCIAL ~~[illegible]~~ 8 Jun 98 ~~[illegible]~~

$$\frac{\partial \phi^*}{\partial y} = \frac{\partial^2 \phi^*}{\partial y^2}$$

$\phi^* = 0$ em $y = 0$

$\phi^* = 1$ em $y = Pe_{\Delta x}$

$$\phi^* = \frac{e^{y} - 1}{e^{Pe_{\Delta x}} - 1}$$

$$Pe_{\Delta x} = \frac{\rho u \Delta x}{\Gamma^{\phi}} = \frac{u \Delta x}{\alpha}$$

$$\alpha_e = \frac{1}{2} - \frac{(e^{Pe_{\Delta x}/2} - 1)}{(e^{Pe_{\Delta x}} - 1)}$$

$$\phi^*_e = (1/2 + \alpha_e)\,\phi^*_P + (1/2 - \alpha_e)\,\phi^*_E$$

$$\beta_e = Pe_{\Delta x} \frac{e^{Pe_{\Delta x}/2}}{(e^{Pe_{\Delta x}} - 1)}$$

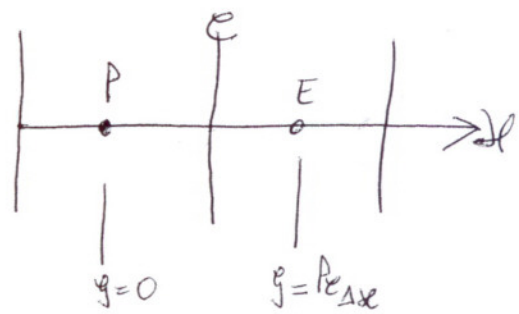
$$\Gamma^{\phi}_e \left.\frac{\partial \phi^*}{\partial y}\right|_e = \beta_e \Gamma^{\phi}_e \frac{(\phi^*_E - \phi^*_P)}{\Delta y}$$

WUDS:

$$\alpha_e = \frac{Pe_{\Delta x}^2}{(10 + 2 Pe_{\Delta x}^2)}$$

$$\phi^* = \frac{T - T_0}{T_L - T_0}$$

$$\beta_e = \frac{(1 + 0{,}005\, Pe_{\Delta x}^2)}{(1 + 0{,}05\, Pe_{\Delta x}^2)}$$



$$\boxed{\rho u \frac{dT}{dx} = \alpha \frac{d^2 T}{dx^2}}$$

$$\frac{d\phi^*}{dx^*} = \frac{1}{Pe} \frac{d^2 \phi^*}{dx^{*2}} \qquad x^* = \frac{x}{L}$$

SOLUÇÃO ANALÍTICA:

$$T = T_0 + \frac{(e^{\frac{x}{L} Pe} - 1)}{(e^{Pe} - 1)} (T_L - T_0)$$

$$Pe = \frac{uL}{\alpha}$$

EXATO

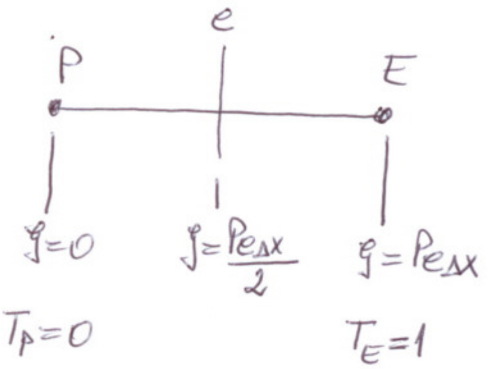
10 Jul 03

COEFICIENTES α E β

$$T_e = (1/2+\alpha)T_P + (1/2-\alpha)T_E = \frac{(T_P+T_E)}{2} + \alpha(T_P - T_E)$$

$$2T_e = T_P + T_E + 2\alpha(T_P - T_E)$$

$$\boxed{\alpha = \frac{(2T_e - T_P - T_E)}{2(T_P - T_E)}}$$



$$T = \frac{e^{\xi} - 1}{e^{Pe_{\Delta x}} - 1}$$

$$T_e = \frac{e^{Pe_{\Delta x}/2} - 1}{e^{Pe_{\Delta x}} - 1} \rightarrow \alpha = \frac{\frac{2(e^{Pe_{\Delta x}/2} - 1)}{(e^{Pe_{\Delta x}} - 1)} - 0 - 1}{2(0-1)} = \boxed{\frac{1}{2} - \frac{(e^{Pe_{\Delta x}/2} - 1)}{(e^{Pe_{\Delta x}} - 1)} = \alpha} \text{ OK}$$

$$\left.\frac{dT}{d\xi}\right| = \frac{e^{\xi}}{e^{Pe_{\Delta x}} - 1}$$

$$\left(\frac{dT}{d\xi}\right)_e = \beta\,\frac{(T_E - T_P)}{\underset{\displaystyle Pe_{\Delta x}}{\underset{\shortparallel}{\Delta x}}} \longrightarrow \beta = \frac{\left(\frac{dT}{d\xi}\right)_e Pe_{\Delta x}}{(T_E - T_P)} = \frac{Pe_{\Delta x}\, e^{Pe_{\Delta x}/2} / (e^{Pe_{\Delta x}} - 1)}{(1-0)} = \boxed{\frac{Pe_{\Delta x}\, e^{Pe_{\Delta x}/2}}{(e^{Pe_{\Delta x}} - 1)} = \beta} \text{ OK}$$

