EXAMPLE 2.1

Consider a turbojet-powered airplane flying at a velocity of 300 m/s at an altitude of 10 km, where the free-stream pressure and density are 2.65×10^4 N/m^2 and 0.414 kg/m^3, respectively. The turbojet engine has inlet and exit areas of 2 m^2 and 1 m^2, respectively. The velocity and pressure of the exhaust gas are 500 m/s and 2.3×10^4 N/m^2 respectively. The fuel-to-air mass ratio is 0.05. Calculate the thrust of the engine.

■ Solution

The mass flow of air through the inlet is

$$\dot{m}_i = \rho_\infty V_\infty A_i = (0.414)(300)(2) = 248.4 \text{ kg/s}$$

Fuel is added and burned inside the engine at the ratio of 0.05 kg of fuel for every kg of air. Hence, the mass flow at the exit, $\dot{m}_e$, is

$$\dot{m}_e = 1.05\, \dot{m}_i = 1.05(248.4) = 260.8 \text{ kg/s}$$

From Eq. (2.45)

$$\begin{aligned} T &= \dot{m}_e V_e - \dot{m}_i V_\infty + (p_e - p_\infty)A_e \\ &= (260.8)(500) - (248.4)(300) + [(2.3 - 2.65) \times 10^4](1) \\ &= 1.304 \times 10^5 - 0.7452 \times 10^5 - 0.35 \times 10^4 \\ &= \boxed{5.238 \times 10^4 \text{ N}} \end{aligned}$$

Since 4.45 N = 1 lb, the thrust in pounds is

$$T = \boxed{11{,}771 \text{ lb}}$$

EXAMPLE 2.2

Consider a liquid-fueled rocket engine burning liquid hydrogen as the fuel and liquid oxygen as the oxidizer. The hydrogen and oxygen are pumped into the combustion chamber at rates of 11 kg/s and 89 kg/s, respectively. The flow velocity and pressure at the exit of the engine are 4000 m/s and 1.2×10^3 N/m^2, respectively. The exit area is 12 m^2. The engine is part of a rocket booster that is sending a payload into space. Calculate the thrust of the rocket engine as it passes through an altitude of 35 km, where the ambient pressure is 0.584×10^3 N/m^2.

■ Solution

For the case of a rocket engine, there is no mass flow of air through an inlet; the propellants are injected directly into the combustion chamber. Hence, for a rocket engine, Eq. (2.45) becomes, with $\dot{m}_i = 0$,

$$T = \dot{m}_e V_e + (p_e - p_\infty)A_e$$

Since the total mass flow of propellants pumped into the combustion chamber is 11 + 89 = 100 kg/s, this is also the mass flow of the burned gases that exhausts through the rocket engine nozzle. That is, $\dot{m}_e = 100$ kg/s. Thus,

$$\begin{aligned} T &= \dot{m}_e V_e + (p_e - p_\infty)A_e \\ &= (100)(4000) + [(1.2 - 0.584) \times 10^3](12) \\ &= 4 \times 10^5 + 7.392 \times 10^3 = \boxed{4.074 \times 10^5 \text{ N}} \end{aligned}$$

In pounds,

$$T = \frac{4.074 \times 10^5}{4.45} = \boxed{91{,}549 \text{ lb}}$$