

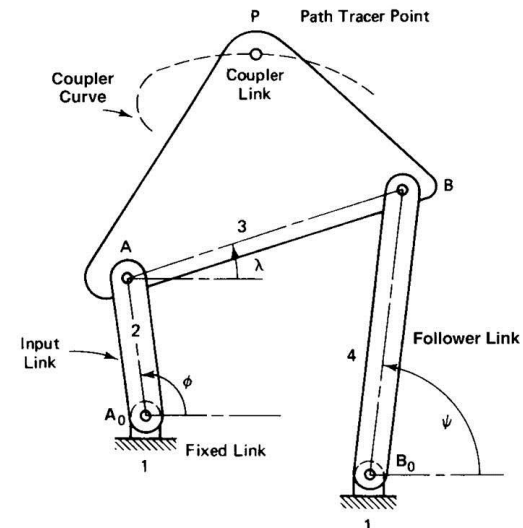
# Mecanismos

## Mecanismos com 1 GL

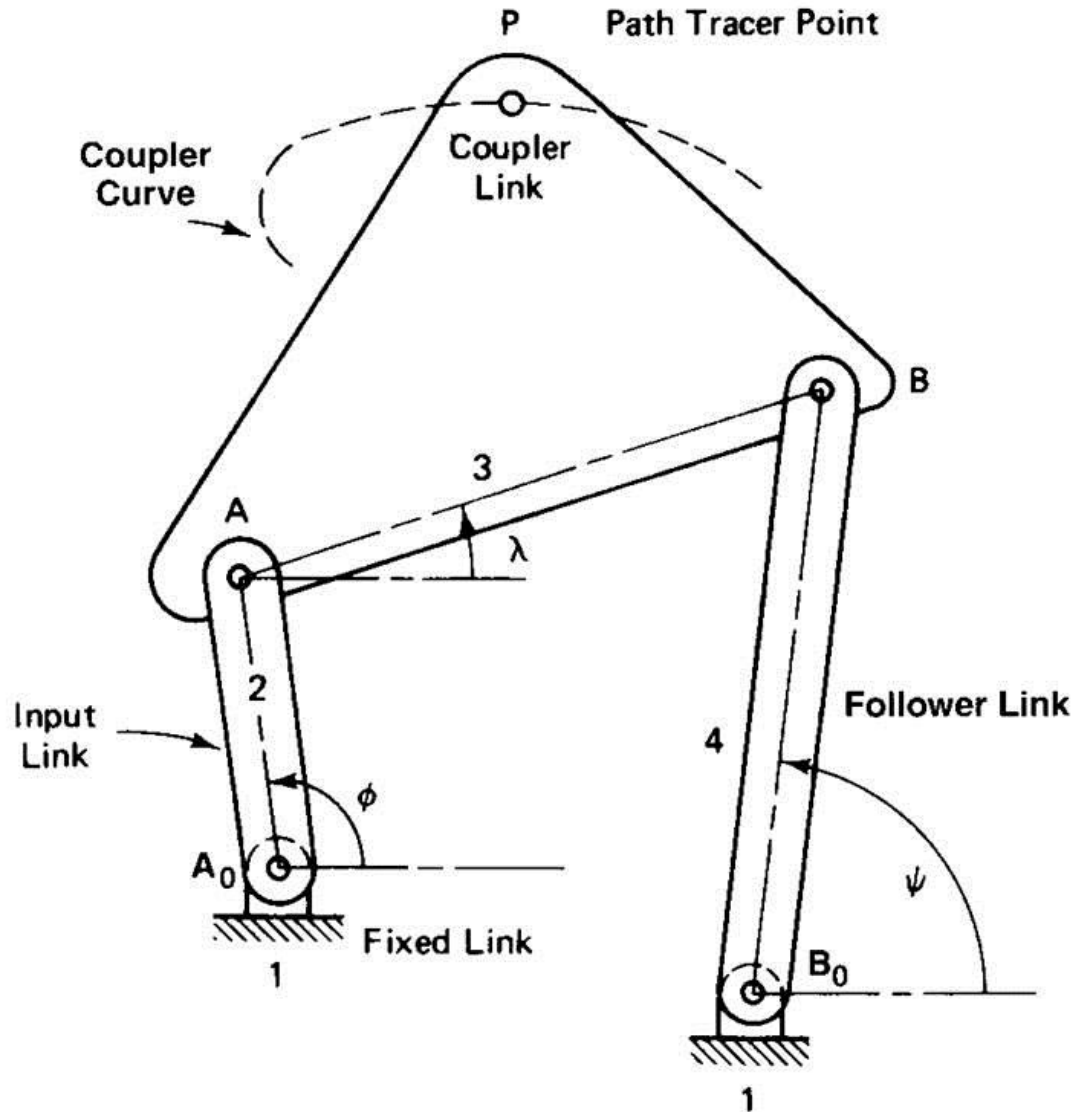
### Mecanismo de 4 barras

- Análise Geral
- Análise de Pontos de Interesse

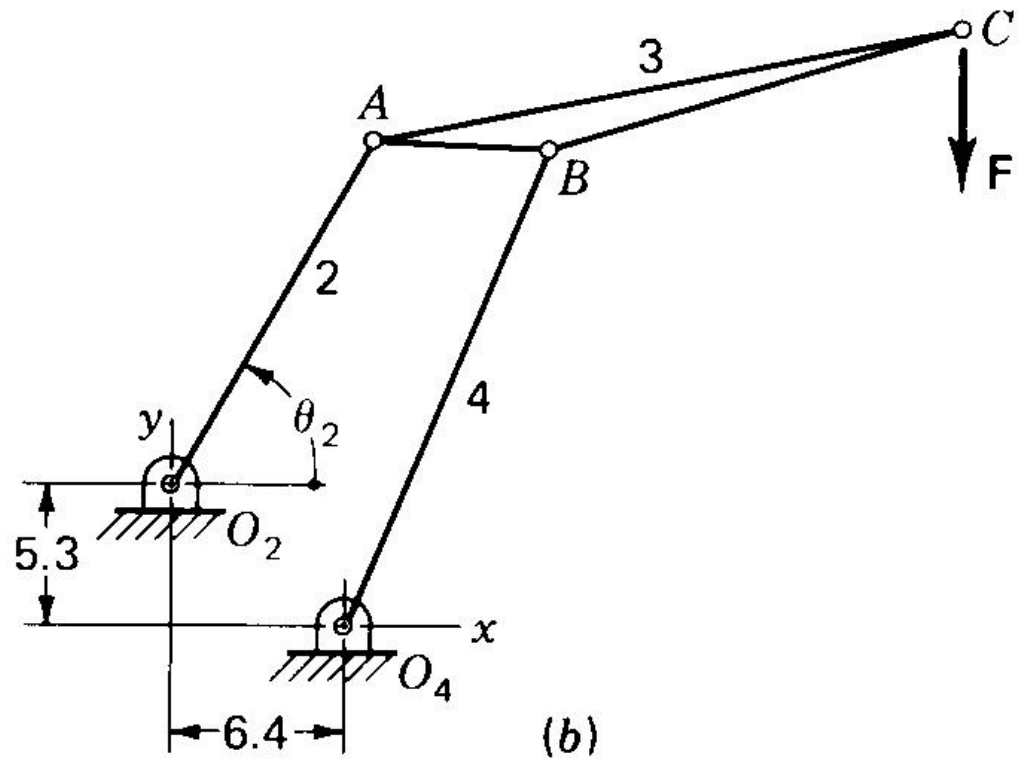
Prof. Jorge Luiz Erthal  
jorgeerthal@gmail.com



# Exemplos

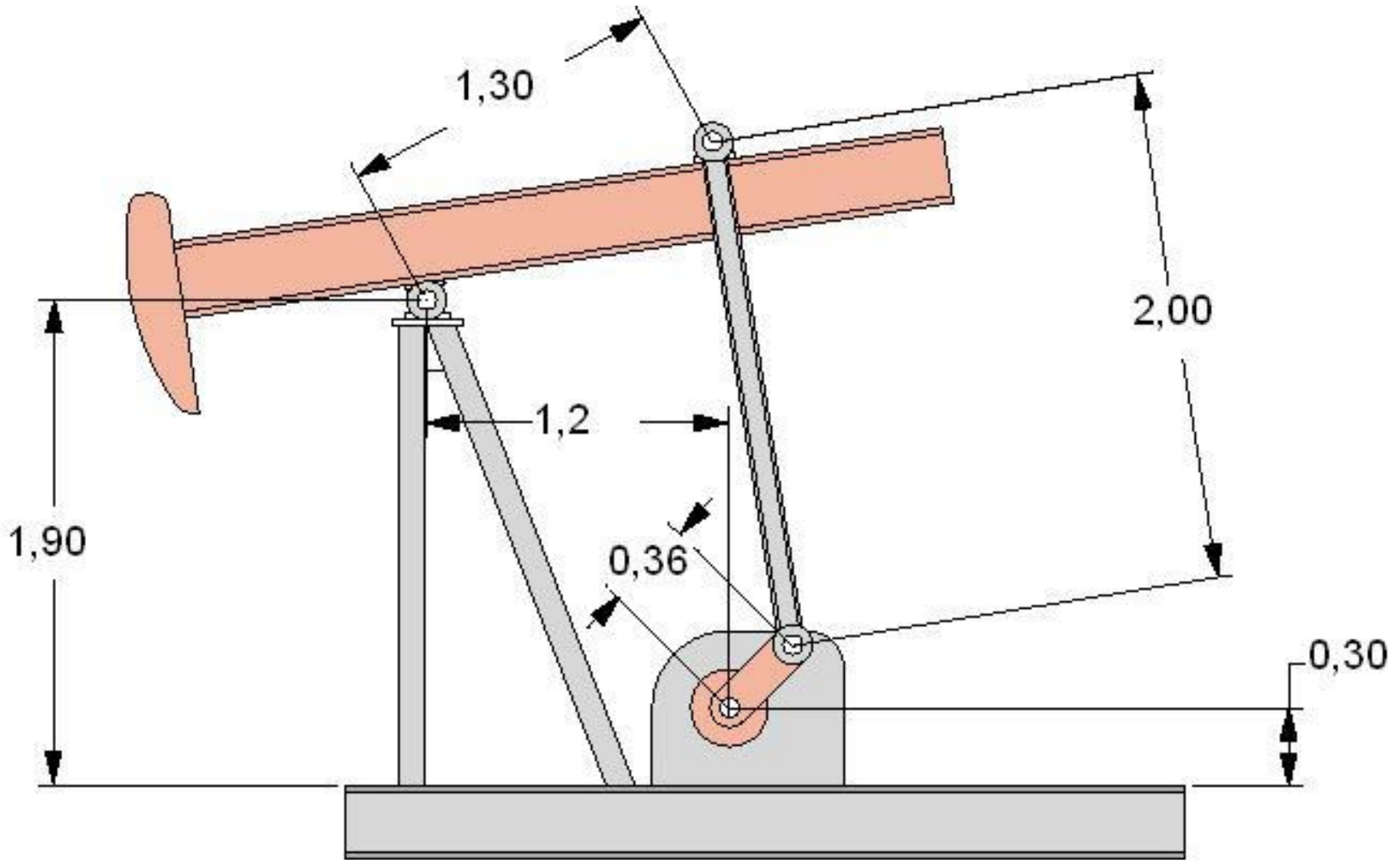


# Guindaste



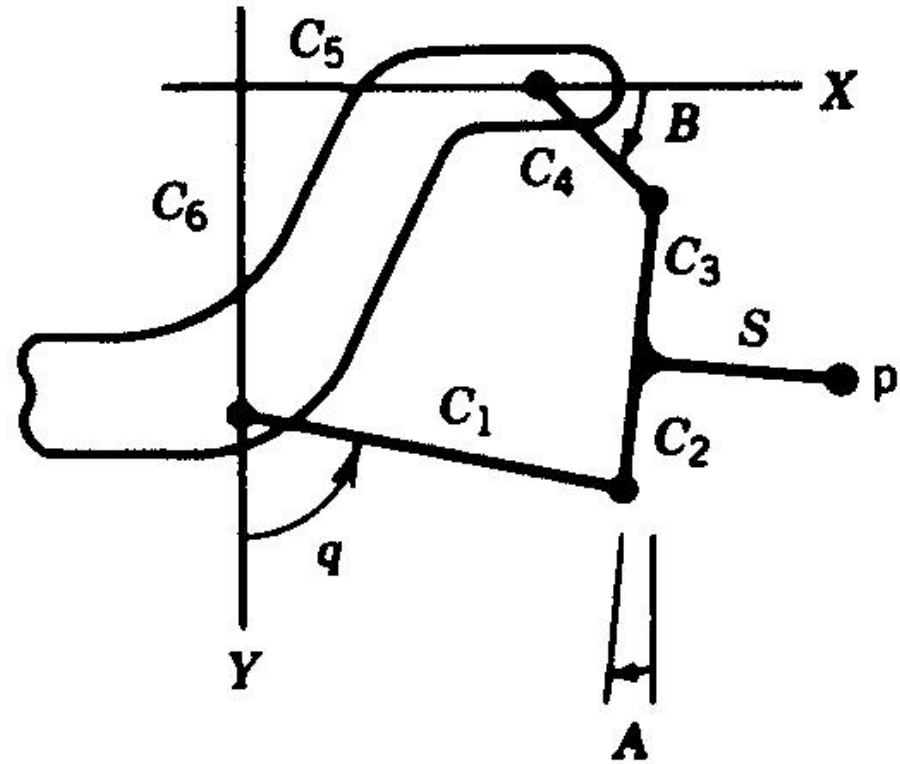
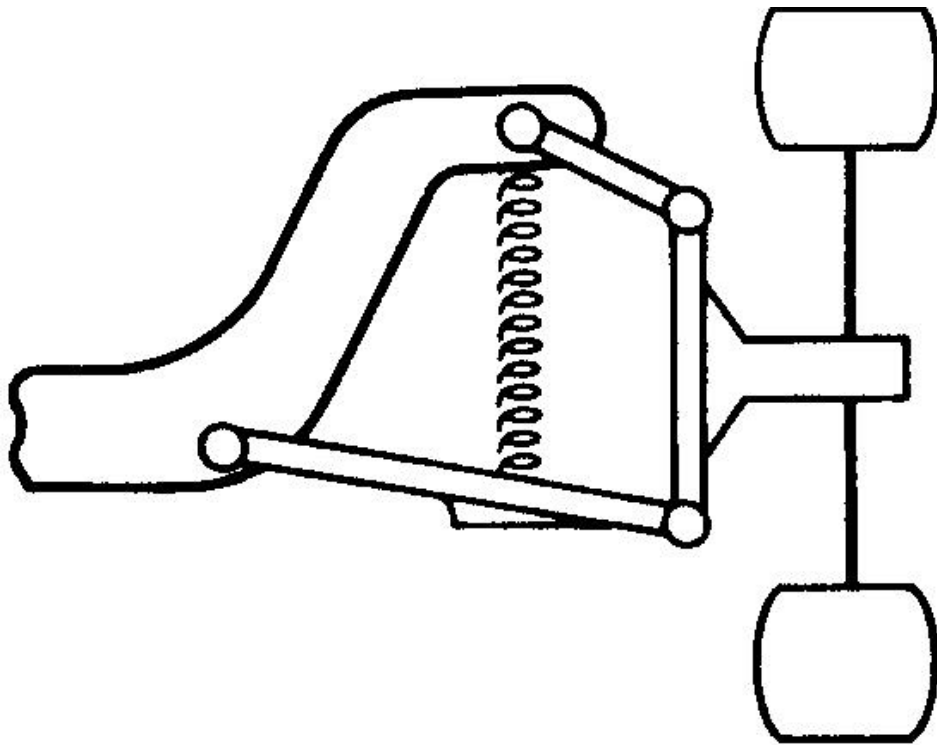
Mecanismo de 4 Barras