EXATO

10 Jul 03

PRIMEIRO VOLUME

$\frac{Pe_{\Delta x}^{\text{original}}}{2}$

$$T = \frac{e^{\xi} - 1}{e^{Pe_{\Delta x}} - 1}$$

$T_w = 0 \quad (\xi = 0)$

$T_P = 1 \quad (\xi = Pe_{\Delta x})$

$$\frac{dT}{d\xi} = \frac{e^{\xi}}{(e^{Pe_{\Delta x}} - 1)}$$

$$\left(\frac{dT}{d\xi}\right)_w = \beta_0 \frac{(T_P - T_w)}{Pe_{\Delta x}} \longrightarrow \beta_0 = \frac{\left(\frac{dT}{d\xi}\right)_w Pe_{\Delta x}}{(T_P - T_w)} = \frac{Pe_{\Delta x}\left(\frac{1}{e^{Pe_{\Delta x}} - 1}\right)}{1 - 0} = \frac{Pe_{\Delta x}}{(e^{Pe_{\Delta x}} - 1)}$$

No original:

$$\boxed{\beta_0 = \frac{Pe_{\Delta x}}{2(e^{Pe_{\Delta x}/2} - 1)}} \quad e \quad \left(\frac{dT}{dx}\right)_w = \beta_0 \frac{(T_P - \overset{0}{T_w})}{\Delta x/2} = \frac{2\beta_0 T_P}{\Delta x}$$

$$P_e(T_e - T_w) = \left(\frac{dT}{dx}\right)_e - \left(\frac{dT}{dx}\right)_w$$

$$P_e\left[(1/2 + \alpha)T_P + (1/2 - \alpha)T_E - 0\right] = \beta\frac{(T_E - T_P)}{\Delta x} - \frac{2\beta_0 T_P}{\Delta x}$$

$$a_w = 0$$

$$a_e = \frac{\beta}{\Delta x} - (1/2 - \alpha)P_e$$

$$a_p = \frac{\beta}{\Delta x} + \frac{2\beta_0}{\Delta x} + (1/2 + \alpha)P_e$$

$$b_p = 0$$

10 Jul 03

ÚLTIMO VOLUME — EXATO

$$T = \frac{e^{\xi}-1}{e^{Pe_{\Delta x}}-1}$$

$\xi = 0$

$\xi = Pe_{\Delta x} \leftarrow \frac{Pe_{\Delta x}^{\text{original}}}{2}$

$T_P = 0$ e $T_e = 1$

$$\frac{dT}{d\xi} = \frac{e^{\xi}}{e^{Pe_{\Delta x}}-1}$$

$$\left(\frac{dT}{d\xi}\right)_e = \beta_1 \frac{(T_e - T_P)}{Pe_{\Delta x}} \rightarrow \beta_1 = \frac{\left(\frac{dT}{d\xi}\right)_e Pe_{\Delta x}}{(T_e - T_P)} = \frac{Pe_{\Delta x}\left(\frac{e^{Pe_{\Delta x}}}{e^{Pe_{\Delta x}}-1}\right)}{(1-0)} = \frac{Pe_{\Delta x}\, e^{Pe_{\Delta x}}}{(e^{Pe_{\Delta x}}-1)}$$

No original: $\boxed{\beta_1 = \frac{Pe_{\Delta x}\, e^{Pe_{\Delta x}/2}}{2\left(e^{Pe_{\Delta x}/2}-1\right)}}$ e $\left(\frac{dT}{dx}\right)_e = \beta_1 \frac{(\overset{1}{T_e} - T_P)}{\Delta x/2} = 2\frac{(1-T_P)}{\Delta x}\beta_1$

$$P_e(T_e - T_w) = \left(\frac{dT}{dx}\right)_e - \left(\frac{dT}{dx}\right)_w$$

$$P_e\left[1 - (1/2+\alpha)T_W - (1/2-\alpha)T_P\right] = \beta_1\, 2\frac{(1-T_P)}{\Delta x} - \beta\frac{(T_P - T_W)}{\Delta x}$$

$$a_w = \frac{\beta}{\Delta x} + (1/2+\alpha) P_e$$

$$a_e = 0$$

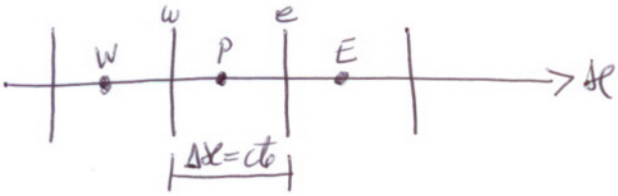
$$a_p = \frac{\beta}{\Delta x} + \frac{2\beta_1}{\Delta x} - (1/2-\alpha) P_e$$

$$b_p = \frac{2\beta_1}{\Delta x} - P_e$$

ALFA

11 Jul 03

VOLUMES INTERNOS



$\Delta x = cte$

$$Pe \frac{dT}{dx} = \frac{d^2T}{dx^2} \qquad T(0)=0,\quad T(1)=1$$

$$\int_{Vol} Pe\frac{dT}{dx}dVol = \int_{Vol}\frac{d^2T}{dx^2}dV$$

$$\boxed{Pe(T_e - T_w)\cancel{A} = \left[\left(\frac{dT}{dx}\right)_e - \left(\frac{dT}{dx}\right)_w\right]\cancel{A}}$$

