GABARITO

1ª QUESTÃO

a) $\bar{U}$ para $h = 0{,}5$ mm

QMx:

$$\rho\left(u\frac{\partial u}{\partial x}^{\,0} + v^{\,0}\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \rho g$$

$$\mu\frac{\partial^2 u}{\partial y^2} = \rho g$$

CC: $y = 0 \quad u(y) = \bar{U}$

$\tau(y=h) = 0 \therefore \left.\frac{du}{dy}\right|_{y=h} = 0$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\rho g}{\mu}$$

$$\frac{du}{dy} = \frac{\rho g}{\mu}y + c_1$$

$$\boxed{u(y) = \frac{\rho g}{2\mu}y^2 + c_1 y + c_2}$$

$c_2 = \bar{U}$

$0 = \frac{\rho g}{\mu}h + c_1$

$c_1 = -\frac{\rho g}{\mu}h$

$$\boxed{u(y) = \frac{\rho g}{2\mu}y^2 - \frac{\rho g}{\mu}hy + \bar{U}}$$ 1

p/ vazão nula $\dot{m} = 0 = \int \rho u \, dA$

$$\int_0^h u w\, dy = 0 = \int_0^h \left(\frac{\rho g}{2\mu}y^2 - \frac{\rho g}{\mu}hy + \bar{U}\right)dy$$

$$\left.\left(\frac{\rho g y^3}{6\mu} - \frac{\rho g h y^2}{2\mu} + \bar{U}y\right)\right|_0^h = 0$$

$$\frac{\rho g h^3}{6\mu} - \frac{\rho g h^3}{2\mu} + \bar{U}h = 0$$

$$\bar{U} = \frac{\rho g h^2}{\mu}\cdot\frac{1}{3} = \frac{940 \cdot 9{,}81 \cdot (0{,}5\times10^{-3})^2}{3 \cdot 0{,}04}$$

$$\boxed{\bar{U} = 0{,}0192 \text{ m/s}}$$ 1,0

a uma
a uma
dio da
s com
locado
o se a

(2)

b) $T_b = 50°C$

$T_\infty = 20°C$

$h = 20\ W/m^2K$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \mu\left\{\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right]\right\} + \dot{q} = 0$$

$$k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 = 0$$

sendo $\frac{\partial u}{\partial y} = \frac{\rho g}{\mu}y - \frac{\rho g}{\mu}h = \frac{\rho g}{\mu}(y-h)$

$$\frac{\partial^2 T}{\partial y^2} = \frac{\mu \rho^2 g^2}{\mu^2 k}(y-h)$$

$$\frac{dT}{dy} = \frac{\rho^2 g^2}{\mu k}\left(\frac{y^2}{2} - hy\right) + c_1$$

$$T(y) = \frac{\rho^2 g^2}{\mu k}\left(\frac{y^3}{6} - \frac{hy^2}{2}\right) + c_1 y + c_2$$

0,5

$T(y=0) = 50 \therefore c_2 = 50$

$$-k\left.\frac{dT}{dy}\right|_{y=h} = h_x[T(y=h) - T_\infty]$$

$$-k\left[\frac{\rho g}{\mu^2}\left(\frac{y^2}{2} - hy\right) + c_1\right]\Big|_h = h_x\left[\frac{\rho g}{\mu}\left(\frac{y^3}{6} - \frac{hy^2}{2}\right) + c_1 y + 50\right]_h$$

$$-k\left[-\frac{\rho g}{\mu^2}\frac{h^2}{2} + c_1\right] = h_x\left[-\frac{\rho g^2}{\mu}\frac{h^3}{3} + c_1 h + 50\right]$$

$$c_1 = \left[\frac{k}{h_x} + h\right]^{-1}\left[\frac{k}{h_x}\frac{\rho^2 g^2}{\mu}\frac{h^2}{2} + \frac{\rho g}{\mu^2}\frac{h^3}{3} - 50\right]$$

$$T(y) = \frac{\rho^2 g^2}{\mu}\left(\frac{y^3}{6} - \frac{hy^2}{2}\right) + c_1 y + 50$$

0,5

p/ $y = h$

$$T(y) = \frac{\rho^2 g^2}{\mu}\left(\frac{h^3}{6} - \frac{h^3}{2}\right) + \left[\frac{k}{h_x} + h\right]^{-1}\left[\frac{k}{h_x}\frac{\rho^2 g^2}{\mu}\frac{h^2}{2} + \frac{\rho^2 g^2}{\mu}\frac{h^3}{3} - 50\right]y + 50$$