# Bomba de extração de petróleo



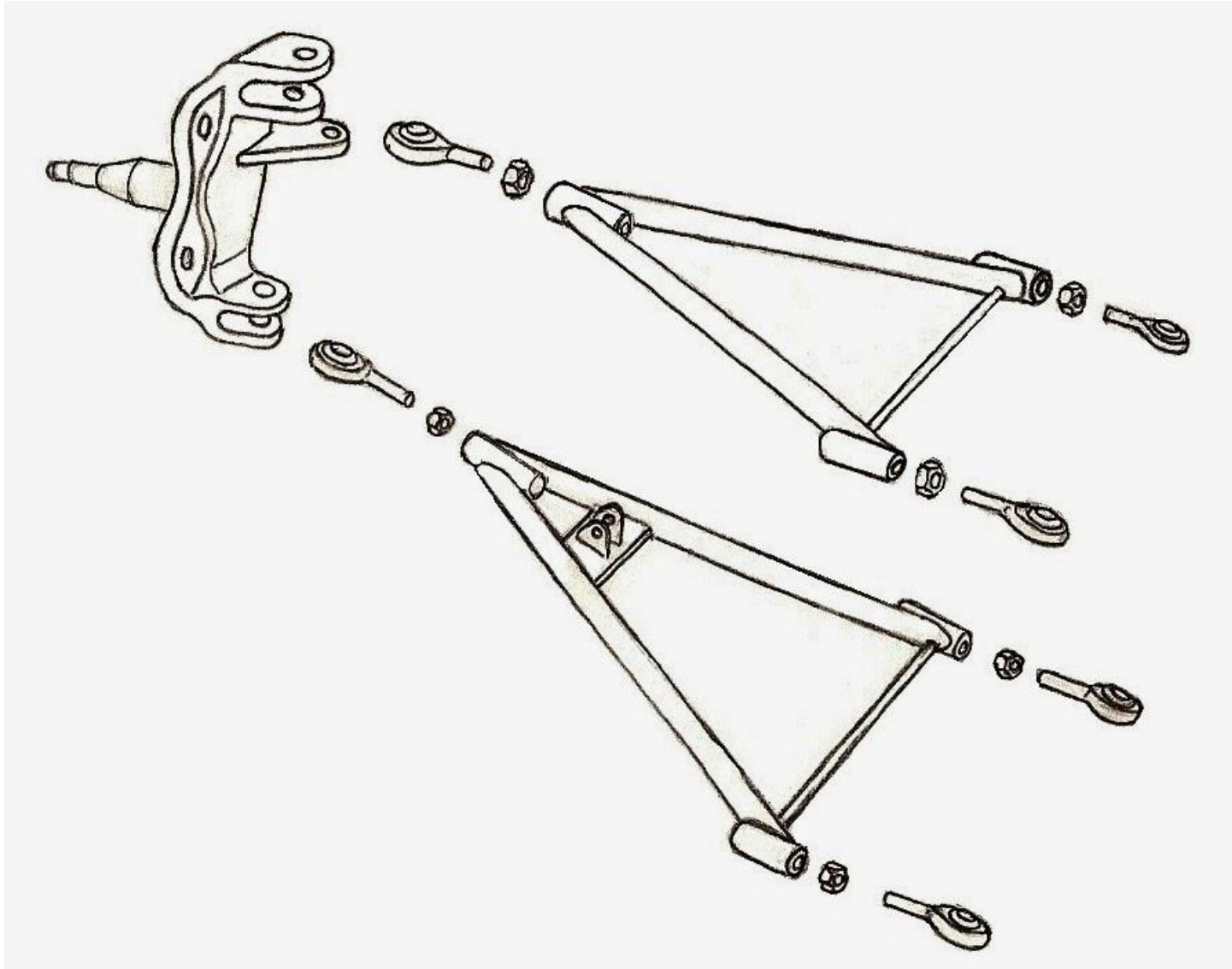
Mecanismo de 4 Barras

# Suspensão duplo A





# Suspensão duplo A



Mecanismo de 4 Barras

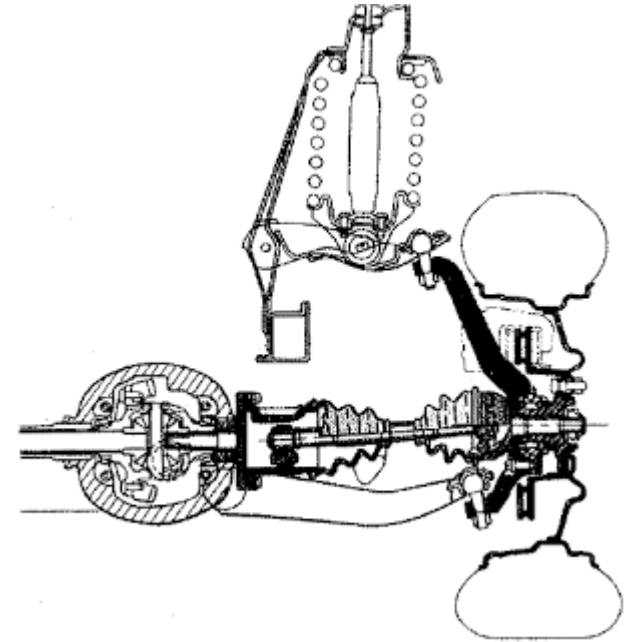
# Suspensão duplo A



Mecanismo de 4 Barras



# Suspensão duplo A



# Sistemas de direção

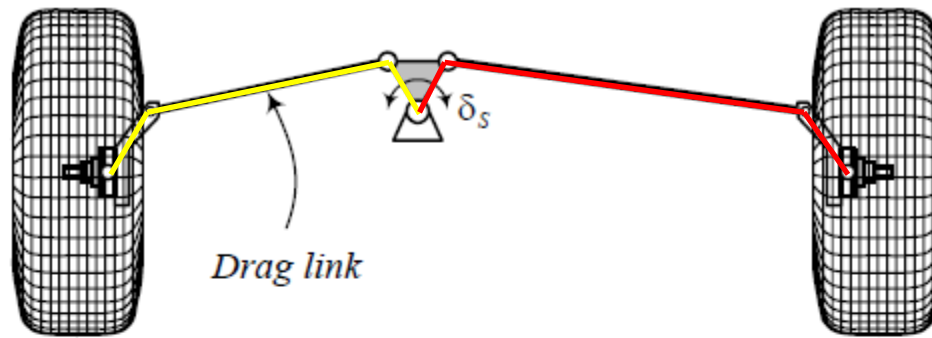


FIGURE 7.25. A lever arm steering system.

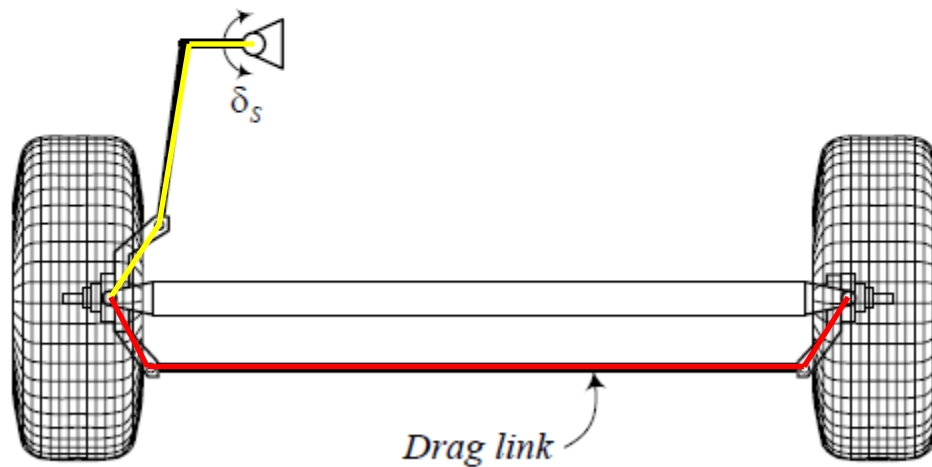
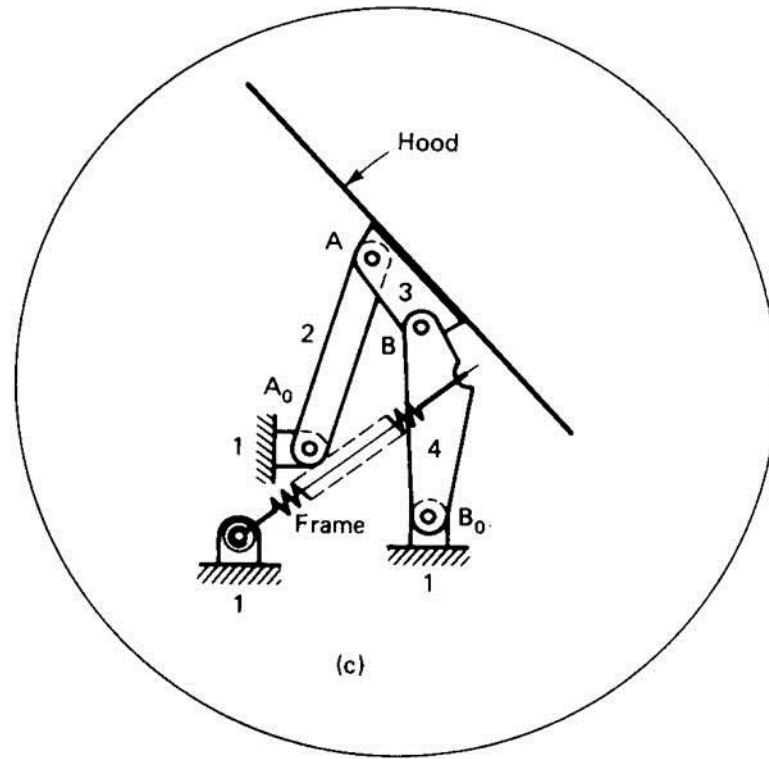
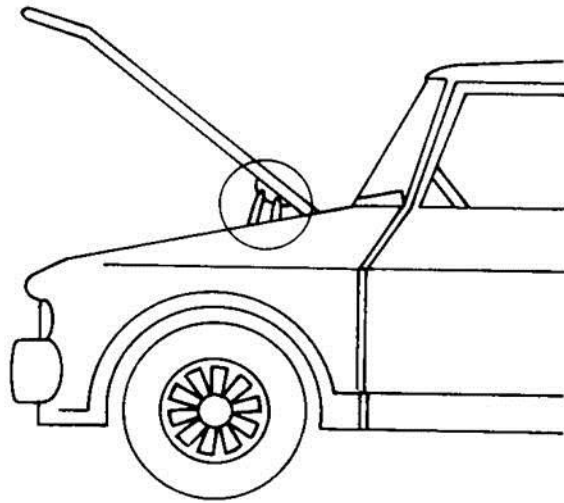


FIGURE 7.26. A drag link steering system.

# Articulação do capô do motor



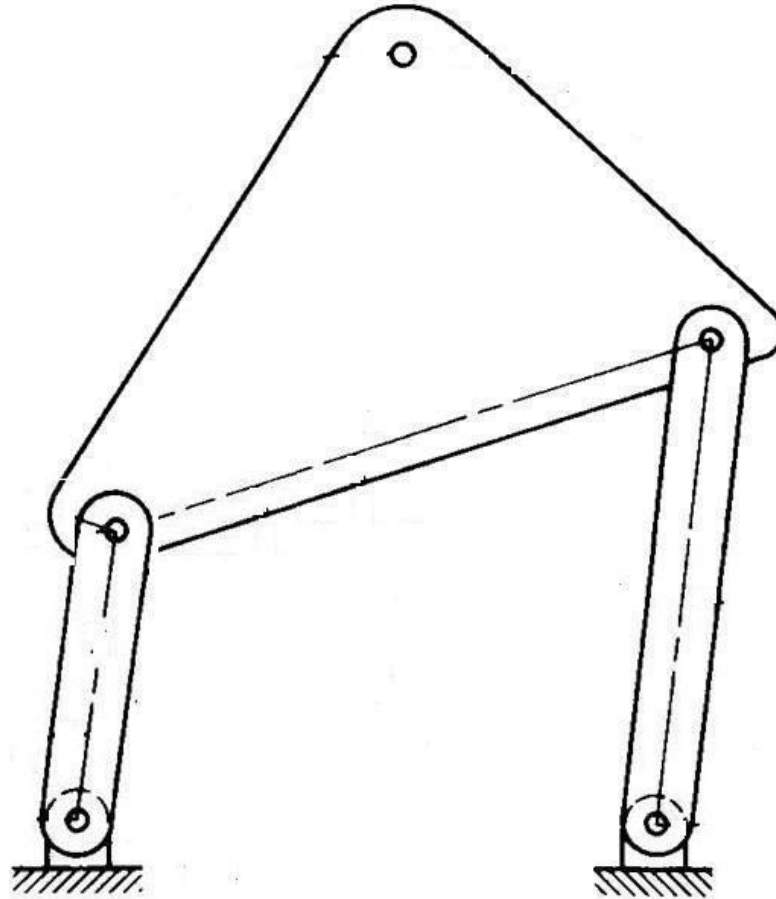


# 1-Análise Geral

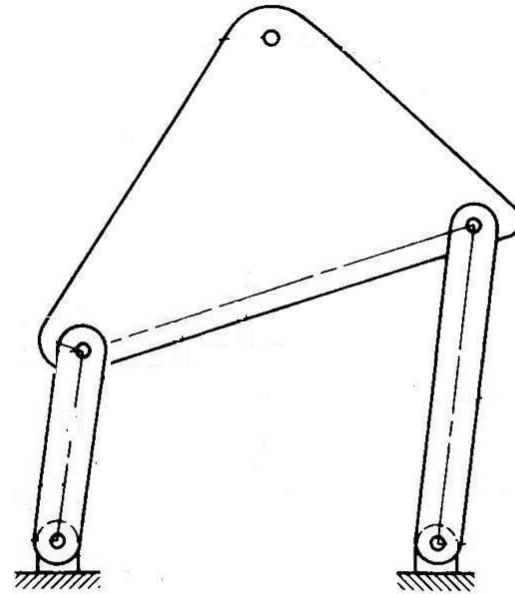
# Conteúdo

- Análise Geral
- Critério de Grashof
- Variações no acionamento
  - Pontos singulares

# Mecanismo de Quatro Barras



# 1-Cálculo do número de graus de liberdade



$$N := 4$$

$$P_1 := 4$$

$$P_2 := 0$$

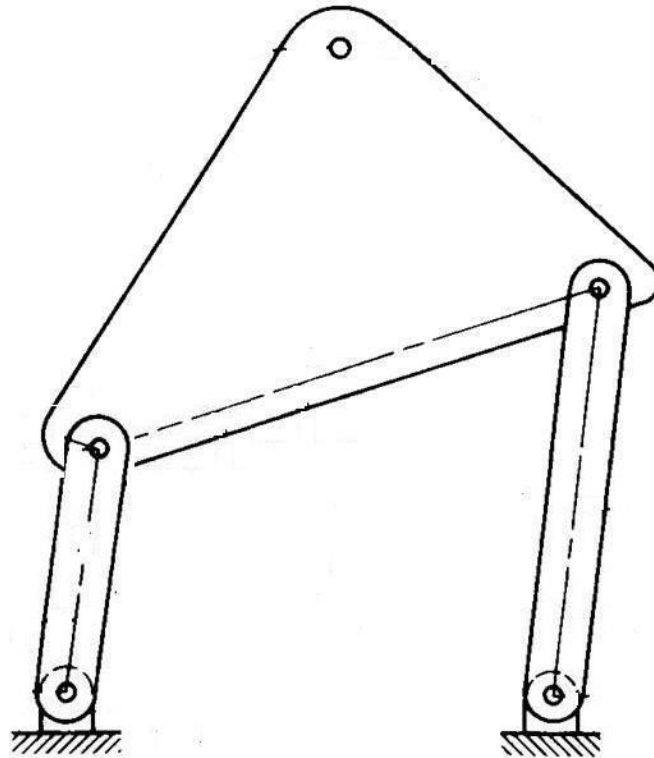
$$F := 3 \cdot (N - 1) - 2 \cdot P_1 - P_2$$