ALFA: $T_e = (1/2+\alpha)T_P + (1/2-\alpha)T_E$

$T_w = (1/2+\alpha)T_W + (1/2-\alpha)T_P$

$\alpha =$ dado

$$\left(\frac{dT}{dx}\right)_e = \frac{(T_E - T_P)}{\Delta x} \qquad \left(\frac{dT}{dx}\right)_w = \frac{(T_P - T_W)}{\Delta x}$$

$$Pe\left[(1/2+\alpha)T_P + (1/2-\alpha)T_E - (1/2+\alpha)T_W - (1/2-\alpha)T_P\right] = \frac{(T_E - T_P)}{\Delta x} - \frac{(T_P - T_W)}{\Delta x}$$

$$\boxed{a_P T_P = a_W T_W + a_E T_E + b_P}$$

$$a_w = \frac{1}{\Delta x} + (1/2+\alpha)Pe$$

$$a_e = \frac{1}{\Delta x} - (1/2-\alpha)Pe$$

$$a_P = a_w + a_e = \frac{2}{\Delta x} + 2\alpha Pe$$

$$b_P = 0$$

Para $a_e \geq 0$:

$\frac{1}{\Delta x} - \left(\frac{1}{2}-\alpha\right)Pe \geq 0$

$\frac{1}{\Delta x} \geq \left(\frac{1}{2}-\alpha\right)Pe$

$1 \geq \left(\frac{1}{2}-\alpha\right)Pe_{\Delta x}$

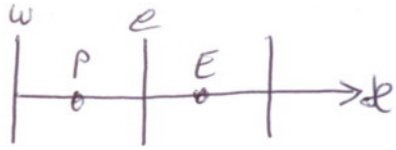
$\alpha \geq \frac{1}{2} - \frac{1}{Pe_{\Delta x}}$

$Pe_{\Delta x}$	α_{MIN}
$\to 0$	$\to -\infty$
1	$\to -1/2$
2	$\to 0$ (CDS)
5	$\to 0{,}3$
10	$\to 0{,}4$
$\to\infty$	$\to 1/2$ (UDS)

$\alpha = 0 \longrightarrow$ CDS (OK)

$\alpha = 1/2 \longrightarrow$ UDS (OK)

1º VOLUME



$T_w = T(0) = 0$

$$\left(\frac{dT}{dx}\right)_w = \frac{(T_P - \overset{=0}{T_w})}{\Delta x/2} = \frac{2T_P}{\Delta x}$$

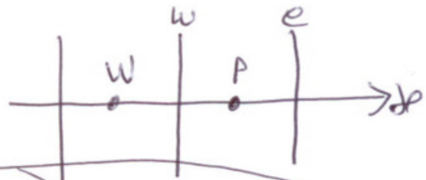
$$Pe\left[(1/2+\alpha)T_P + (1/2-\alpha)T_E - 0\right] = \frac{(T_E - T_P)}{\Delta x} - \frac{2T_P}{\Delta x}$$

$$a_w = 0, \qquad a_e = \frac{1}{\Delta x} - (1/2-\alpha)Pe$$

$$b_P = 0, \qquad a_P = \frac{3}{\Delta x} + (1/2+\alpha)Pe$$

$\alpha_{MIN} =$ vols. internos

ÚLTIMO VOLUME



$T_e = T(1) = 1$

$$\left(\frac{dT}{dx}\right)_e = \frac{(\overset{=1}{T_e} - T_P)}{\Delta x/2} = \frac{2(1-T_P)}{\Delta x}$$

$$T_e = T_P + 2(1/2-\alpha)(1-T_P)$$

$$Pe\left[\cancel{X} - (1/2+\alpha)T_W - (1/2-\alpha)T_P\right] = \frac{2(1-T_P)}{\Delta x} - \frac{(T_P - T_W)}{\Delta x}$$

$$a_w = \frac{1}{\Delta x} + (1/2+\alpha)Pe, \qquad a_e = 0$$

$$a_P = \frac{3}{\Delta x} - (1/2-\alpha)Pe + 2\alpha Pe$$

$$b_P = \frac{2}{\Delta x} - Pe + 2\alpha Pe$$

α_{MIN}: $a_P \geq 0$,

$$\frac{3}{\Delta x} - \frac{Pe}{2} + \underbrace{\alpha Pe + 2\alpha Pe}_{3\alpha Pe} \geq 0$$

$$\frac{3}{\Delta x} \geq Pe\left(\frac{1}{2} - 3\alpha\right)$$

$$3 \geq \left(\frac{1}{2} - 3\alpha\right)Pe_{\Delta x}$$

C.C. direita: $\alpha \geq \frac{1}{2} - \frac{3}{Pe_{\Delta x}}$

Menos restritivo

Se $Pe_{\Delta x} = 2$

$\alpha \geq \frac{1}{2} - \frac{3}{2} = -\frac{1}{2}$

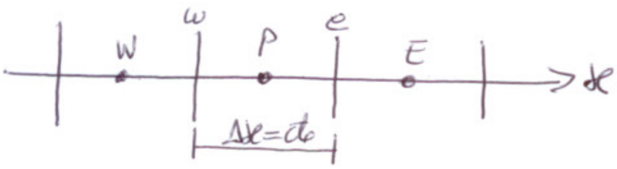
$$T_e = T_P + 2(1/2-\alpha)(1-T_P)$$
$$= T_P[1 - 1 + 2\alpha] + 1 - 2\alpha = 2\alpha T_P + 1 - 2\alpha$$

