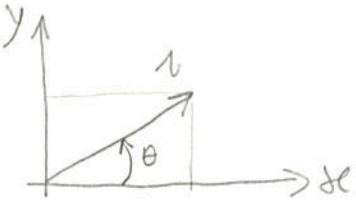
1 ENERGIA, POTÊNCIA, CALOR, TRABALHO, ÁREA, VOLUME

2 PESO, TORQUE, MOMENTO DE FORÇA, FLUXO DE CALOR

3



$x = r\cos\theta$

$y = r\,\mathrm{sen}\,\theta$

$z = z$

4 $\vec{r} = 3\hat{i} + 2\hat{j}$ $\quad \vec{p} = \hat{i} + 5\hat{j}$ $\quad \vec{q} = -5\hat{i} - 2\hat{j} + 3\hat{k}$

a] $\vec{r} + \vec{p} = 4\hat{i} + 7\hat{j}$

b] $\vec{r} + \vec{p} + \vec{q} = -\hat{i} + 5\hat{j} + 3\hat{k}$

5 a] $\vec{r} \cdot \vec{p} = 3 + 10 = 13$

b] $\vec{r} \cdot \vec{q} = -15 - 4 + 0 = -19$

c] $\vec{p} \cdot \vec{q} = -5 - 10 + 0 = -15$

d] $\vec{r} \cdot \vec{p}\, \vec{q} = 13(-5\hat{i} - 2\hat{j} + 3\hat{k}) = -65\hat{i} - 26\hat{j} + 39\hat{k}$

6 a] $\vec{r} \times \vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 0 \\ 1 & 5 & 0 \end{vmatrix} = (15-2)\hat{k} = 13\hat{k}$

b] $\vec{p} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 5 & 0 \\ 3 & 2 & 0 \end{vmatrix} = (2-15)\hat{k} = -13\hat{k}$

c] $\vec{r} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 0 \\ -5 & -2 & 3 \end{vmatrix} = (6-0)\hat{i} + (0-9)\hat{j} + (-6+10)\hat{k} = 6\hat{i} - 9\hat{j} + 4\hat{k}$

d] $\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 5 & 0 \\ -5 & -2 & 3 \end{vmatrix} =$

$= (15-0)\hat{i} + (0-3)\hat{j} + (-2+25)\hat{k}$

$= 15\hat{i} - 3\hat{j} + 23\hat{k}$

7 $\vec{p} = \hat{i} + 5\hat{j}$

$\|\vec{p}\| = \sqrt{1^2 + 5^2} = \sqrt{26}$

$\hat{n}_{\vec{p}} = \dfrac{\vec{p}}{\|\vec{p}\|} = \dfrac{1}{\sqrt{26}}\hat{i} + \dfrac{5}{\sqrt{26}}\hat{j}$

EXERCICIOS CAP. 1 (2)

$\boxed{8}$ $f = 3x^3y^2 + 2x\sqrt{y}$ $\quad x = \sqrt{t}$ $\quad y = t^2$

a] $\dfrac{\partial f}{\partial x} = 9x^2y^2 + 2\sqrt{y}$ $\qquad$ b] $\dfrac{\partial f}{\partial y} = 6x^3y + \dfrac{x}{\sqrt{y}}$

c] $\dfrac{df}{dt} = ?$ $\qquad \dfrac{dx}{dt} = \dfrac{1}{\sqrt{t}}$ $\qquad \dfrac{dy}{dt} = 2t$

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} = \left(9x^2y^2 + 2\sqrt{y}\right)\frac{1}{\sqrt{t}} + \left(6x^3y + \frac{x}{\sqrt{y}}\right)2t$$

$$= \frac{(9tt^4 + 2t)}{\sqrt{t}} + \left(6t^{3/2}t^2 + \frac{\sqrt{t}}{t}\right)2t$$

$$= 9t^{9/2} + 2\sqrt{t} + 6t^{9/2} + 2\sqrt{t} = 15t^{9/2} + 4\sqrt{t}$$

$\boxed{9}$ a] $\vec{\nabla} f = \dfrac{\partial f}{\partial x}\hat{\imath} + \dfrac{\partial f}{\partial y}\hat{\jmath} = (9x^2y^2 + 2\sqrt{y})\,\hat{\imath} + (6x^3y + x/\sqrt{y})\,\hat{\jmath}$

b] $\nabla^2 f = \dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} = 18xy^2 + 6x^3 - \dfrac{x}{2}y^{-3/2}$

$\boxed{10}$ $\vec{\nabla}\cdot\vec{q} = ?$ $\qquad \vec{q} = -5\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

$$\vec{\nabla}\cdot\vec{q} = \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} = 0$$

$\boxed{11}$ PROVAR QUE $\nabla^2 f = \vec{\nabla} \cdot (\vec{\nabla} f)$

$$\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{\imath} + \frac{\partial f}{\partial y}\hat{\jmath}$$

$$\vec{\nabla} \cdot (\vec{\nabla} f) = \hat{\imath} \cdot \frac{\partial}{\partial x}\left(\vec{\nabla} f|_x\right) + \hat{\jmath} \cdot \frac{\partial}{\partial y}\left(\vec{\nabla} f|_y\right)$$

$$= \hat{\imath} \cdot \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\hat{\imath}\right) + \hat{\jmath} \cdot \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\hat{\jmath}\right)$$

$$= \hat{\imath} \cdot \hat{\imath}\,\frac{\partial^2 f}{\partial x^2} + \hat{\jmath} \cdot \hat{\jmath}\,\frac{\partial^2 f}{\partial y^2}$$

$$= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \nabla^2 f$$