$$F = 1$$

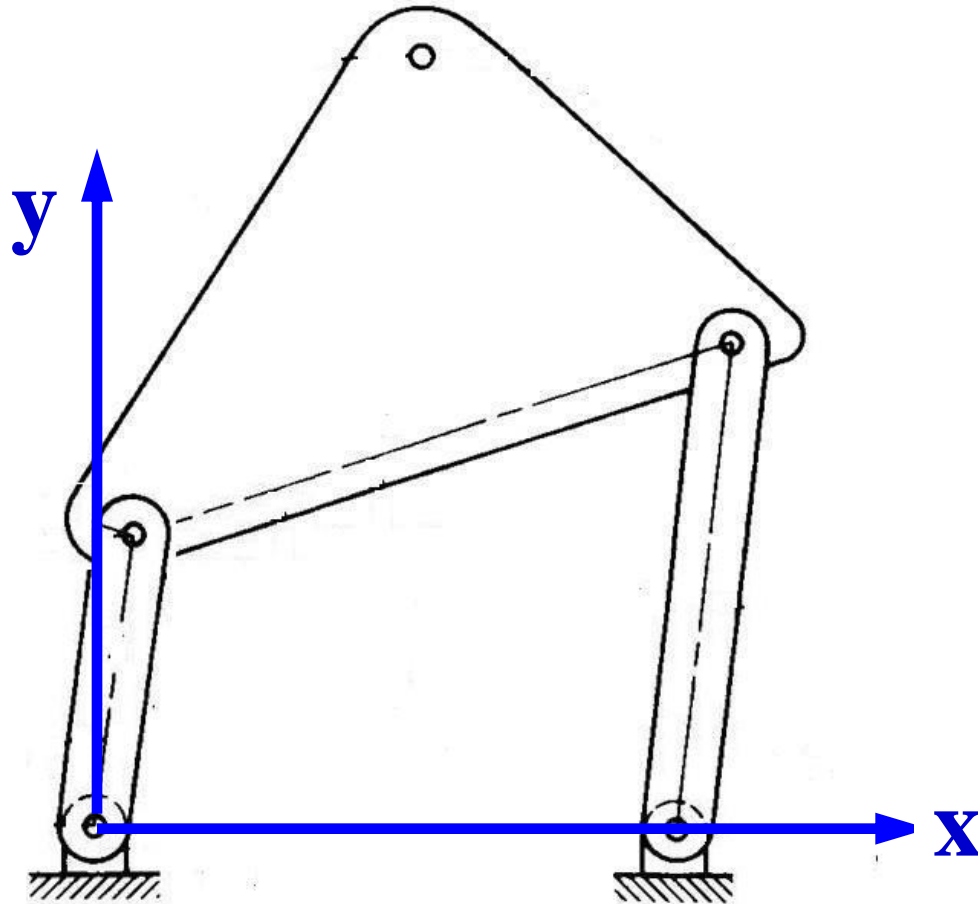


## 2-Decomposição dos pares superiores

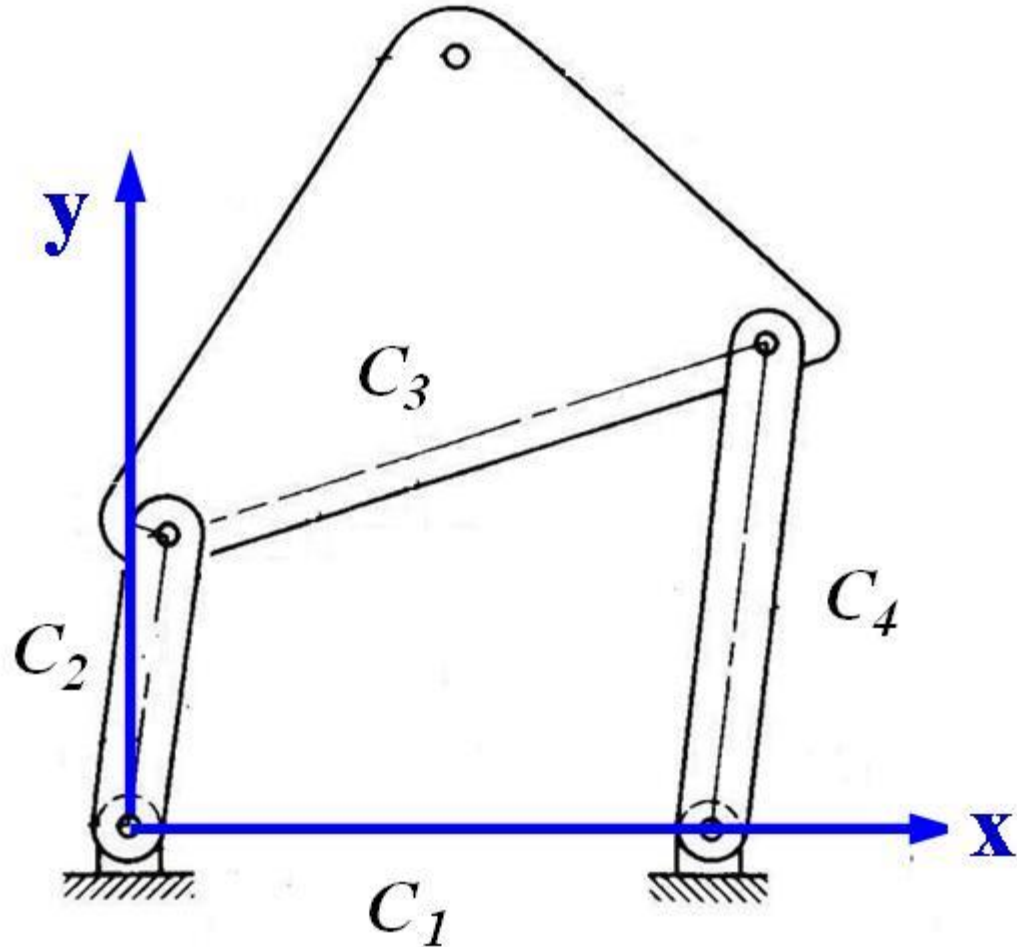
***Não há pares superiores***



### 3-Sistema GLOBAL de coordenadas



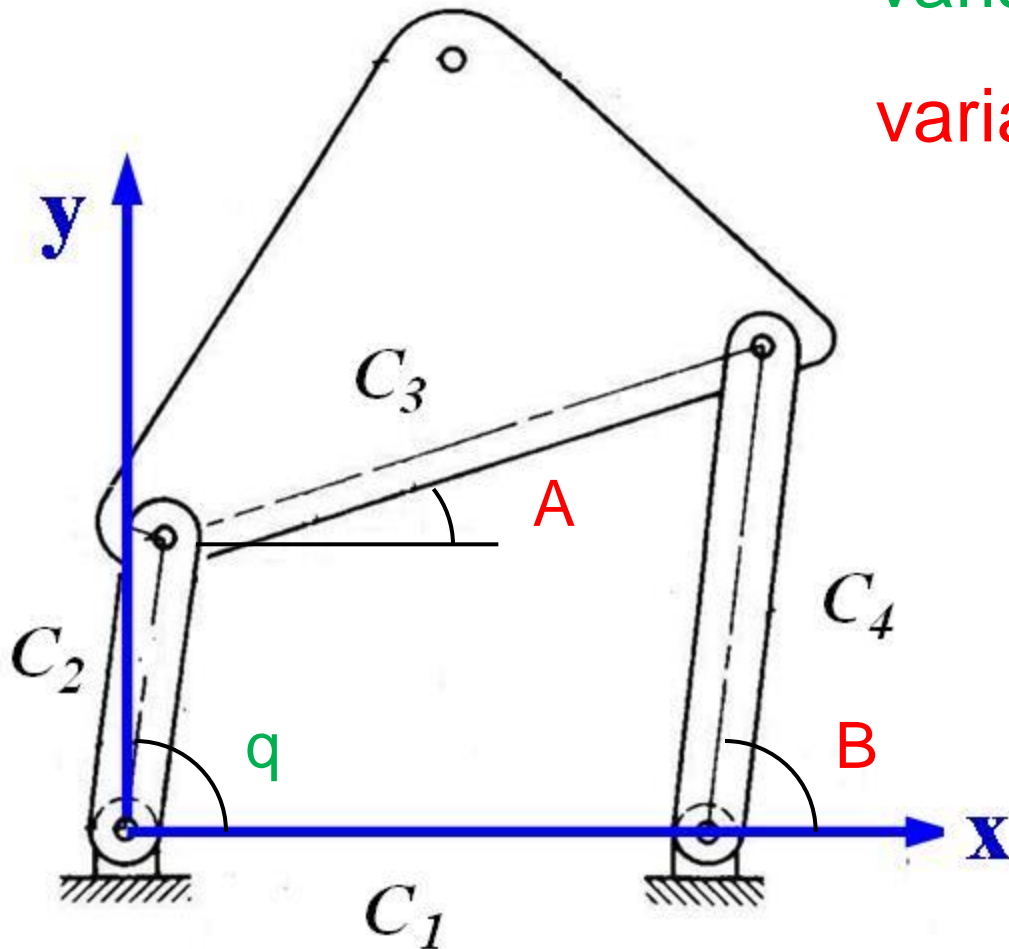
# 4-Identificação das medidas constantes



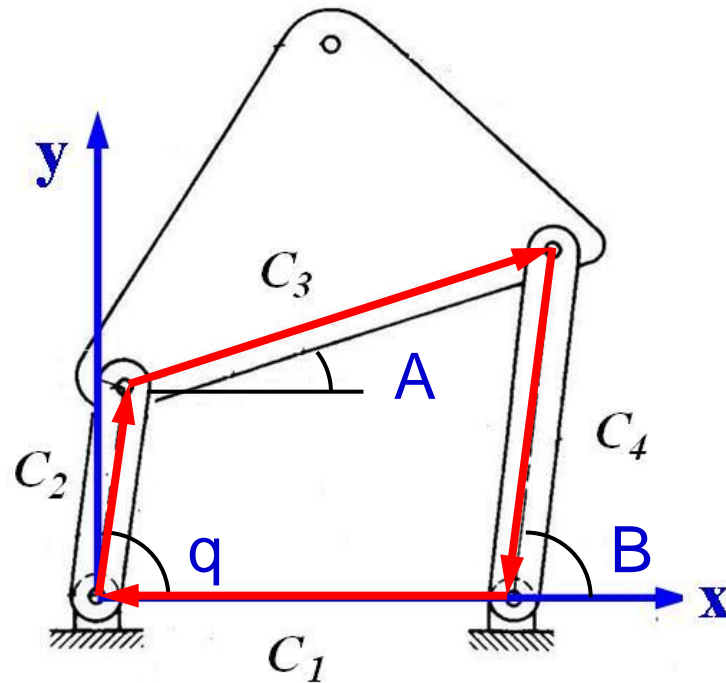
# 5-Definição das variáveis primárias e secundárias

variável primária:  $q$

variáveis secundárias:  $A, B$



## 6-Equações cinemáticas de posição

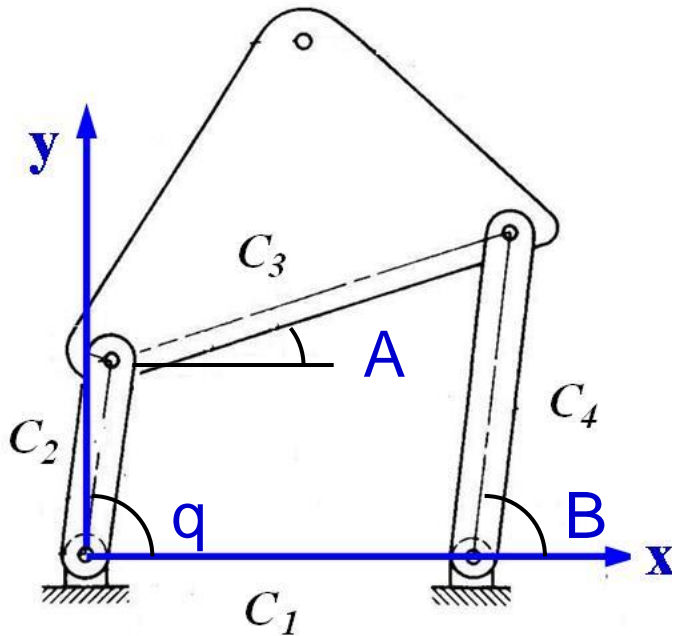


$$C_2 \cdot \cos(q) + C_3 \cdot \cos(A) - C_4 \cdot \cos(B) - C_1 = 0$$

$$C_2 \cdot \sin(q) + C_3 \cdot \sin(A) - C_4 \cdot \sin(B) = 0$$

# 7-Solução do sistema de equações

## Solução numérica



Variável primária:

$$q := 60\text{deg}$$

Valores iniciais:  $\underline{A} := 30\text{deg}$      $B := 90\text{deg}$

Solução numérica:

Given

$$C_2 \cdot \cos(q) + C_3 \cdot \cos(A) - C_4 \cdot \cos(B) - C_1 = 0$$

$$C_2 \cdot \sin(q) + C_3 \cdot \sin(A) - C_4 \cdot \sin(B) = 0$$

$$\begin{pmatrix} \underline{A} \\ \underline{B} \end{pmatrix} := \text{Find}(A, B)$$

$$A = 26.311 \text{ deg}$$

$$B = 75.229 \text{ deg}$$

# 7-Solução para um intervalo de valores de q

Valores constantes:  $C_1 := 60$     $C_2 := 20$     $C_3 := 70$     $C_4 := 50$

Variável primária:  $q := 0\text{deg}, 10\text{deg}.. 360\text{deg}$

Valores iniciais:  $A := 30\text{deg}$     $B := 90\text{deg}$

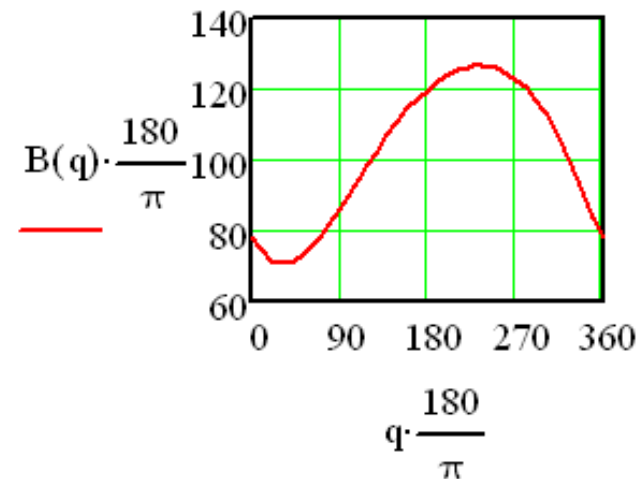
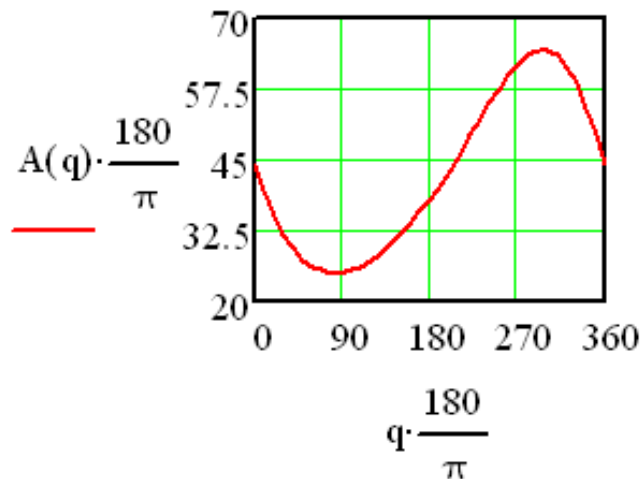
Solução numérica:

Given

$$C_2 \cdot \cos(q) + C_3 \cdot \cos(A) - C_4 \cdot \cos(B) - C_1 = 0$$

$$C_2 \cdot \sin(q) + C_3 \cdot \sin(A) - C_4 \cdot \sin(B) = 0$$

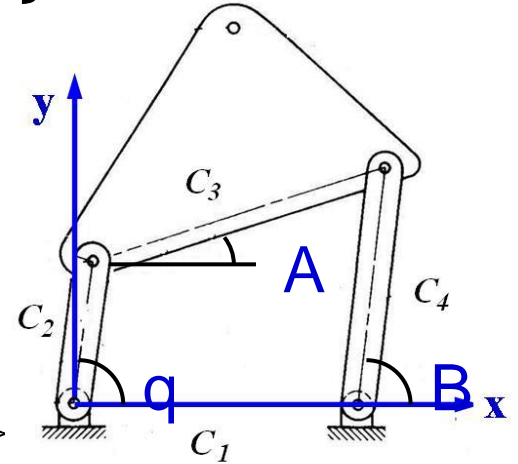
$$\begin{pmatrix} A(q) \\ B(q) \end{pmatrix} := \text{Find}(A, B)$$



## 7-Solução do sistema de equações

vetor das equações cinemáticas :

$$f = \begin{Bmatrix} C_2 \cdot \cos(q) + C_3 \cdot \cos(a) - C_4 \cdot \cos(B) - C_1 \\ C_2 \cdot \sin(q) + C_3 \cdot \sin(a) - C_4 \cdot \sin(B) \end{Bmatrix}$$



vetor das variáveis secundárias :

$$S = \begin{Bmatrix} A \\ B \end{Bmatrix}$$

objetivo :

obter  $\bar{S}$  de modo que  $f(\bar{S}) = 0$

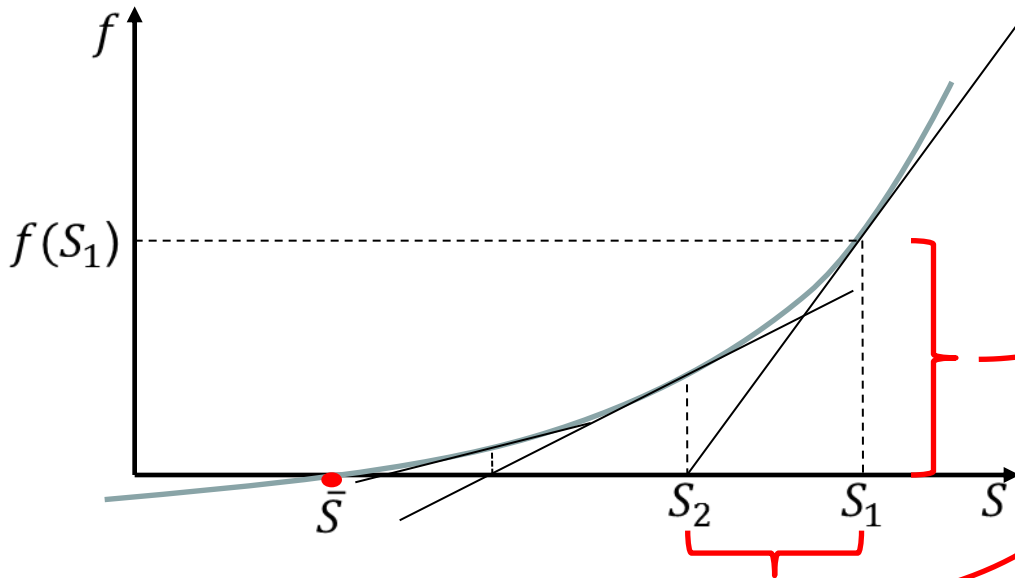
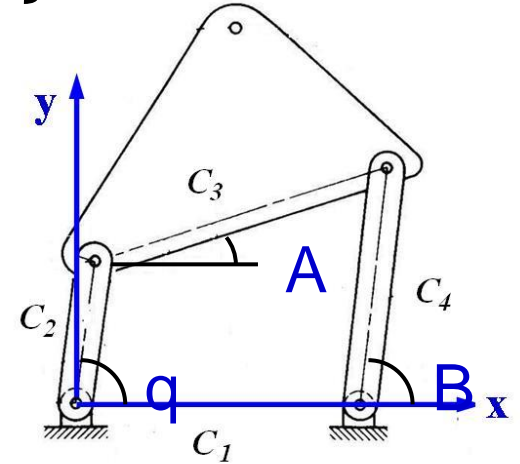
**Solução numérica**  
(as variáveis secundárias  
são obtidas numericamente)



# 7-Solução do sistema de equações

Solução numérica

Método de Newton-Raphson



$$\frac{f(S_1)}{S_1 - S_2} = f'(S_1)$$

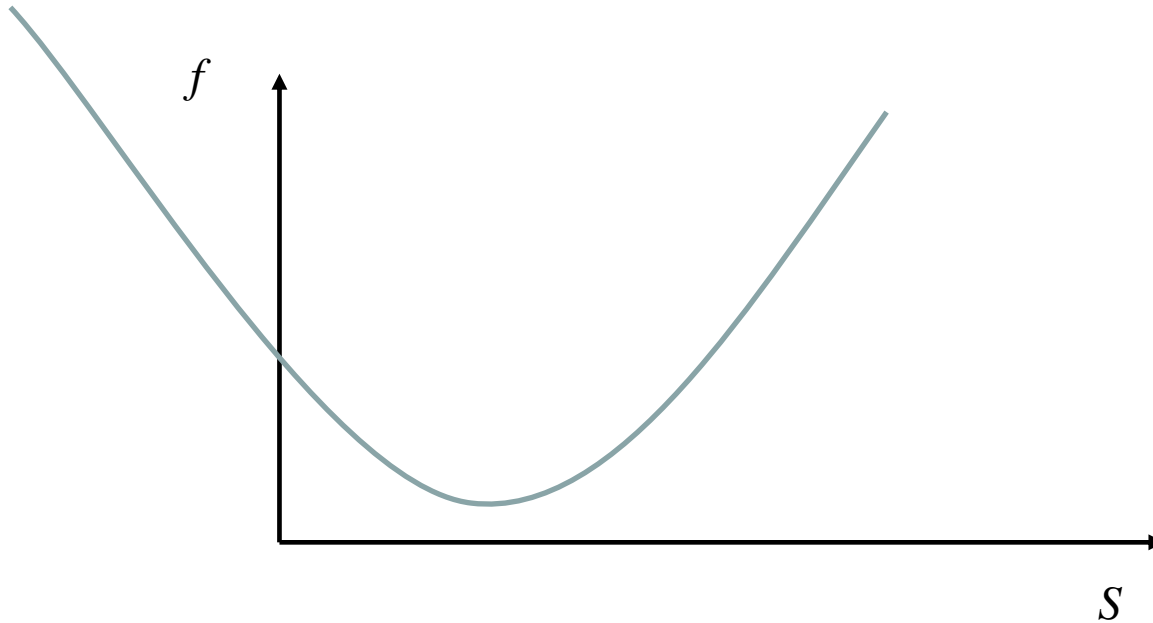
$$S_2 = -\frac{f(S_1)}{f'(S_1)} + S_1$$

$$f(S_1) \cong 0$$

## 7-Problemas com a solução numérica:

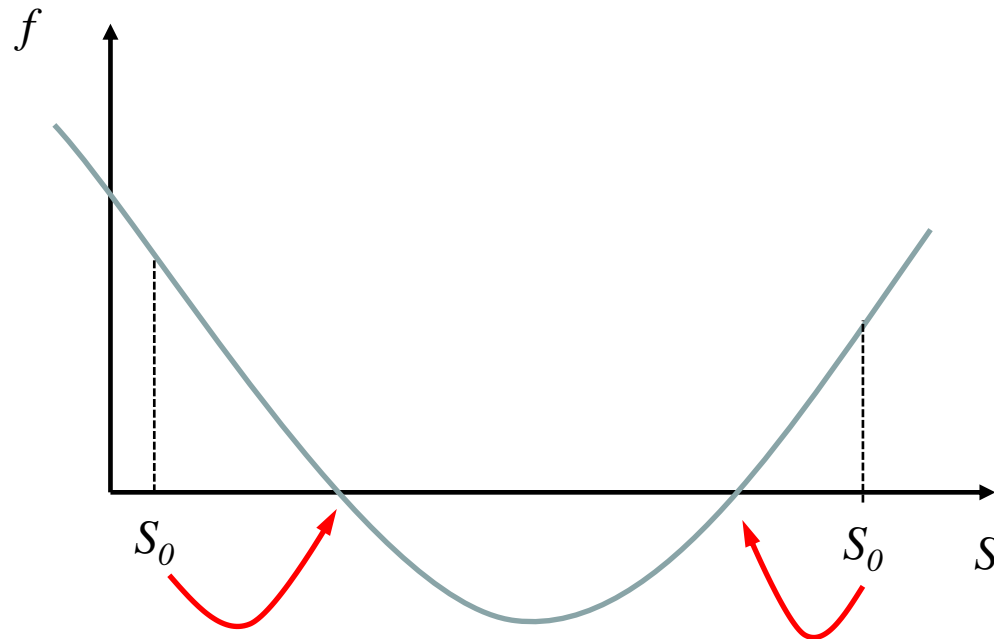
- ***Existência***
- ***Unicidade***
- ***Intervalo de aplicação***