Se $\alpha = 1/2 \longrightarrow T_e = T_P$ (UDS)

Se $\alpha = 0 \longrightarrow T_e = 1 = T(1)$ (CDS)

TVD/SUPERBEE/$P_e>0$ 11 Jul 03

VOLUMES INTERNOS



$$P_e \frac{dT}{dx} = \frac{d^2T}{dx^2} \qquad T(0)=0, \quad T(1)=1$$

$$\int_{Vol} P_e \frac{dT}{dx}\, dV = \int_{Vol} \frac{d^2T}{dx^2}\, dV$$

$$\boxed{P_e(T_e - T_w)\cancel{A} = \left[\left(\frac{dT}{dx}\right)_e - \left(\frac{dT}{dx}\right)_w\right]\cancel{A}}$$

TVD: $T_e = (1/2+\alpha_e)T_P + (1/2-\alpha_e)T_E$

$T_w = (1/2+\alpha_w)T_W + (1/2-\alpha_w)T_P$

Adaptação p/ usar α

$\alpha_e = \frac{1}{2}(1-\Psi_e) \qquad \alpha_w = \frac{1}{2}(1-\Psi_w)$

superbee (Roe, 1983)

$\Psi_e = Max\left[0;\ Min(2r_e;1);\ Min(r_e;2)\right]$ [0,2]

$\Psi_e = 0 \to \alpha_e = 1/2$ (UDS); $\Psi_e = 1 \to \alpha_e = 0$ (CDS); $\Psi_e = 2 \to \alpha_e = -1/2$ (DDS)

$r_e = \frac{(T_P - T_W)}{(T_E - T_P)}$ (para $P = 2, 3, \ldots N-1$ e $P_e > 0$) ITERATIVO

~~Se $(T_E - T_P) = 0$, $r_e \to \infty$, $\Psi_e \to 2$, $\alpha_e \to -1/2$~~

$\left(\frac{dT}{dx}\right)_e = \frac{(T_E - T_P)}{\Delta x} \qquad \left(\frac{dT}{dx}\right)_w = \frac{(T_P - T_W)}{\Delta x}$

Se $T_E - T_P = 0$ → Mas fazer $\alpha_e = 1/2$ c/ $r_e = 0$, $\Psi_e = 0$

$$P_e\left[(1/2+\alpha_e)T_P + (1/2-\alpha_e)T_E - (1/2+\alpha_w)T_W - (1/2-\alpha_w)T_P\right] = \frac{(T_E - T_P)}{\Delta x} - \frac{(T_P - T_W)}{\Delta x}$$

$$a_P T_P = a_w T_W + a_e T_E + b_P$$

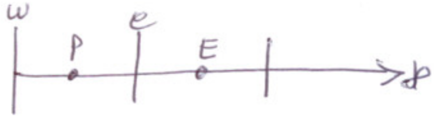
$a_w = \frac{1}{\Delta x} + (1/2+\alpha_w)P_e, \quad b_P = 0$

$a_e = \frac{1}{\Delta x} - (1/2-\alpha_e)P_e$

$a_P = \frac{2}{\Delta x} + (1/2+\alpha_e)P_e - (1/2-\alpha_w)P_e$

$= \frac{2}{\Delta x} + (\alpha_w + \alpha_e)P_e = a_w + a_e$

1º VOLUME



$T_w = T(0) = 0$

$\left(\frac{dT}{dx}\right)_w = \frac{(T_P - T_w)}{\Delta x/2} = \frac{2T_P}{\Delta x}$

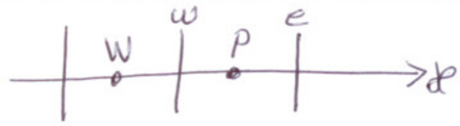
$r_e = \frac{2(T_P - T_w)}{(T_E - T_P)} = \frac{2T_P}{(T_E - T_P)}$ | Se $T_E = T_P \to r_e = 0 \to \Psi = 0 \to \alpha_e = 1/2$

$$P_e\left[(1/2+\alpha_e)T_P + (1/2-\alpha_e)T_E - 0\right] = \frac{(T_E - T_P)}{\Delta x} - \frac{2T_P}{\Delta x}$$

$a_w = 0, \quad a_e = \frac{1}{\Delta x} - (1/2-\alpha_e)P_e$

$b_P = 0, \quad a_P = \frac{3}{\Delta x} + (1/2+\alpha_e)P_e$

ÚLTIMO VOLUME



$T_e = T(1) = 1$

$\left(\frac{dT}{dx}\right)_e = \frac{(T_e - T_P)}{\Delta x/2} = \frac{2(1-T_P)}{\Delta x}$

$T_e = T_P + 2(1/2-\alpha_e)(1-T_P) = 2\alpha_e T_P + 1 - 2\alpha_e$

$r_e = \frac{(T_P - T_W)}{2(T_e - T_P)} = \frac{(T_P - T_W)}{2(1-T_P)}$ | Se $T_P = 1 \to r_e = 0 \to \Psi = 0 \to \alpha_e = 1/2$.

$$P_e\left[2\alpha_e T_P + 1 - 2\alpha_e - (1/2+\alpha_w)T_W - (1/2-\alpha_w)T_P\right] = 2\frac{(1-T_P)}{\Delta x} - \frac{(T_P - T_W)}{\Delta x}$$

$a_w = \frac{1}{\Delta x} + (1/2+\alpha_w)P_e, \quad a_e = 0$

$a_P = \frac{3}{\Delta x} - (1/2-\alpha_w)P_e + 2\alpha_e P_e$

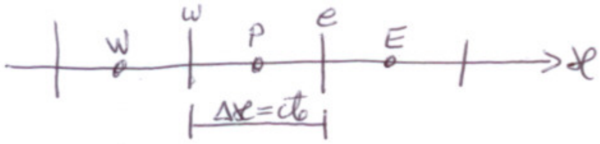
$b_P = \frac{2}{\Delta x} - P_e + 2\alpha_e P_e$

cálculo de α é explícito → $P_{e\,\Delta x} = \frac{P_e}{N} < 2$; se não, α_{TVD} não converge

É IDÊNTICO AO ESQUEMA ALFA MAS COM α VARIÁVEL EM CADA NÓ E OBTIDO EM FUNÇÃO DE T (NÃO-LINEARIDADE).

ADS

VOLUMES INTERNOS



$$Pe \frac{dT}{dx} = \frac{d^2T}{dx^2} \qquad T(0)=0,\; T(1)=1$$

$$\int_{vol} Pe \frac{dT}{dx} dV = \int_{vol} \frac{d^2T}{dx^2} dV$$

$$\boxed{Pe\,(T_e - T_w)\cancel{A} = \left[\left(\frac{dT}{dx}\right)_e - \left(\frac{dT}{dx}\right)_w\right]\cancel{A}}$$

ADS: $T_e = (1/2+\alpha_e)T_P + (1/2-\alpha_e)T_E$

$T_w = (1/2+\alpha_w)T_W + (1/2-\alpha_w)T_P$

$$\alpha_e^* = \frac{(2T_e^* - T_P^* - T_E^*)}{2\,(T_P^* - T_E^*)} \qquad \text{ITERATIVO}$$

$\rightarrow$ Se $(T_P^* = T_E^*)$, $\alpha_e^* = 1/2$

$$\alpha_e = Min\left[\frac{1}{2};\ Max\left(-\frac{1}{2};\ \alpha_e^*\right)\right] \Rightarrow \alpha_e = [-1/2,\ 1/2]$$

$$T_e^* = Max\left[Min(T_e^-; T_e^m); Min(T_e^-; T_e^+); Min(T_e^m; T_e^+)\right]$$

$$T_e^- = T_P^* + \frac{(T_P^* - T_W^*)}{2} \qquad T_e^m = \frac{(T_P^* + T_E^*)}{2}$$

$$T_e^+ = T_E^* - \frac{(T_{EE}^* - T_E^*)}{2} \qquad \text{para } P = 2, 3, \dots N-2$$

$$\left(\frac{dT}{dx}\right)_e = \frac{(T_E - T_P)}{\Delta x} \qquad \left(\frac{dT}{dx}\right)_w = \frac{(T_P - T_W)}{\Delta x}$$

$$Pe\left[(1/2+\alpha_e)T_P + (1/2-\alpha_e)T_E - (1/2+\alpha_w)T_W - (1/2-\alpha_w)T_P\right] = \frac{(T_E - T_P)}{\Delta x} - \frac{(T_P - T_W)}{\Delta x}$$

$$\boxed{a_P T_P = a_W T_W + a_E T_E + b_P}$$ (5 volumes envolvidos)

$$a_w = \frac{1}{\Delta x} + (1/2+\alpha_w)Pe \qquad b_P = 0$$

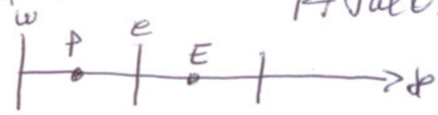
$$a_e = \frac{1}{\Delta x} - (1/2-\alpha_e)Pe$$

$$a_P = a_w + a_e$$

Parece que não tem limite de $Pe_{\Delta x}$ para convergir cálculo de α.

14 Jul 03

1º VOLUME (alterações)



$T_w = T(0) = 0$

$$\left(\frac{dT}{dx}\right)_w = \frac{(T_P - T_w)}{\Delta x/2} = \frac{2T_P}{\Delta x}$$

(3 volumes)

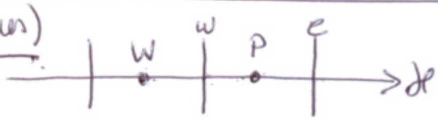
$$T_e^- = T_P^* + \frac{(T_P^* - \cancel{T_w})}{1} = 2T_P^* \quad \Big| \quad \text{Se } T_P^* = T_E^* \rightarrow \alpha_e = 1/2$$

$$Pe\left[(1/2+\alpha_e)T_P + (1/2-\alpha_e)T_E - 0\right] = \frac{(T_E - T_P)}{\Delta x} - \frac{2T_P}{\Delta x}$$

$$a_w = 0, \qquad a_e = \frac{1}{\Delta x} - (1/2-\alpha_e)Pe$$

$$b_P = 0, \qquad a_P = \frac{3}{\Delta x} + (1/2+\alpha_e)Pe$$

ÚLTIMO VOLUME (alterações) (2 volumes)