## 7-Existência da solução para um intervalo de $q$



***Relacionado com o “alcance” da variável***

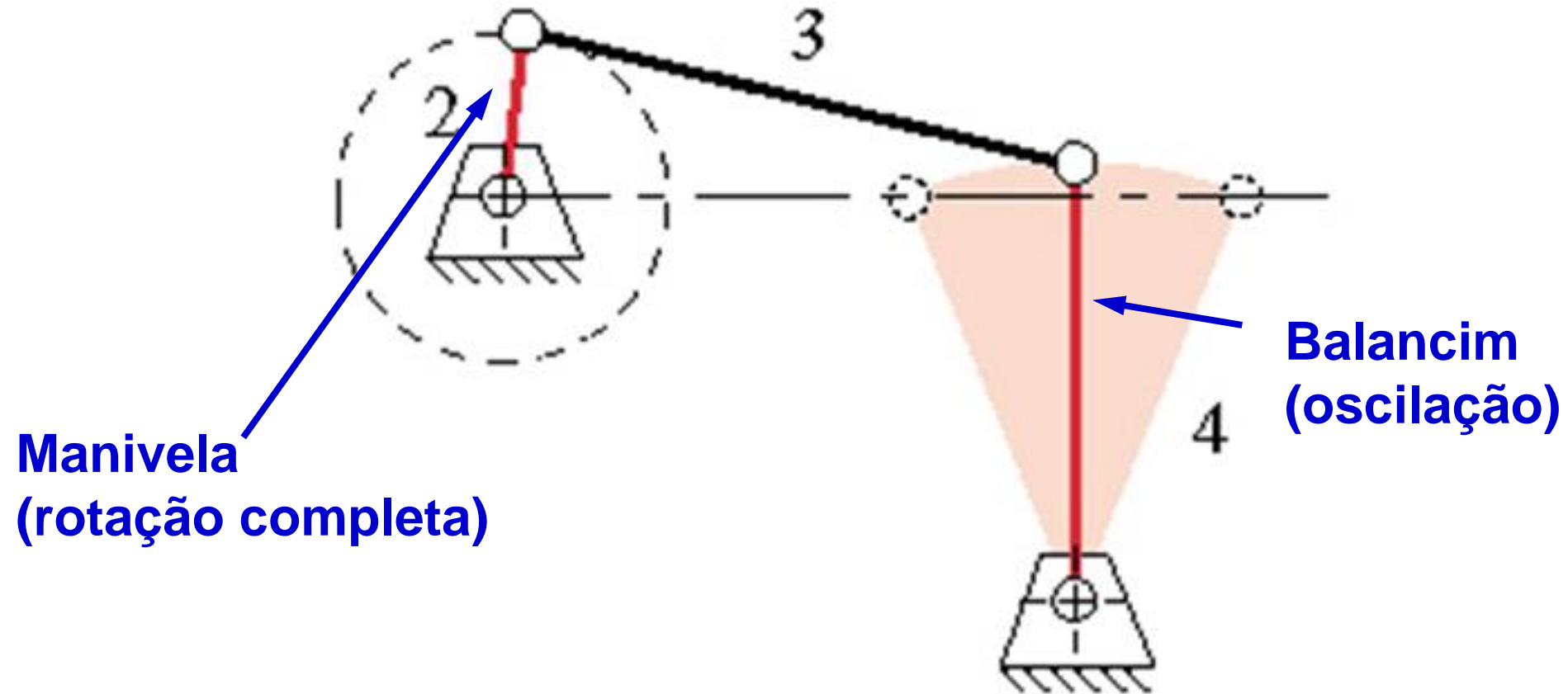
## 7-Unicidade (número de soluções)



***Valor inicial apropriado para atingir a solução desejada***

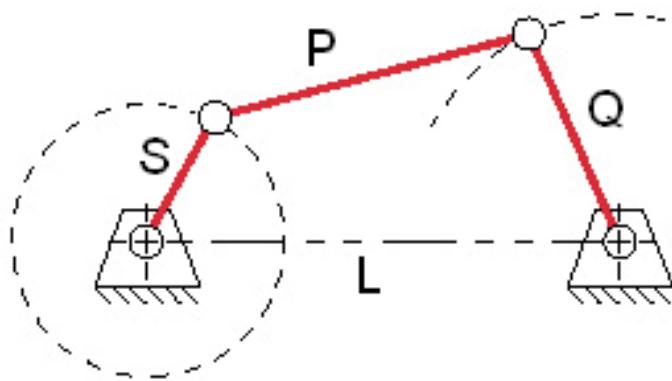
# 7-Intervalo de aplicação

## Movimentos dos elos



**Mecanismo manivela-balancim**

# Critério de *Grashof*:



S – comprimento do elo menor  
L – comprimento do elo maior  
P,Q – comprimentos dos elos restantes

Se  $S+L < P+Q \Rightarrow$  Classe I (mecanismo de Grashof)

Se  $S+L > P+Q \Rightarrow$  Classe II (mecanismo não-Grashof)

Nenhum elo é capaz de girar completamente.

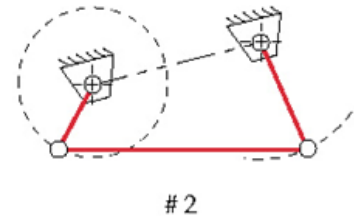
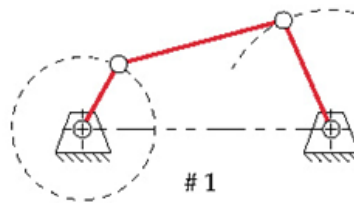
Se  $S+L = P+Q \Rightarrow$  Classe III

# Critério de *Grashof*:

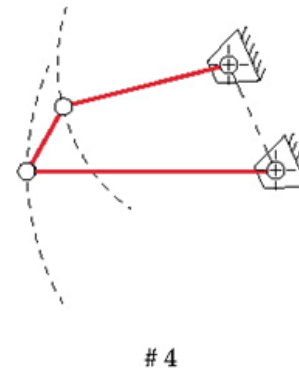
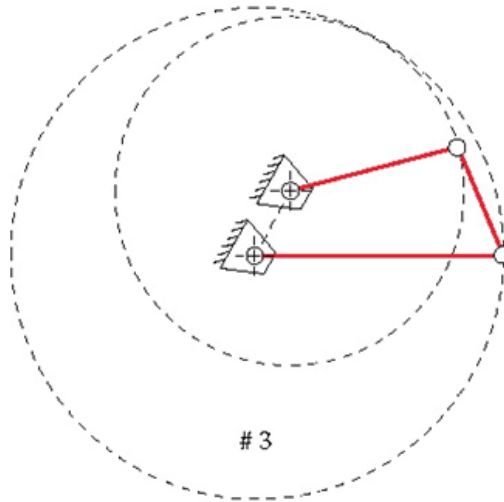
## Classe I ( $S+L < P+Q$ )

exemplo: quatrobarras\_CI.dv

1. Dupla manivela se S for fixo
2. Duplo balancim se S for oposto ao fixo
3. Manivela-balancim se S for ligado ao fixo



**Mecanismo manivela-balancim**



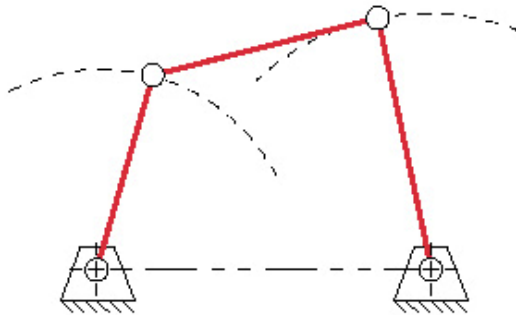
**Mecanismo  
dupla manivela**

**Mecanismo  
duplo balancim**

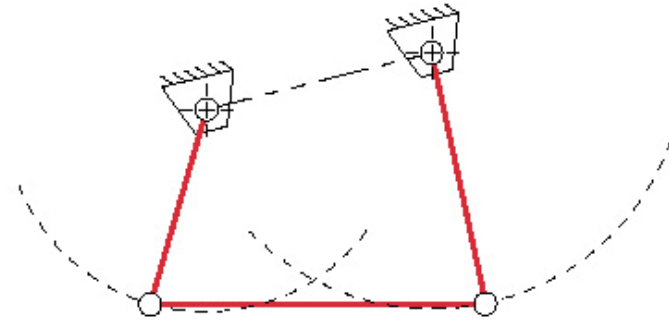
# Critério de *Grashof*:

**Classe II (  $S+L > P+Q$  )**

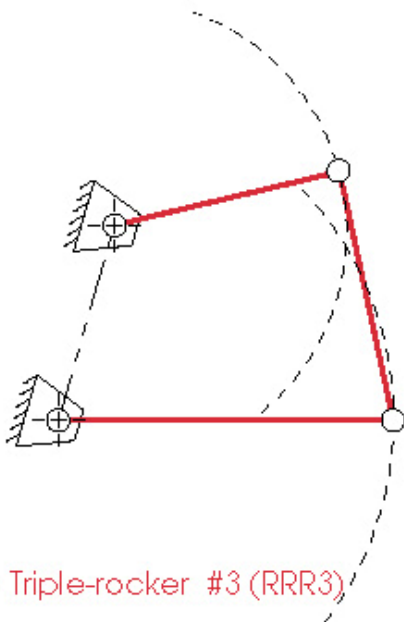
Duplo balancim



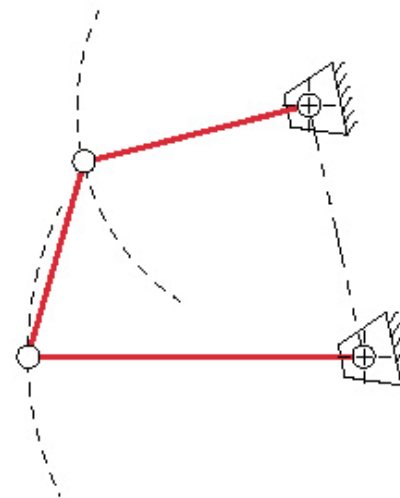
(a) Triple-rocker #1 (RRR1)



(b) Triple-rocker #2 (RRR2)



(c) Triple-rocker #3 (RRR3)



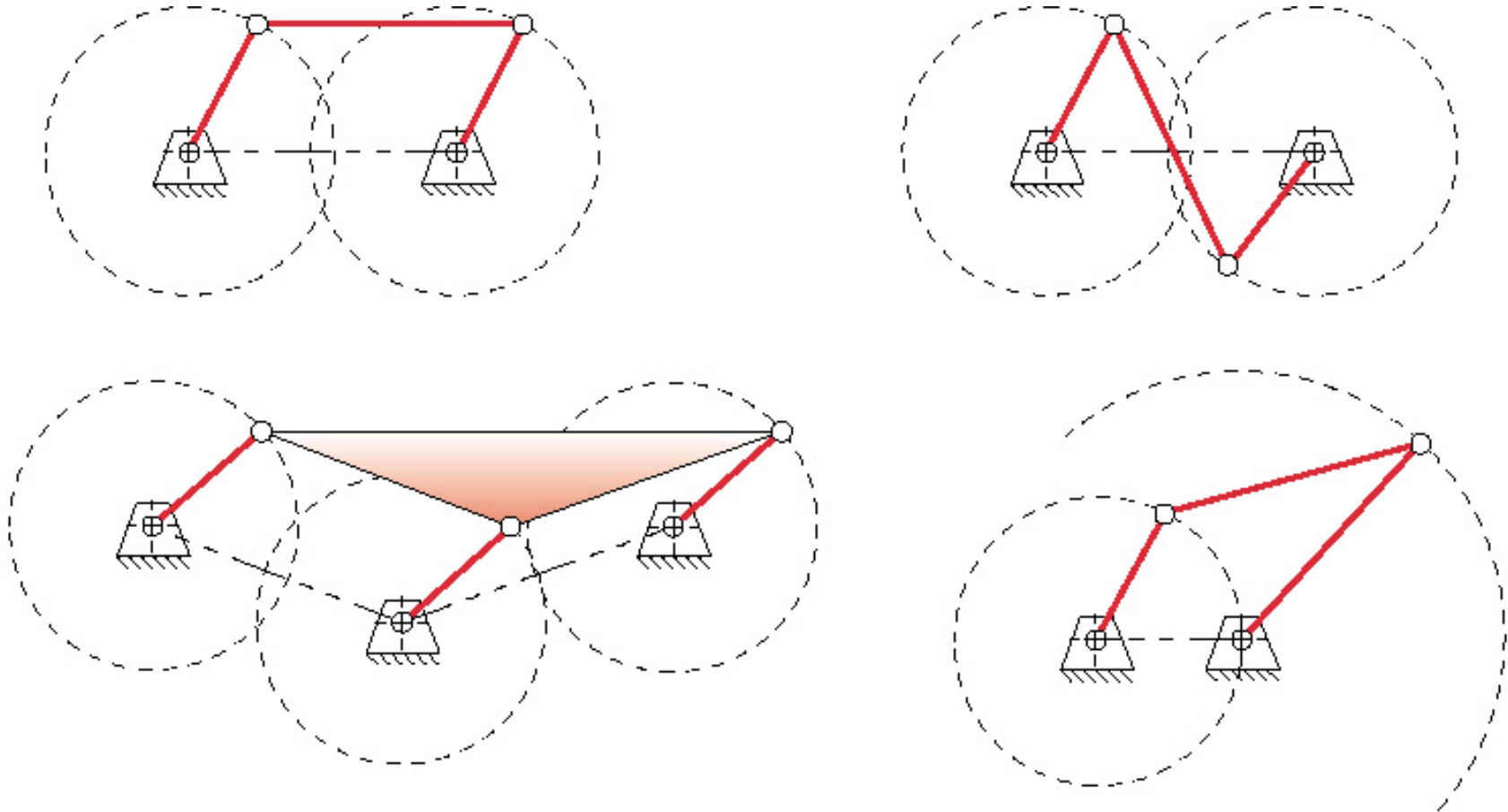
(d) Triple-rocker #4 (RRR4)



# Critério de *Grashof*:

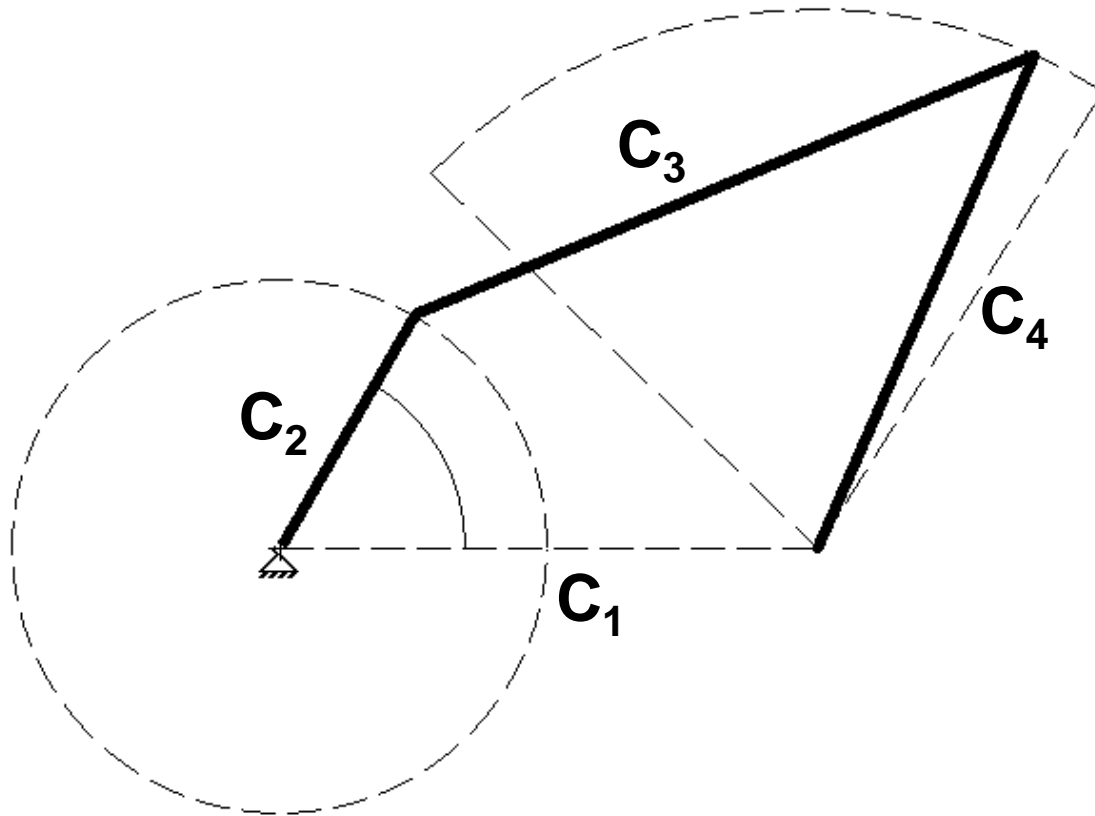
Classe III (  $S+L = P+Q$  )

Configuração incerta



# Limites do mecanismo manivela-balancim

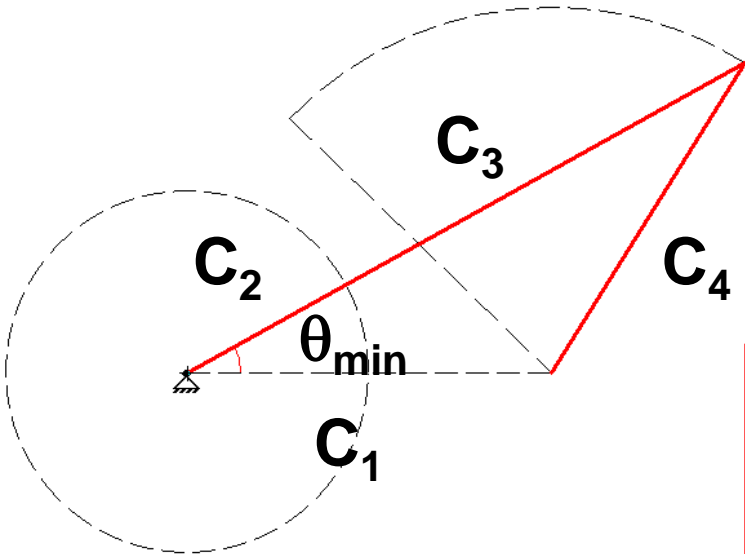
posições limites do elo de saída (não caracteriza travamento)



exemplo: [limites\\_4bMB.dv](#)

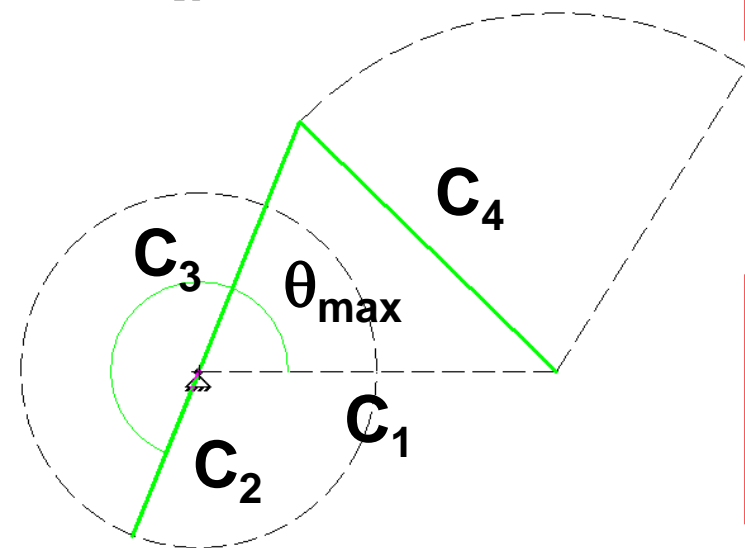
# Limites do mecanismo manivela-balancim

*Alinhamento do elo de entrada ( $C_2$ ) com o intermediário ( $C_3$ ).*



$$C_4^2 = C_1^2 + (C_3 + C_2)^2 - 2.C_1.(C_3 + C_2).cos \theta_{\min}$$

$$\theta_{\min} = \arccos\left(\frac{C_1^2 + (C_3 + C_2)^2 - C_4^2}{2.C_1.(C_3 + C_2)}\right)$$

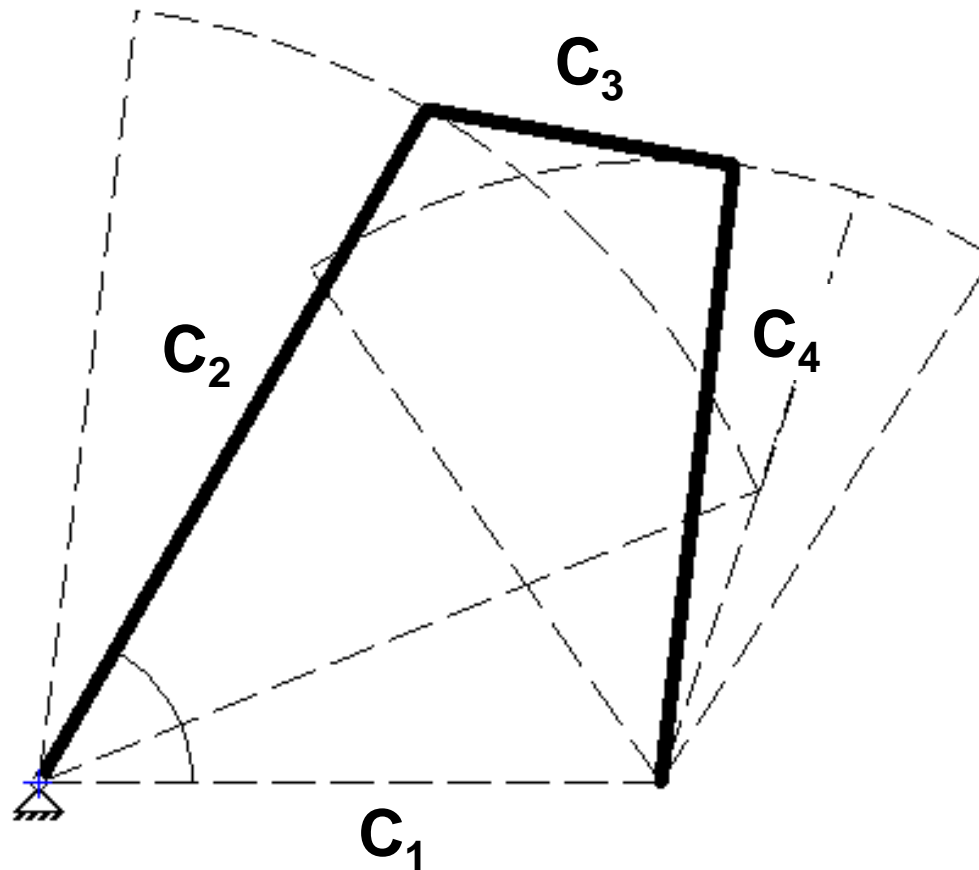


$$C_4^2 = C_1^2 + (C_3 - C_2)^2 - 2.C_1.(C_3 - C_2).cos \theta_{\max}$$

$$\theta_{\max} = \arccos\left(\frac{C_1^2 + (C_3 - C_2)^2 - C_4^2}{2.C_1.(C_3 - C_2)}\right) + 180^\circ$$

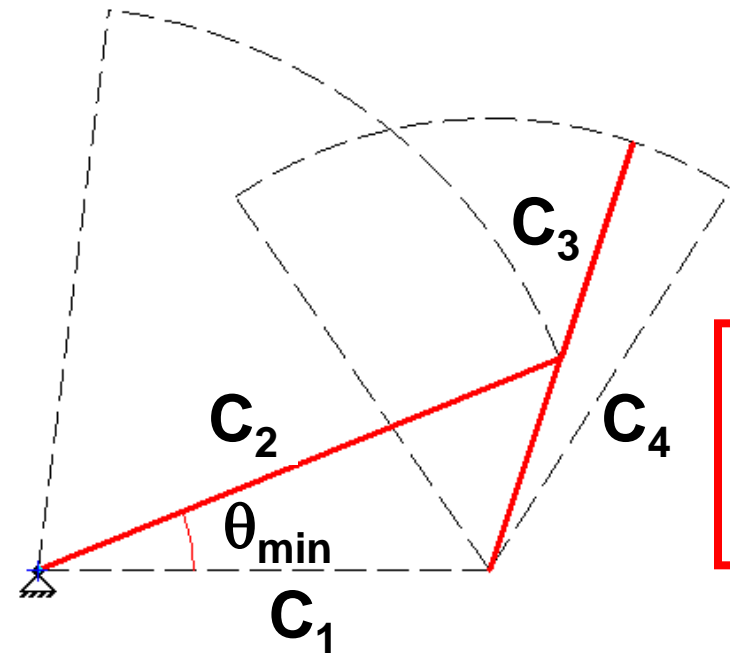
# Limites do mecanismo duplo-balancim

posições limites do elo de *entrada* (caracteriza travamento)



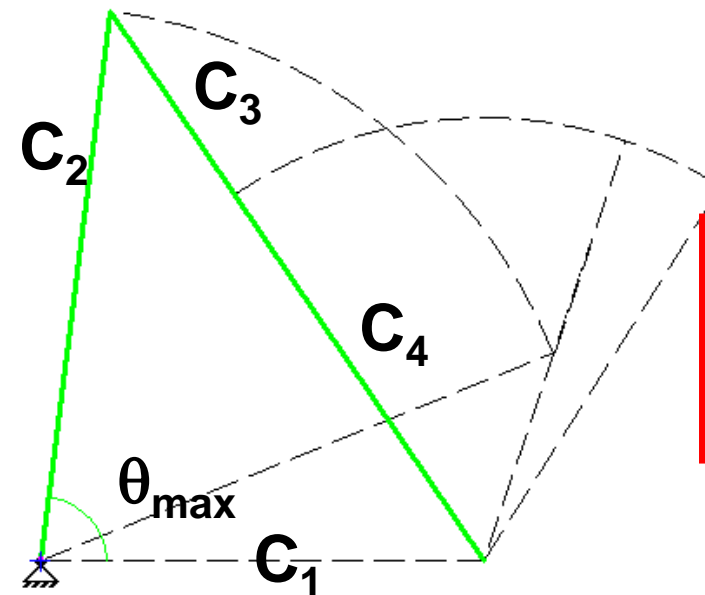
exemplo: [limites\\_4bBB.dv](#)

# Limites do mecanismo duplo-balancim



$$(C_4 - C_3)^2 = C_1^2 + C_2^2 - 2.C_1.C_2.\cos \theta_{\min}$$

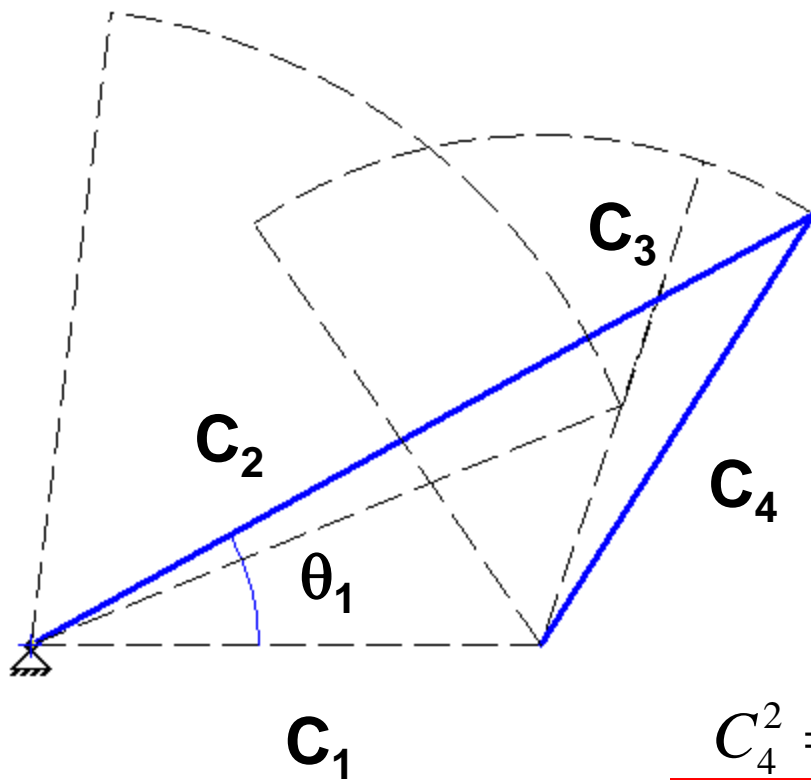
$$\theta_{\min} = \arccos\left(\frac{C_1^2 + C_2^2 - (C_4 - C_3)^2}{2.C_1.C_2}\right)$$



$$(C_4 + C_3)^2 = C_1^2 + C_2^2 - 2.C_1.C_2.\cos \theta_{\max}$$

$$\theta_{\max} = \arccos\left(\frac{C_1^2 + C_2^2 - (C_4 + C_3)^2}{2.C_1.C_2}\right)$$

# Limites do mecanismo duplo-balancim



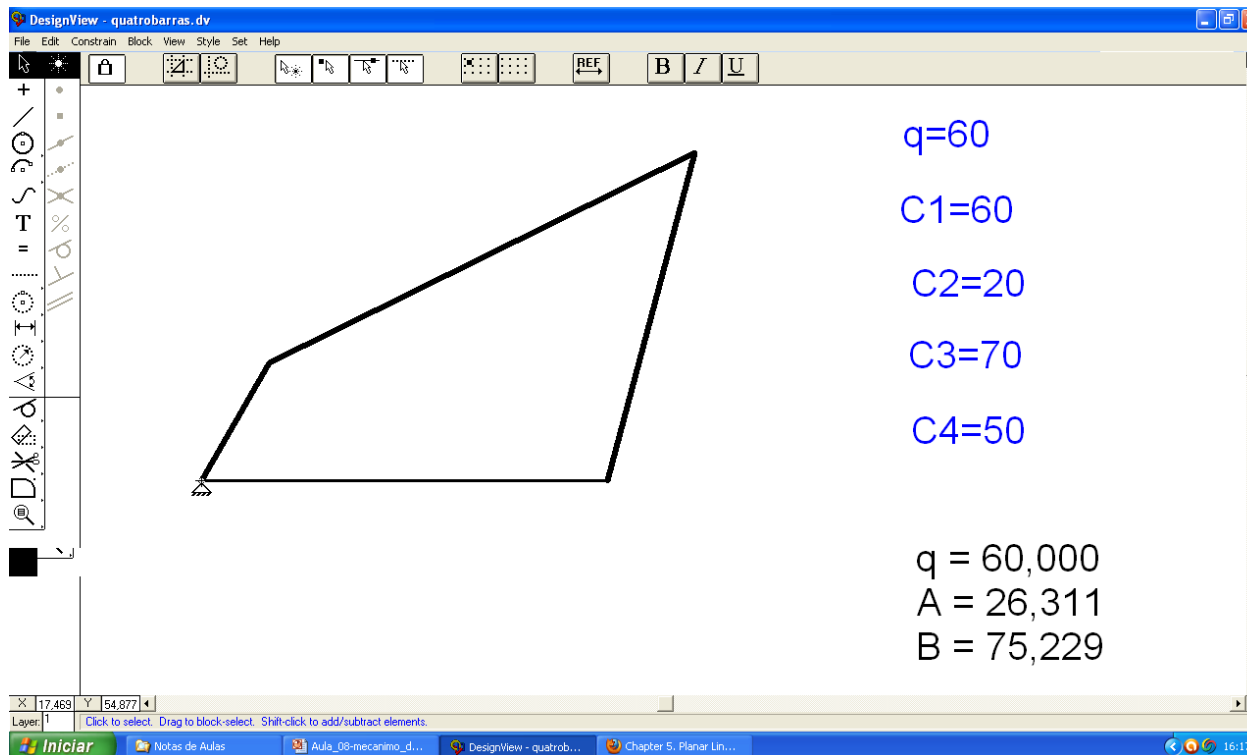
*Posição limite do elo de saída*

$$C_4^2 = C_1^2 + (C_3 + C_2)^2 - 2.C_1.(C_3 + C_2). \cos \theta_1$$

$$\theta_1 = \arccos\left(\frac{C_1^2 + (C_3 + C_2)^2 - C_4^2}{2.C_1.(C_3 + C_2)}\right)$$

# Exemplo

Arq.: quatro Barras.dv (Design View)



The screenshot shows a CAD software window titled "DesignView - quatro Barras.dv". The window contains a quadrilateral design and a list of parameters. The parameters are:

- $q=60$
- $C1=60$
- $C2=20$
- $C3=70$
- $C4=50$

Below these parameters, the following values are displayed:

- $q = 60,000$
- $A = 26,311$
- $B = 75,229$

The CAD window also shows a toolbar with various tools and a status bar at the bottom with the text "Click to select. Drag to block-select. Shift-click to add/subtract elements."