$T_e = T(1) = 1$

$$\left(\frac{dT}{dx}\right)_e = \frac{(T_e - T_P)}{\Delta x/2} = \frac{2(1 - T_P)}{\Delta x} \quad \Big| \quad \alpha_e^* = \frac{(T_e^* - 1)}{2(T_P^* - 1)}$$

Do esquema ALFA:

$$T_e^* = T_P + 2(1/2-\alpha_e)(1-T_P) = 2\alpha_e T_P + 1 - 2\alpha_e$$

$$T_e^* = T_e = T(1) = 1; \quad T_e^m = T_P^* \;\Big|\; \cancel{\alpha_e^* = 1/2 \ldots}$$

? Se $T_P^* = 1 \rightarrow \alpha_e^* = 1/2$

$$Pe\left[2\alpha_e T_P + 1 - 2\alpha_e - (1/2+\alpha_w)T_W - (1/2-\alpha_w)T_P\right] = \frac{2(1-T_P)}{\Delta x} - \frac{(T_P - T_W)}{\Delta x}$$

$$a_w = \frac{1}{\Delta x} + (1/2+\alpha_w)Pe, \qquad a_e = 0$$

$$a_P = \frac{3}{\Delta x} - (1/2-\alpha_w)Pe + 2\alpha_e Pe$$

$$b_P = \frac{2}{\Delta x} - Pe + 2\alpha_e Pe$$

É IDÊNTICO AO ESQUEMA ALFA MAS COM α VARIÁVEL EM CADA NÓ E OBTIDO EM FUNÇÃO DE T (NÃO-LINEARIDADE).

PENÚLTIMO VOLUME (N−1)

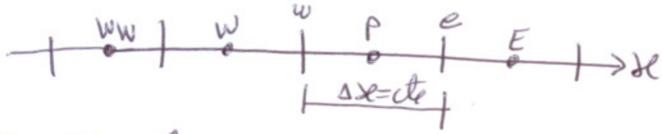
$$T_e^+ = T_E^* - (1 - T_E^*) = 2T_E^* - 1$$

C.C.

QUICK $(Pe>0)$

15 Jul 03

VOLUMES INTERNOS



$$Pe\frac{dT}{dx} = \frac{d^2T}{dx^2} \qquad T(0)=0 \quad T(1)=1$$

$$\int_{Vol} Pe\frac{dT}{dx}dV = \int_{Vol}\frac{d^2T}{dx^2}dV$$

$$\boxed{Pe(T_e - T_w)\cancel{A} = \left[\left(\frac{dT}{dx}\right)_e - \left(\frac{dT}{dx}\right)_w\right]\cancel{A}}$$

QUICK: $T_e = T_P + \Delta T_e = \frac{6}{8}T_P + \frac{3}{8}T_E - \frac{T_W}{8}$

$Pe>0 \quad T_w = T_W + \Delta T_w = \frac{6}{8}T_W + \frac{3}{8}T_P - \frac{T_{WW}}{8}$

$$\left.\begin{aligned}\Delta T_e &= \frac{(3T_E - 2T_P - T_W)}{8}\\ \Delta T_w &= \frac{(3T_P - 2T_W - T_{WW})}{8}\end{aligned}\right\}$$ correção da parábola p/ a reta

$$\left(\frac{dT}{dx}\right)_e = \frac{(T_E - T_P)}{\Delta x} \qquad \left(\frac{dT}{dx}\right)_w = \frac{(T_P - T_W)}{\Delta x}$$

$$Pe\left[\overbrace{T_P + \Delta T_e}^{T_e} - \overbrace{T_W - \Delta T_w}^{T_w}\right] = \frac{(T_E - T_P)}{\Delta x} - \frac{(T_P - T_W)}{\Delta x}$$

$$Pe\left[\frac{6}{8}T_P + \frac{3}{8}T_E - \frac{T_W}{8} - \frac{6}{8}T_W - \frac{3}{8}T_P + \frac{T_{WW}}{8}\right]$$

$$\boxed{a_P T_P = a_w T_W + a_e T_E + b_P} \quad P = 3, 4, \ldots N-1$$

$$a_w = \frac{1}{\Delta x} + Pe\frac{7}{8} \qquad \underbrace{a_e = \frac{1}{\Delta x} - \frac{3}{8}Pe}$$

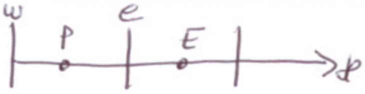
$$a_P = \frac{2}{\Delta x} + Pe\frac{3}{8}$$

$$b_P = \cancel{\Delta T_e - \Delta T_w} = -\frac{Pe}{8}T_{WW}$$

ITERATIVO

(4 volumes)