# Equações das velocidades

Derivadas das equações de posição em relação ao tempo

$$-C_2 \cdot \sin(q) \cdot \dot{q} - C_3 \cdot \sin(A) \cdot \dot{A} + C_4 \cdot \sin(B) \cdot \dot{B} = 0$$

$$C_2 \cdot \cos(q) \cdot \dot{q} + C_3 \cdot \cos(A) \cdot \dot{A} - C_4 \cdot \cos(B) \cdot \dot{B} = 0$$

$$\begin{pmatrix} -C_3 \cdot \sin(A) & C_4 \cdot \sin(B) \\ C_3 \cdot \cos(A) & -C_4 \cdot \cos(B) \end{pmatrix} \cdot \begin{pmatrix} \dot{A} \\ \dot{B} \end{pmatrix} + \begin{pmatrix} -C_2 \cdot \sin(q) \\ C_2 \cdot \cos(q) \end{pmatrix} \cdot \dot{q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

**Matriz  
jacobiana**

**Vetor das velocidades  
secundárias**

**Vetor dos  
coeficientes  
constantes**

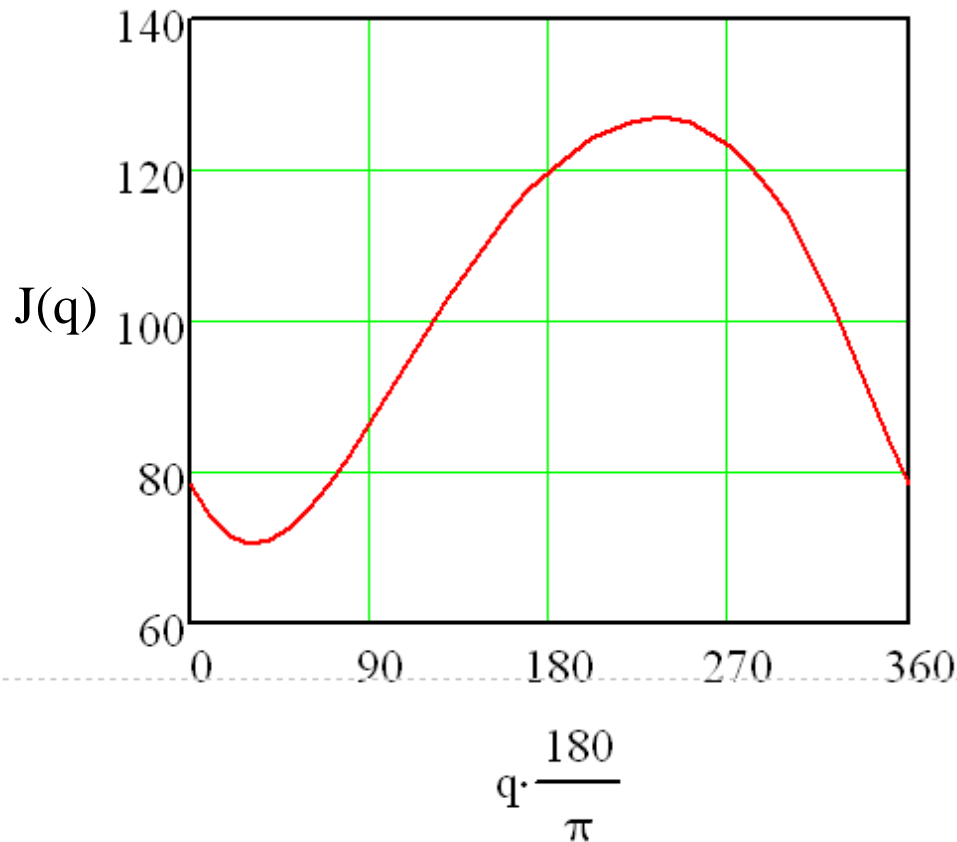
**Velocidade  
primária**

$$J \cdot \dot{S} + Q \cdot \dot{q} = 0$$



# Determinante da matriz Jacobiana

$$\underline{\underline{J}}(q) := \begin{pmatrix} -C_3 \cdot \sin(A(q)) & C_4 \cdot \sin(B(q)) \\ C_3 \cdot \cos(A(q)) & -C_4 \cdot \cos(B(q)) \end{pmatrix} \quad \det J(q) := C_3 \cdot C_4 \cdot \sin(A(q) - B(q))$$



# Solução para as velocidades secundárias

$$\begin{pmatrix} \dot{A} \\ \dot{B} \end{pmatrix} = - \begin{pmatrix} -C_3 \cdot \sin(A) & C_4 \cdot \sin(B) \\ C_3 \cdot \cos(A) & -C_4 \cdot \cos(B) \end{pmatrix}^{-1} \cdot \begin{pmatrix} -C_2 \cdot \sin(q) \\ C_2 \cdot \cos(q) \end{pmatrix} \cdot \dot{q}$$

$$\dot{S} = -J^{-1} \cdot Q \cdot \dot{q}$$

$$\begin{pmatrix} \dot{A} \\ \dot{B} \end{pmatrix} = \begin{pmatrix} \frac{C_2 \cdot \sin(B - q)}{C_3 \cdot \sin(A - B)} \\ \frac{C_2 \cdot \sin(A - q)}{C_4 \cdot \sin(A - B)} \end{pmatrix} \cdot \dot{q}$$

$$\dot{S} = K \cdot \dot{q}$$

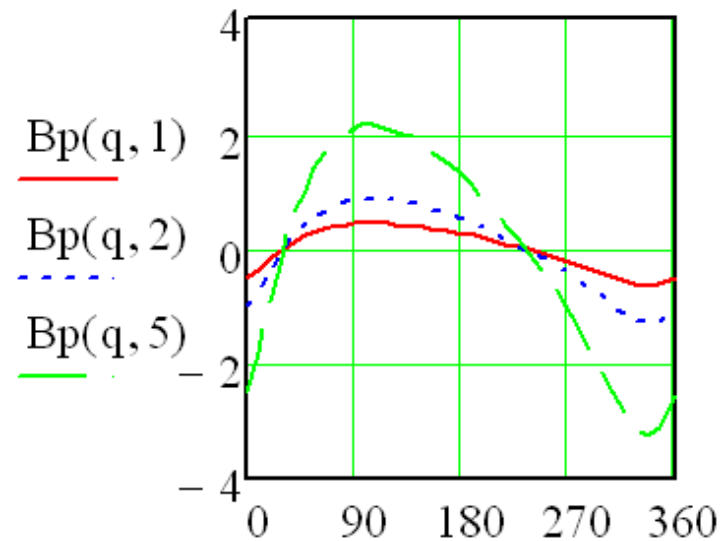
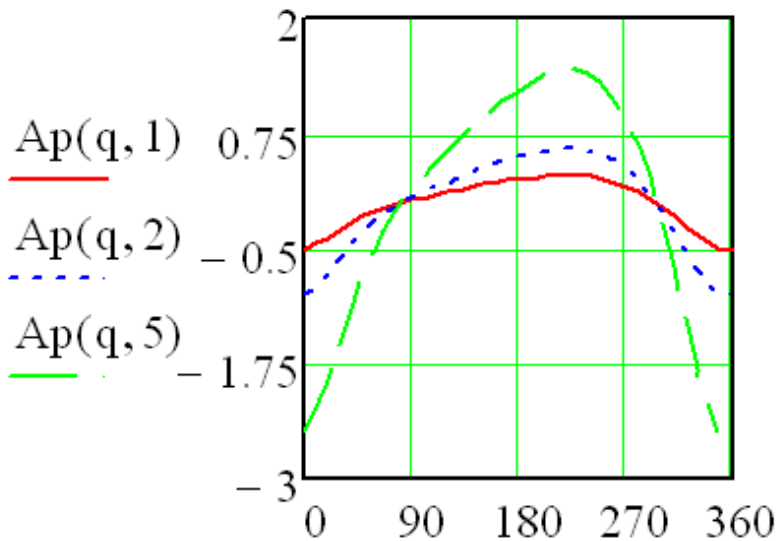
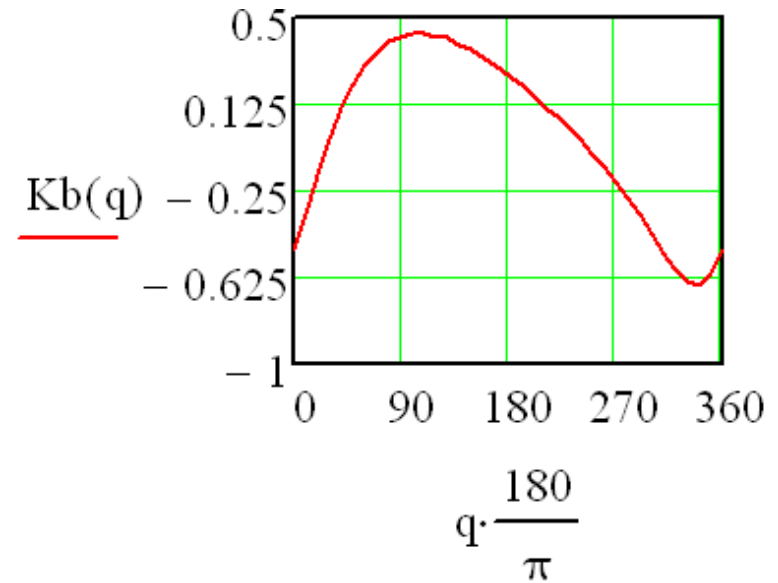
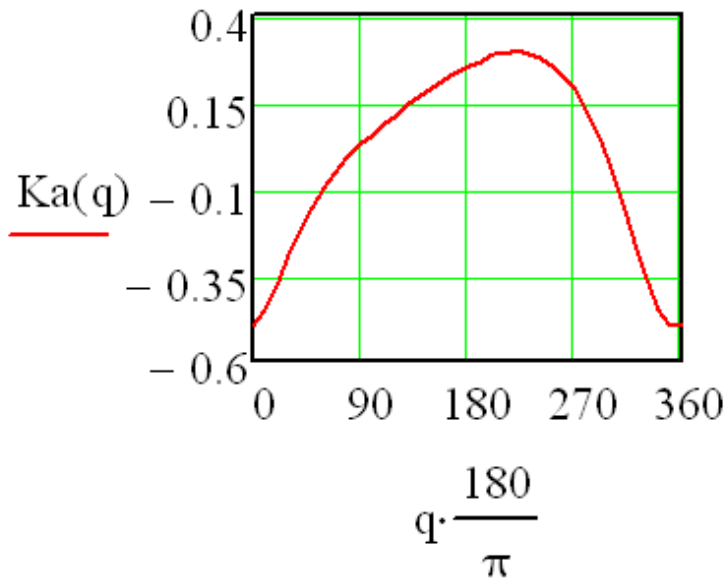
$$K_a(q) = \frac{C_2 \cdot \sin(B(q) - q)}{C_3 \cdot \sin(A(q) - B(q))}$$

$$K_b(q) = \frac{C_2 \cdot \sin(A(q) - q)}{C_4 \cdot \sin(A(q) - B(q))}$$

$$\dot{A}(q) = K_a(q) \cdot \dot{q}$$

$$\dot{B}(q) = K_b(q) \cdot \dot{q}$$

# Velocidades secundárias



# Acelerações secundárias

$$\ddot{A}(q) = K_a(q) \cdot \ddot{q} + L_a(q) \cdot \dot{q}^2$$

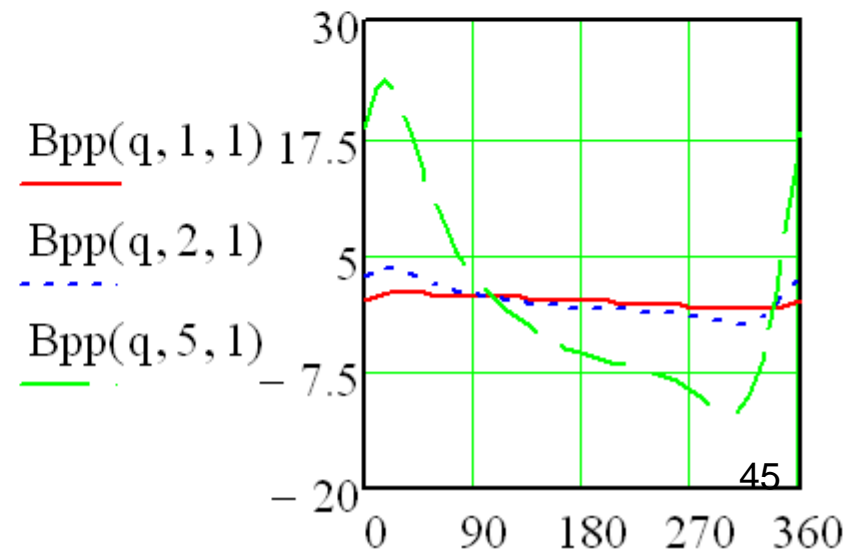
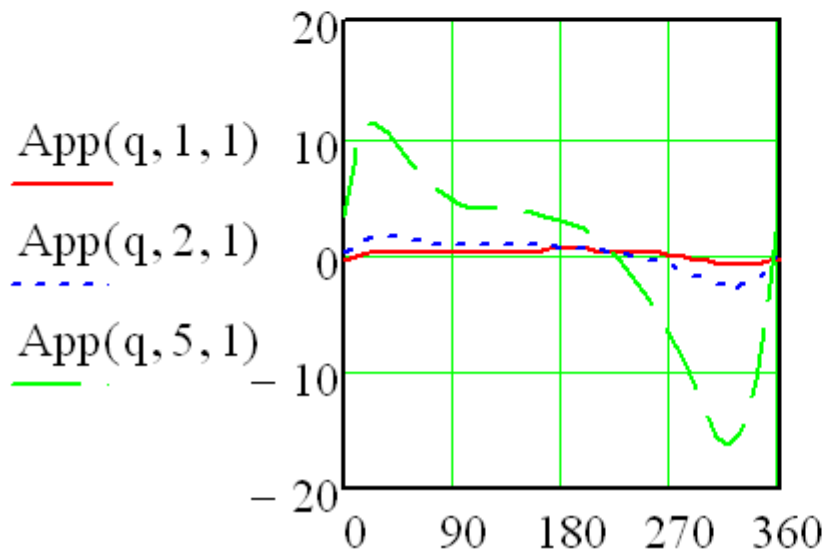
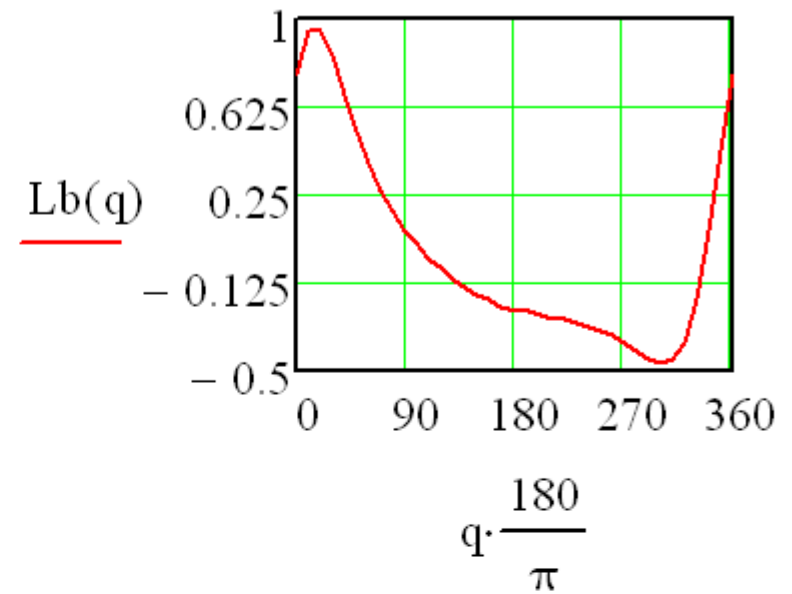
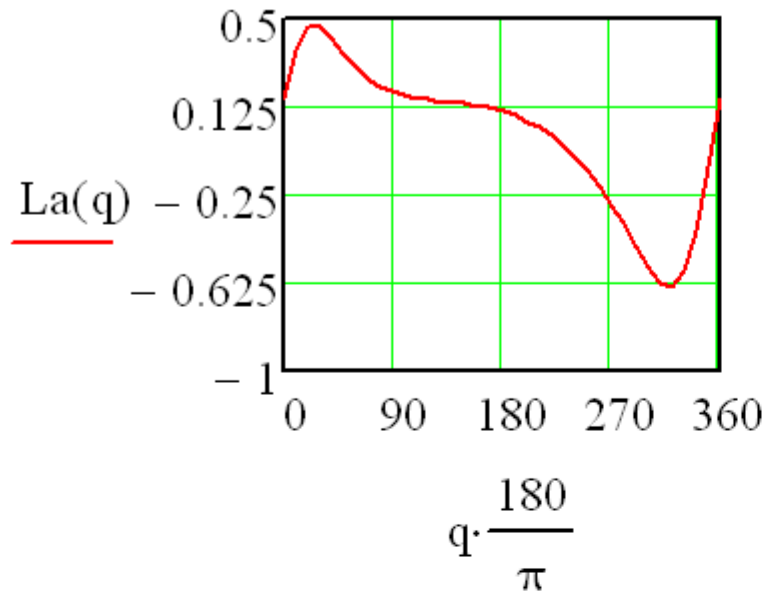
$$\ddot{B}(q) = K_b(q) \cdot \ddot{q} + L_b(q) \cdot \dot{q}^2$$

$$L_a(q) = \frac{d}{dq} K_a(q) \qquad L_b(q) = \frac{d}{dq} K_b(q)$$

$$L_a(q) := \frac{C_2 \cdot \cos(B(q) - q) \cdot (K_b(q) - 1)}{C_3 \cdot \sin(A(q) - B(q))} - \frac{C_2 \cdot \sin(B(q) - q) \cdot \cos(A(q) - B(q)) \cdot (K_a(q) - K_b(q))}{C_3 \cdot \sin(A(q) - B(q))^2}$$

$$L_b(q) := \frac{C_2 \cdot \cos(A(q) - q) \cdot (K_a(q) - 1)}{C_4 \cdot \sin(A(q) - B(q))} - \frac{C_2 \cdot \sin(A(q) - q) \cdot \cos(A(q) - B(q)) \cdot (K_a(q) - K_b(q))}{C_4 \cdot \sin(A(q) - B(q))^2}$$