1º VOLUME



$T_w = T(0) = 0$

$$\Delta T_e = \frac{(3T_E - T_P - 2T_w)}{8}$$ considera $T_W = 2T_w - T_P$ (extrapolação linear c/ T_P e T_w p/ fictício)

$$T_e = \frac{7}{8}T_P + \frac{3}{8}T_E - \frac{2}{8}T_w$$

$$\left(\frac{dT}{dx}\right)_w = \frac{(9T_P - 8T_w - T_E)}{3\Delta x}$$

$$Pe\left[\underbrace{T_P + \Delta T_e}_{T_e} - \overset{0}{T_w}\right] = \frac{(T_E - T_P)}{\Delta x} - \frac{(9T_P - 8\overset{0}{T_w} - T_E)}{3\Delta x}$$

$$Pe\left[\frac{7}{8}T_P + \frac{3}{8}T_E\right] =$$

$$a_w = 0, \quad a_e = \frac{1}{\Delta x} + \frac{1}{3\Delta x} - \frac{3}{8}Pe = \underbrace{\frac{4}{3\Delta x} - \frac{3}{8}Pe}$$

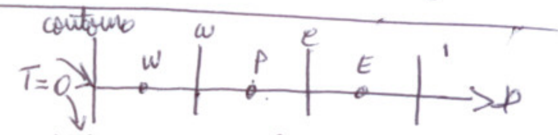
$$b_P = 0, \quad a_P = \frac{4}{\Delta x} + \frac{7}{8}Pe$$

$$Pe_{\Delta x} \le \frac{32}{9} \simeq 3{,}55\ldots$$

2 volumes

2º VOLUME

T_e do 1º volume



$$T_w = \frac{7}{8}T_W + \frac{3}{8}T_P - \frac{2}{8}T(0) = \frac{7}{8}T_W + \frac{3}{8}T_P$$

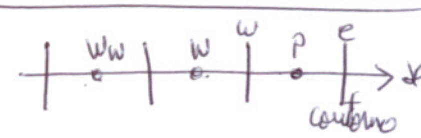
$$Pe\left[\frac{6}{8}T_P + \frac{3}{8}T_E - \frac{T_W}{8} - \frac{7}{8}T_W - \frac{3}{8}T_P\right] = \frac{(T_E - T_P)}{\Delta x} - \frac{(T_P - T_W)}{\Delta x}$$

$$a_w = \frac{1}{\Delta x} + Pe \qquad \underbrace{a_e = \frac{1}{\Delta x} - Pe\frac{3}{8}}$$

$$a_P = \frac{2}{\Delta x} + \frac{Pe\,3}{8} \qquad b_P = 0 \qquad Pe_{\Delta x} \le \frac{8}{3}$$

3 volumes

ÚLTIMO VOLUME



$T_e = T(1) = 1$

$$\left(\frac{dT}{dx}\right)_e = \frac{(8T_e - 9T_P + T_W)}{3\Delta x}$$

$$Pe\left[\overset{T_e}{1} - \overbrace{\frac{6}{8}T_W - \frac{3}{8}T_P + \frac{T_{WW}}{8}}^{T_w}\right] = \frac{(8 - 9T_P + T_W)}{3\Delta x} - \frac{(T_P - T_W)}{\Delta x}$$

$$a_e = 0, \quad a_w = \frac{1}{\Delta x} + \frac{1}{3\Delta x} + \frac{6Pe}{8} = \frac{4}{3\Delta x} + \frac{6}{8}Pe$$

$$a_P = \frac{1}{\Delta x} + \frac{3}{\Delta x} - \frac{3Pe}{8} = \underbrace{\frac{4}{\Delta x} - \frac{3Pe}{8}}$$

$$b_P = \frac{8}{3\Delta x} - Pe - \frac{Pe}{8}T_{WW}$$

$$Pe_{\Delta x} \le \frac{32}{3} \simeq 10{,}66\ldots$$

3 volumes

Precisa de pelo menos 3 volumes no domínio!

para $a_e \geq 0 \rightarrow \frac{1}{\Delta x} - \frac{3}{8}Pe \geq 0$

$$\frac{1}{\Delta x} \geq \frac{3}{8}Pe \quad \text{ou} \quad \frac{8}{3} \geq Pe_{\Delta x}$$

$$Pe_{\Delta x} \le \frac{8}{3} \simeq 2{,}66\ldots$$

$\forall N$

~~3 volumes~~

$N \geq 40$ volumes $\rightarrow$ diverge

$N = 10$ ou 20 $\rightarrow$ converge

chutes testados: $T = x$ e $T = T_{analítico}$

QUICK com correção adiada sobre UDS / 8 Ago 03

(UDS implícito e correção p/ QUICK, explícito)

$P = 3, 4, \ldots, N-1$

$$P_e\left[\underbrace{T_P + \Delta T_e^*}_{T_e} - \underbrace{T_W - \Delta T_w^*}_{-T_w}\right] = \frac{(T_E - T_P)}{\Delta x} - \frac{(T_P - T_W)}{\Delta x}$$

$$\boxed{a_P T_P = a_W T_W + a_e T_E + b_P}$$

$a_w = \frac{1}{\Delta x} + P_e \qquad a_e = \frac{1}{\Delta x}$

ITERATIVO

$a_P = a_w + a_e \qquad b_P = P_e(\Delta T_w^* - \Delta T_e^*)$

$\Delta T_e^* = \frac{(3T_E^* - 2T_P^* - T_W^*)}{8}$

$\Delta T_w^* = \frac{(3T_P^* - 2T_W^* - T_{WW}^*)}{8}$

correções s/ UDS, de forma explícita

$$\Delta T_w^* - \Delta T_e^* = \frac{(3T_P^* - 2T_W^* - T_{WW}^* - 3T_E^* + 2T_P^* + T_W^*)}{8}$$

$$b_P = \frac{P_e}{8}(5T_P^* - T_W^* - T_{WW}^* - 3T_E^*)$$

$P = 2$

(usa T_e do 1º volume) que aqui é o T_w

$$P_e\left[\underbrace{T_P + \Delta T_e^*}_{T_e} - \underbrace{T_W - \Delta T_w^*}_{-T_w}\right] = \frac{(T_E - T_P)}{\Delta x} - \frac{(T_P - T_W)}{\Delta x}$$