# Acelerações secundárias

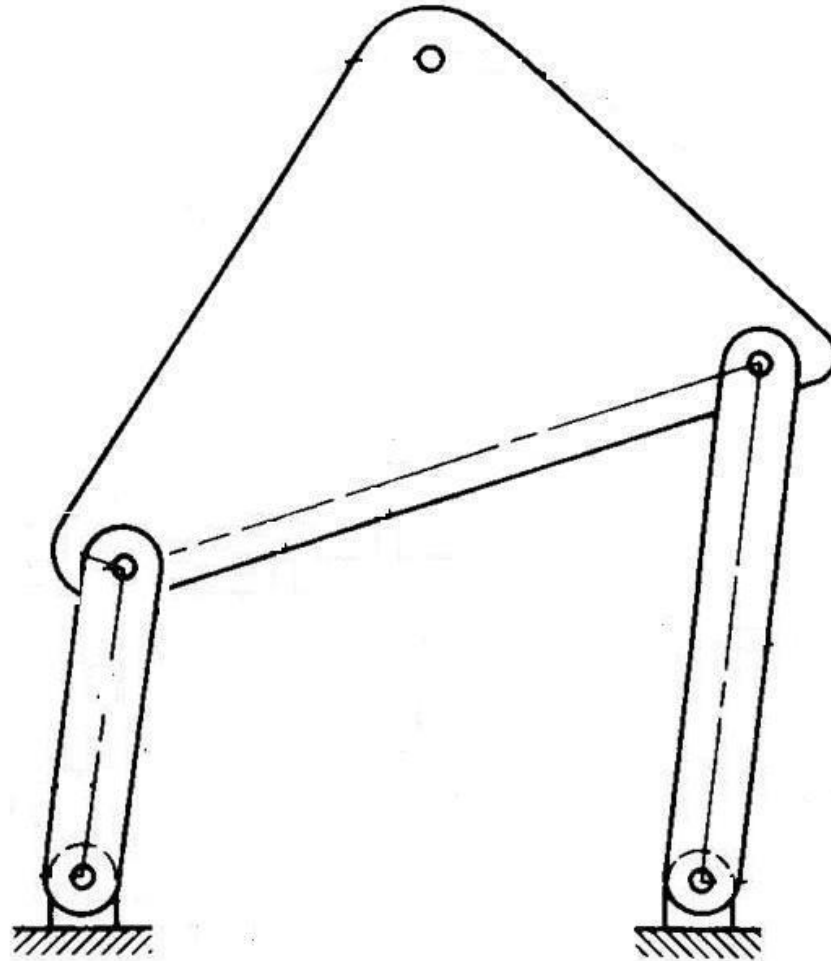


# 2-Análise de Pontos de Interesse

# Conteúdo

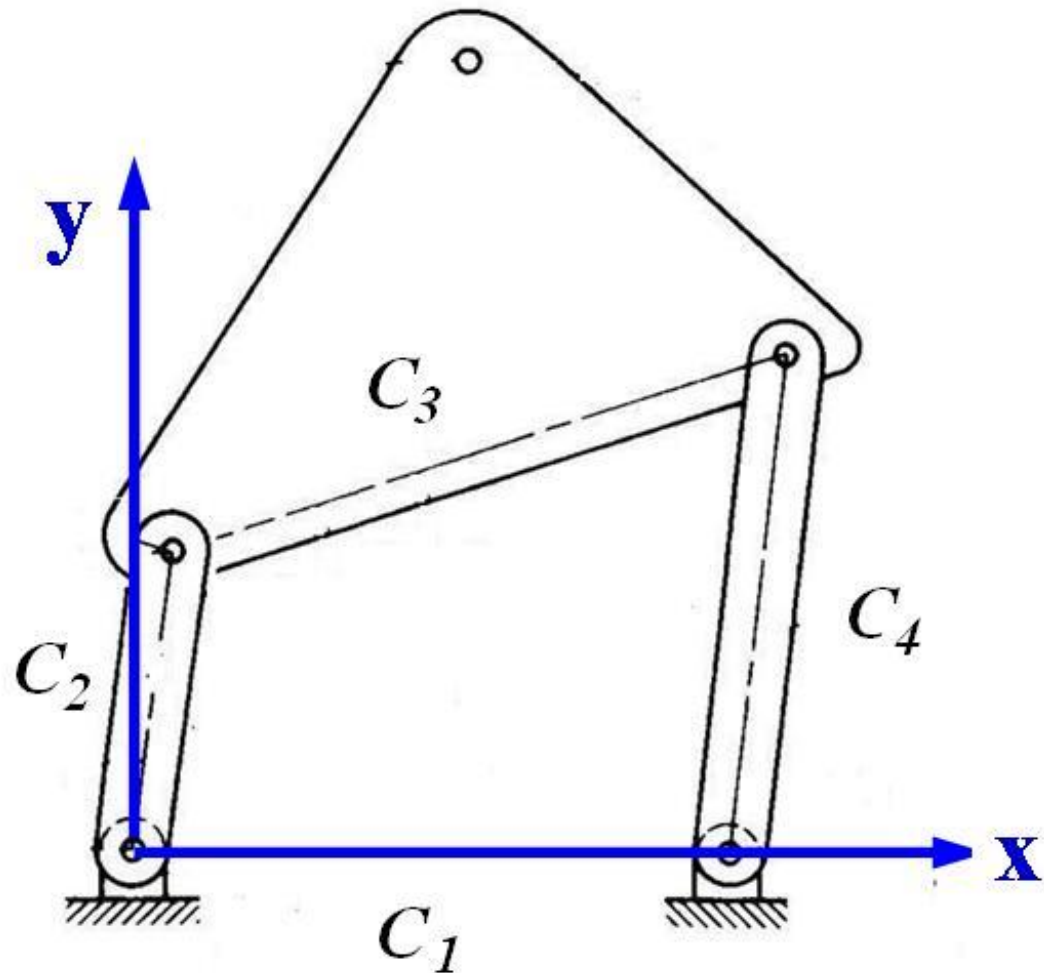
- Escolha do ponto de interesse
- Cálculo da posição, velocidade e aceleração do ponto
- Solução no MathCAd
- Desenho de deslocamentos
- Exemplo: guindaste

# Exemplo

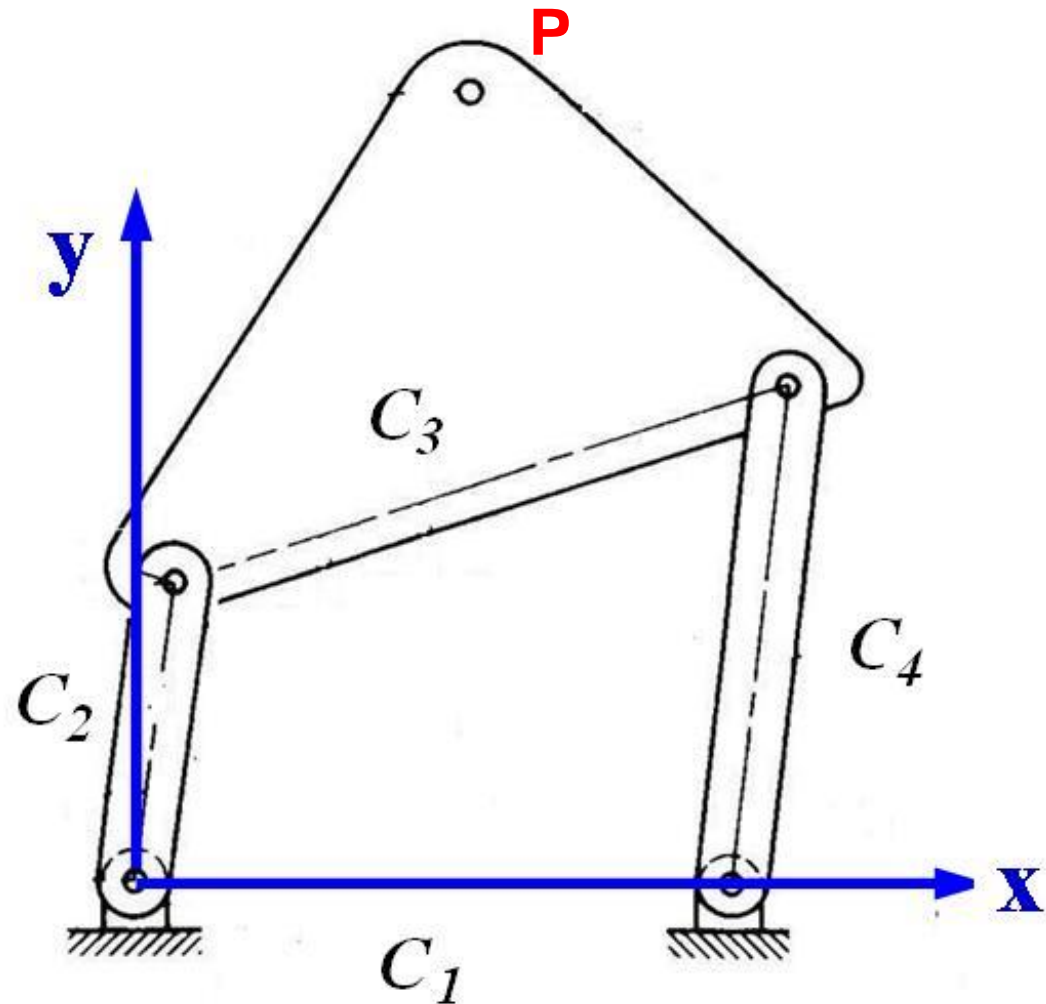




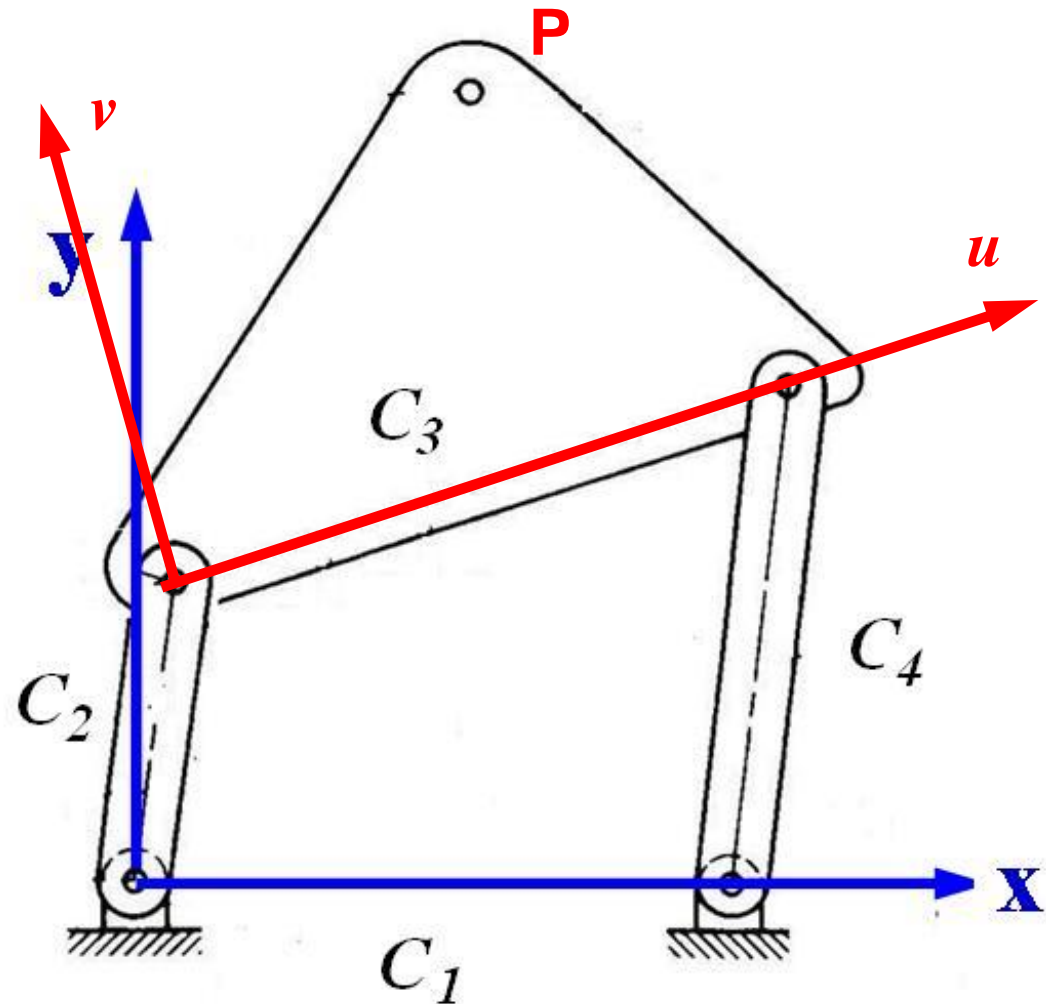
# 1- Sistema global (x,y)



## 2- Seleção do ponto de interesse (P)

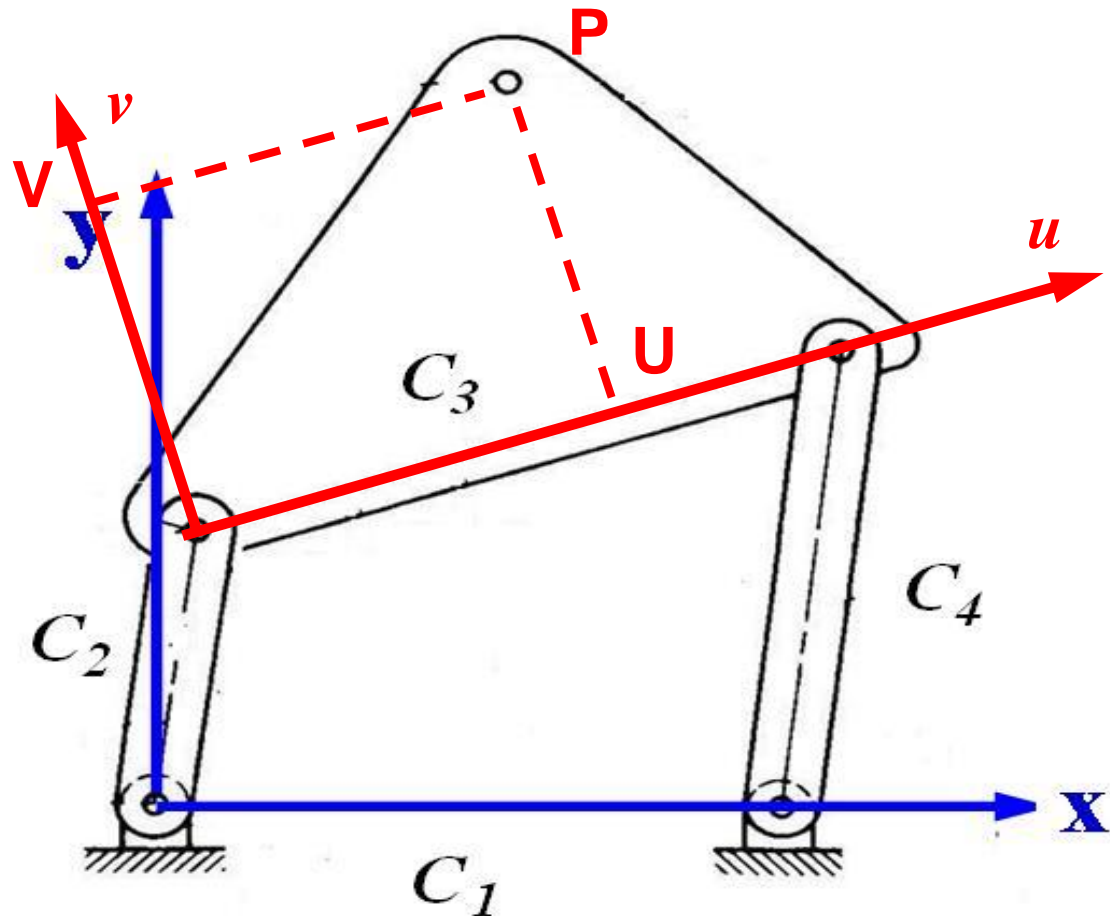


# 3-Sistema LOCAL (u,v)

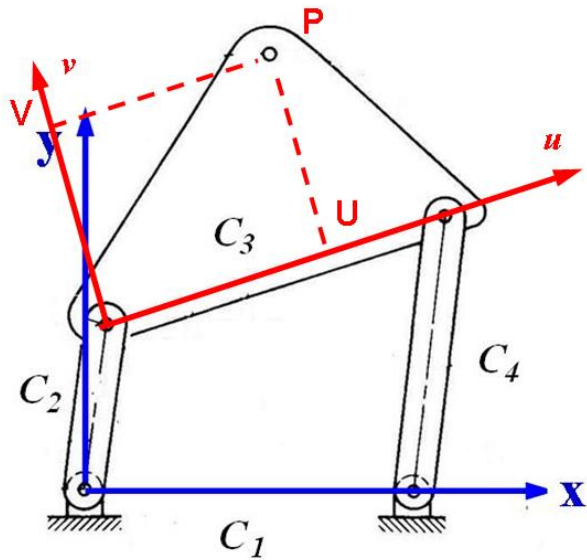


# 4-Coordenadas locais (U,V)

$$U_P := 50 \quad V_P := 50$$



# 5-Coordenadas globais (X,Y)



COORDENADAS LOCAIS

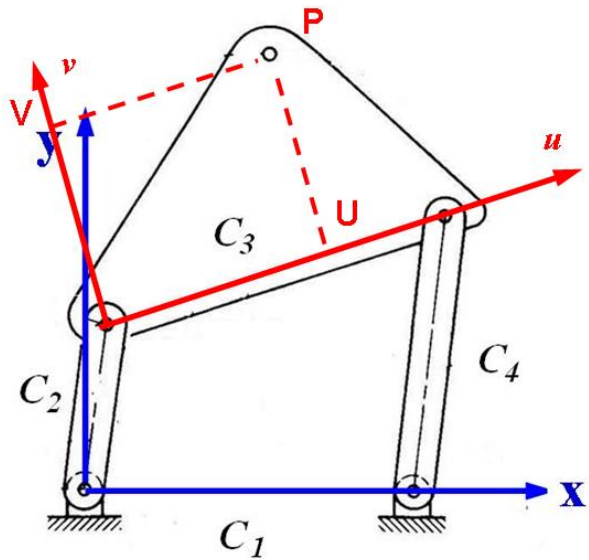
$$\begin{pmatrix} X_P \\ Y_P \end{pmatrix} = \begin{pmatrix} O_x \\ O_y \end{pmatrix} + \begin{pmatrix} u_x & v_x \\ u_y & v_y \end{pmatrix} \cdot \begin{pmatrix} U_P \\ V_P \end{pmatrix}$$

COORDENADAS GLOBAIS

ORIGEM DO SISTEMA LOCAL

MATRIZ DE ROTAÇÃO

# 5-Coordenadas globais (X,Y) (posição)



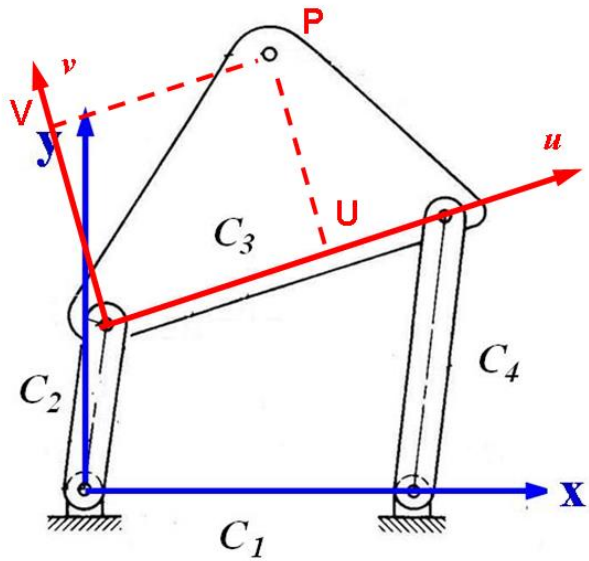
$$\begin{pmatrix} X_P \\ Y_P \end{pmatrix} = \begin{pmatrix} O_x \\ O_y \end{pmatrix} + \begin{pmatrix} u_x & v_x \\ u_y & v_y \end{pmatrix} \cdot \begin{pmatrix} U_P \\ V_P \end{pmatrix}$$

$$\begin{pmatrix} X(q) \\ Y(q) \end{pmatrix} = \begin{pmatrix} C_2 \cdot \cos(q) \\ C_2 \cdot \sin(q) \end{pmatrix} + \begin{pmatrix} \cos(A(q)) & -\sin(A(q)) \\ \sin(A(q)) & \cos(A(q)) \end{pmatrix} \cdot \begin{pmatrix} U_P \\ V_P \end{pmatrix}$$