$\Delta T_e^* = \frac{(3T_E^* - 2T_P^* - T_W^*)}{8} \qquad \Delta T_w^* = \Delta T_e^*\big|_{P=1} = \frac{(3T_P^* - T_W^*)}{8}$

$a_w = \frac{1}{\Delta x} + P_e \qquad a_e = \frac{1}{\Delta x}$

$a_P = \frac{2}{\Delta x} + P_e = a_w + a_e$

$b_P = P_e(\Delta T_w^* - \Delta T_e^*)$

$$\Delta T_w^* - \Delta T_e^* = \frac{3T_P - \cancel{T_W} - 3T_E + 2T_P + \cancel{T_W}}{8} = \frac{5T_P - 3T_E}{8}$$

$$b_P = \frac{P_e}{8}(5T_P^* - 3T_E^*)$$

$P = 1$

$\left(\frac{dT}{dx}\right)_w$ parábola c/ T_w

$$P_e\left[\underbrace{T_P + \Delta T_e^*}_{T_e} - \overset{0}{T_W}\right] = \frac{(T_E - T_P)}{\Delta x} - \frac{(9T_P - 8\overset{0}{T_W} - T_E)}{3\Delta x}$$

$$\Delta T_e^* = \frac{(3T_E^* - T_P^* - 2\overset{0}{T_W^*})}{8} = \frac{(3T_E^* - T_P^*)}{8}$$

ΔT_e considera que $T_W = 2T_w - T_P$ ↑ fictício extrapolação linear s/ T_w e T_P

$a_w = 0, \qquad a_e = \frac{1}{\Delta x} + \frac{1}{3\Delta x} = \frac{4}{3\Delta x}$

$a_P = \frac{1}{\Delta x} + \frac{3}{\Delta x} + P_e = \frac{4}{\Delta x} + P_e$

$$b_P = -P_e\Delta T_e^* = \frac{P_e}{8}(T_P^* - 3T_E^*)$$

CONVERGE $\forall N$

$P = N$

$\left(\frac{dT}{dx}\right)_e$ (parábola)

$$P_e\left[\overset{T_e}{1} - \underbrace{T_W - \Delta T_w^*}_{T_w}\right] = \frac{(8T_e - 9T_P + T_W)}{3\Delta x} - \frac{(T_P - T_W)}{\Delta x}$$

$$\Delta T_w^* = \frac{(3T_P^* - 2T_W^* - T_{WW}^*)}{8}$$

$a_w = \frac{1}{\Delta x} + \frac{1}{3\Delta x} + P_e = \frac{4}{3\Delta x} + P_e$

$a_e = 0$

$a_P = \frac{1}{\Delta x} + \frac{3}{\Delta x} = \frac{4}{\Delta x}$

$$b_P = \frac{8}{3\Delta x} + P_e(\Delta T_w^* - 1)$$

18 Jul 03

PROG5_CFD1

$$Pe_{\Delta x} = \frac{Pe}{N}$$

ESQUEMA	QUANDO OSCILA	QUANDO CONVERGE α	N MÍNIMO	Ver
UDS	NUNCA	—	2	
CDS	$Pe_{\Delta x} > 2$	—	2	$Pe=10$, $N=5$, $Pe_{\Delta x}=2$ (limite); $Pe=20$, $N=5$, $Pe_{\Delta x}=4$ oscilações; $Pe=10$, $N=10$, $Pe_{\Delta x}=1$ OK (não oscila)
Exato	NUNCA	—	2	
Alfa	$\alpha < \frac{1}{2} - \frac{1}{Pe_{\Delta x}}$	—	2	$Pe=10$, $N=2$, $Pe_{\Delta x}=5$: $\alpha=0 \rightarrow$ oscila = CDS; $\alpha=0{,}33 \rightarrow$ não oscila pois $\alpha > \alpha_{MIN}=0{,}3$; $\alpha=0{,}5 \rightarrow$ " " " = UDS
TVD	$Pe_{\Delta x} > 2$	$Pe_{\Delta x} < 2$	2	
ADS	NUNCA (?)	$\forall\, Pe_{\Delta x}$	3	
QUICK	$Pe_{\Delta x} > \frac{8}{3} = 2{,}66\ldots7$	—	3	$Pe=10$, $N=4$, $Pe_{\Delta x}=2{,}5 \rightarrow$ OK não oscila; $Pe=12$, $N=4$, $Pe_{\Delta x}=3$ oscila

Ver casos:

- $Pe=100$, $N=10 \rightarrow Pe_{\Delta x}=10$
- $Pe=100$, $N=5 \rightarrow Pe_{\Delta x}=20$
- $Pe=100$, $N=3 \rightarrow Pe_{\Delta x}=33{,}3$