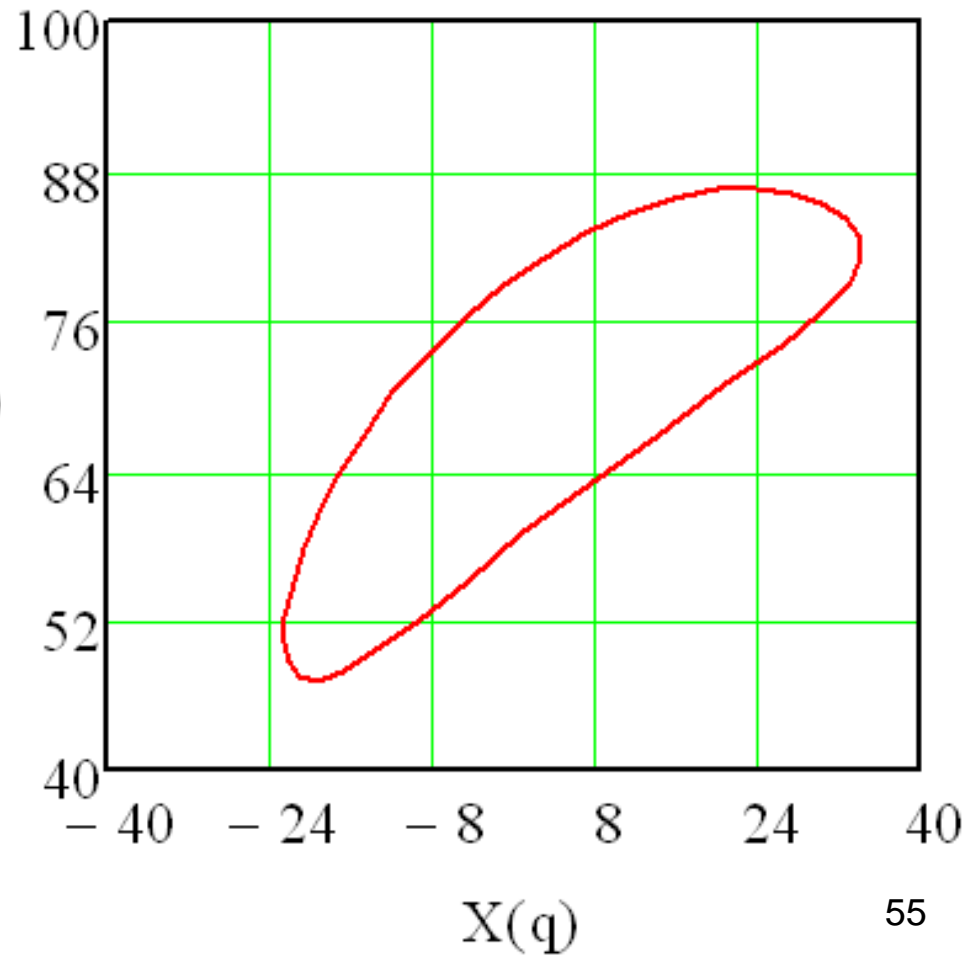
$$X(q) := C_2 \cdot \cos(q) + \cos(A(q)) \cdot U_P - \sin(A(q)) \cdot V_P$$

$$Y(q) := C_2 \cdot \sin(q) + \sin(A(q)) \cdot U_P + \cos(A(q)) \cdot V_P$$

# 5-Coordenadas globais (X,Y) (posição)



Y(q)



# Equações das velocidades

$$X(q) := C_2 \cdot \cos(q) + \cos(A(q)) \cdot U_P - \sin(A(q)) \cdot V_P$$

$$Y(q) := C_2 \cdot \sin(q) + \sin(A(q)) \cdot U_P + \cos(A(q)) \cdot V_P$$

Derivadas das equações de posição em relação ao tempo

$$K_{px}(q) := -C_2 \cdot \sin(q) - \sin(A(q)) \cdot K_a(q) \cdot U_P - \cos(A(q)) \cdot K_a(q) \cdot V_P$$

$$K_{py}(q) := C_2 \cdot \cos(q) + \cos(A(q)) \cdot K_a(q) \cdot U_P - \sin(A(q)) \cdot K_a(q) \cdot V_P$$

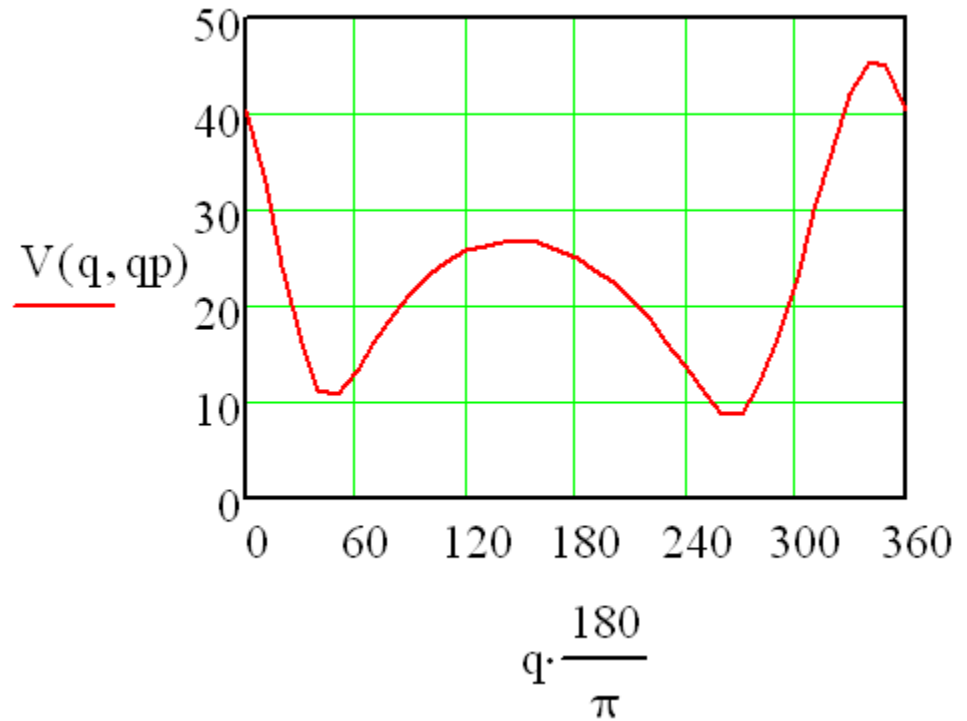
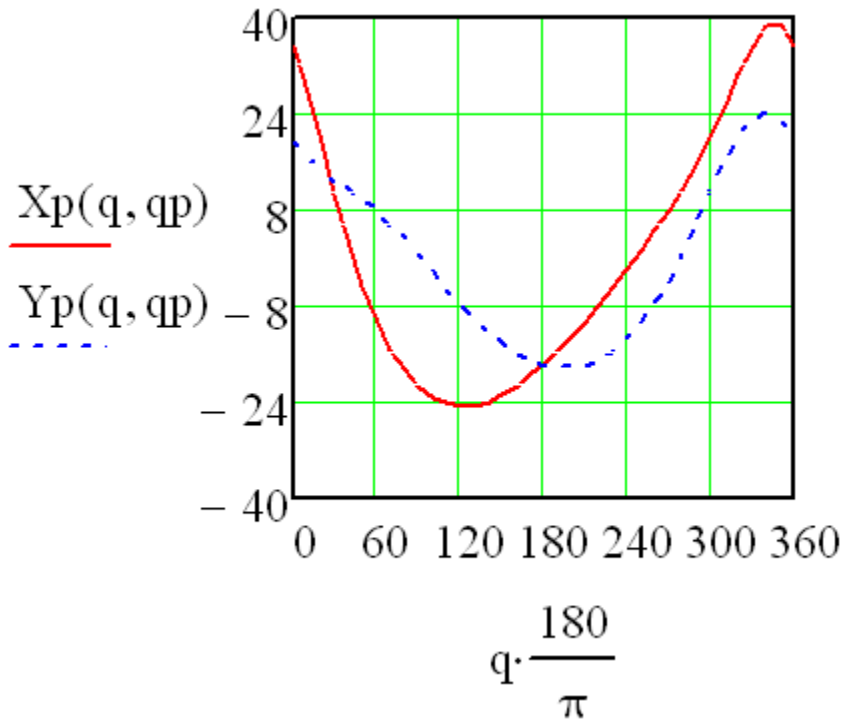
$$\dot{X}(q, \dot{q}) := K_{px}(q) \cdot \dot{q}$$

$$\dot{Y}(q, \dot{q}) := K_{py}(q) \cdot \dot{q}$$

$$V(q, \dot{q}) := \sqrt{\dot{X}(q, \dot{q})^2 + \dot{Y}(q, \dot{q})^2}$$



# Velocidade do ponto P



# Equações das acelerações

$$L_{Px}(q) := -Ka(q)^2 \cdot (U_P \cdot \cos(A(q)) - V_P \cdot \sin(A(q))) - C_2 \cdot \cos(q) - La(q) \cdot (U_P \cdot \sin(A(q)) + V_P \cdot \cos(A(q)))$$

$$L_{Py}(q) := (-U_P \cdot \sin(A(q)) - V_P \cdot \cos(A(q))) \cdot Ka(q)^2 - C_2 \cdot \sin(q) + (U_P \cdot \cos(A(q)) - V_P \cdot \sin(A(q))) \cdot La(q)$$

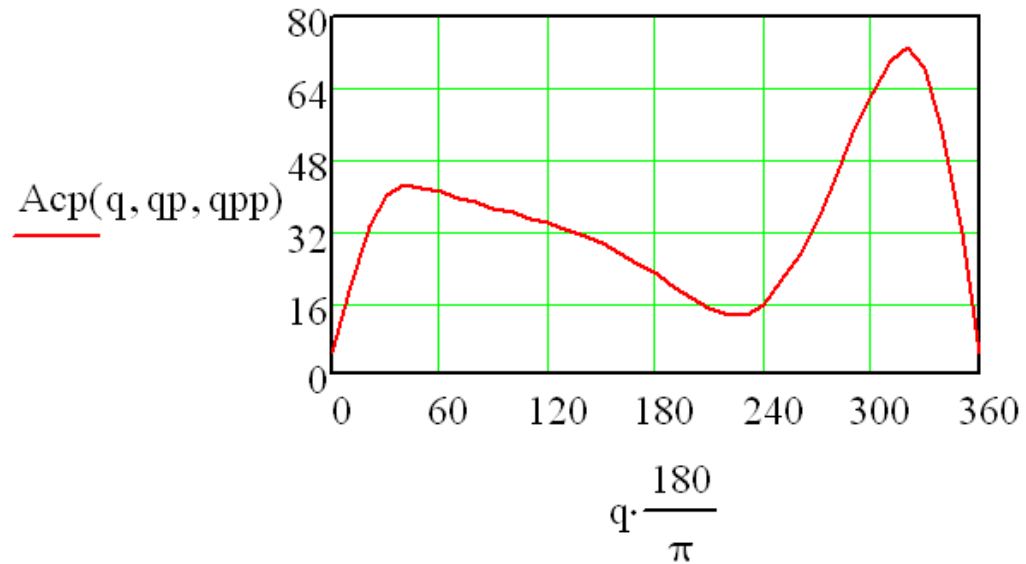
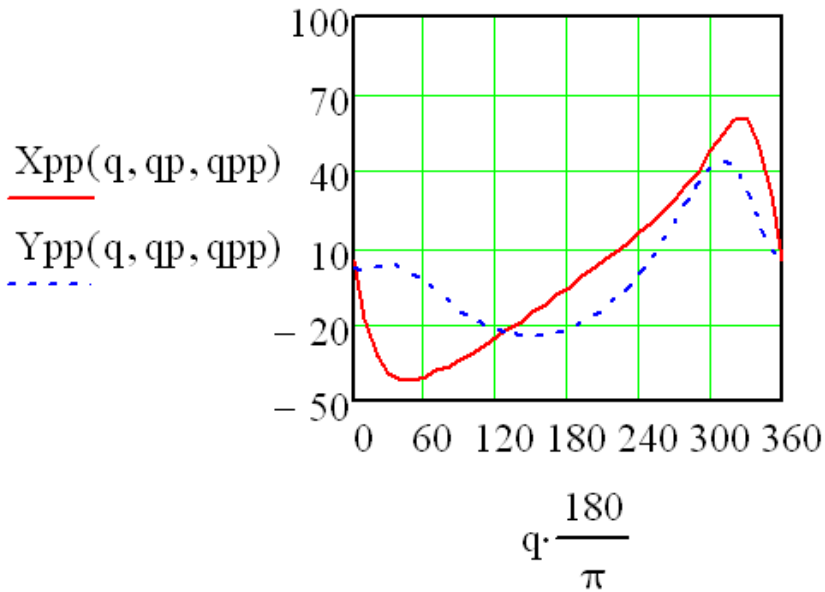
$$L_{Px} = \frac{d}{dq} K_{Px} \quad L_{Py} = \frac{d}{dq} K_{Py}$$

$$\ddot{X}_P = K_{Px} \cdot \ddot{q} + L_{Px} \cdot \dot{q}^2$$

$$\ddot{Y}_P = K_{Py} \cdot \ddot{q} + L_{Py} \cdot \dot{q}^2$$

$$A_P = \sqrt{\ddot{X}_P^2 + \ddot{Y}_P^2}$$

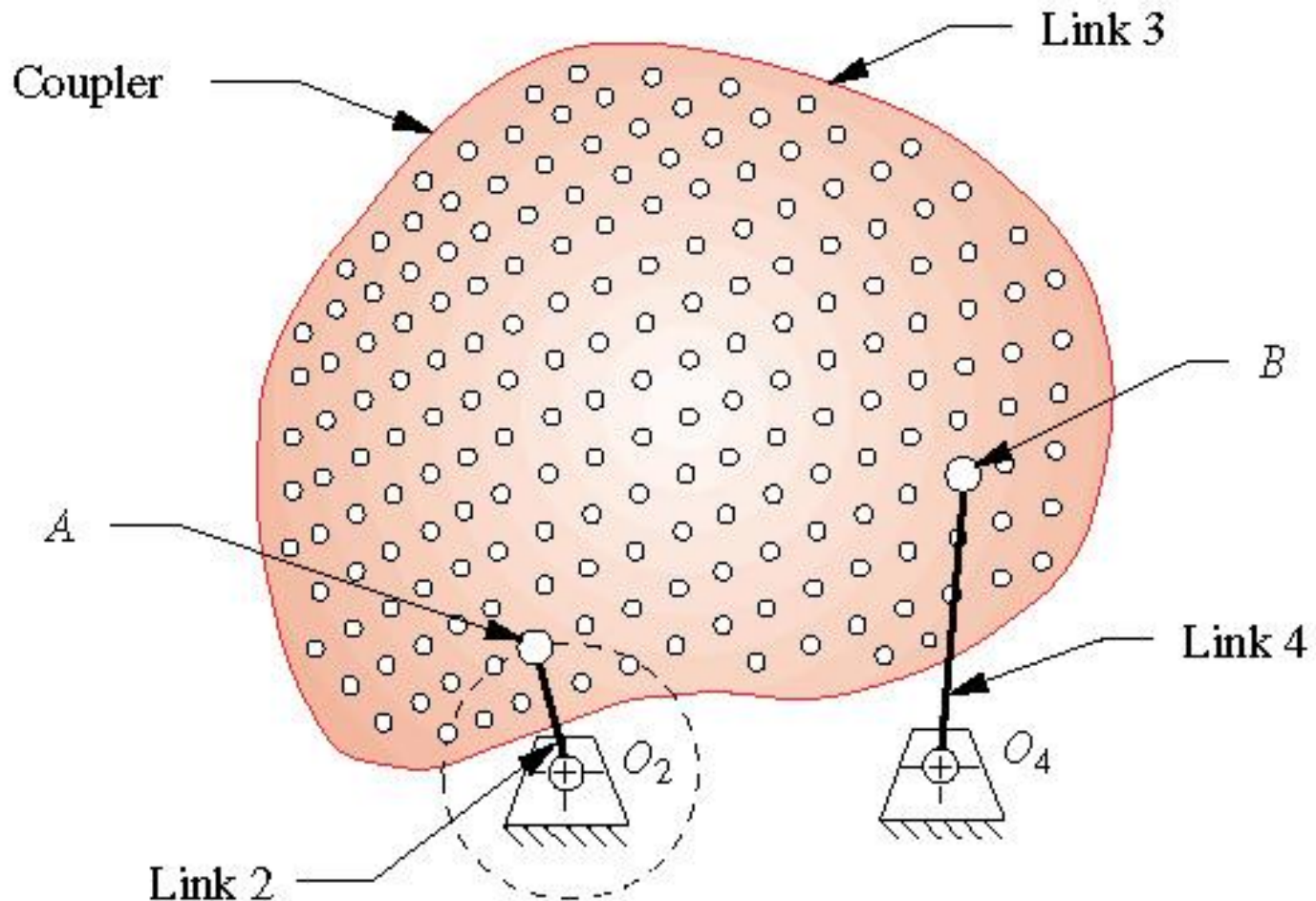
# Aceleração do ponto P



# Solução no Matlab

[Arquivo: quatrobarras.m](#)

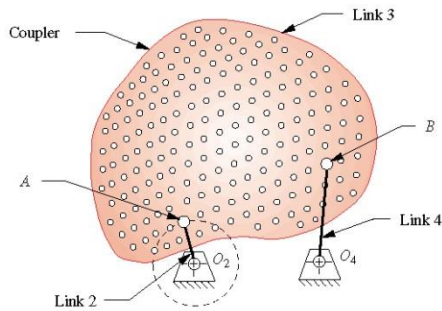
# Desenhos de deslocamentos



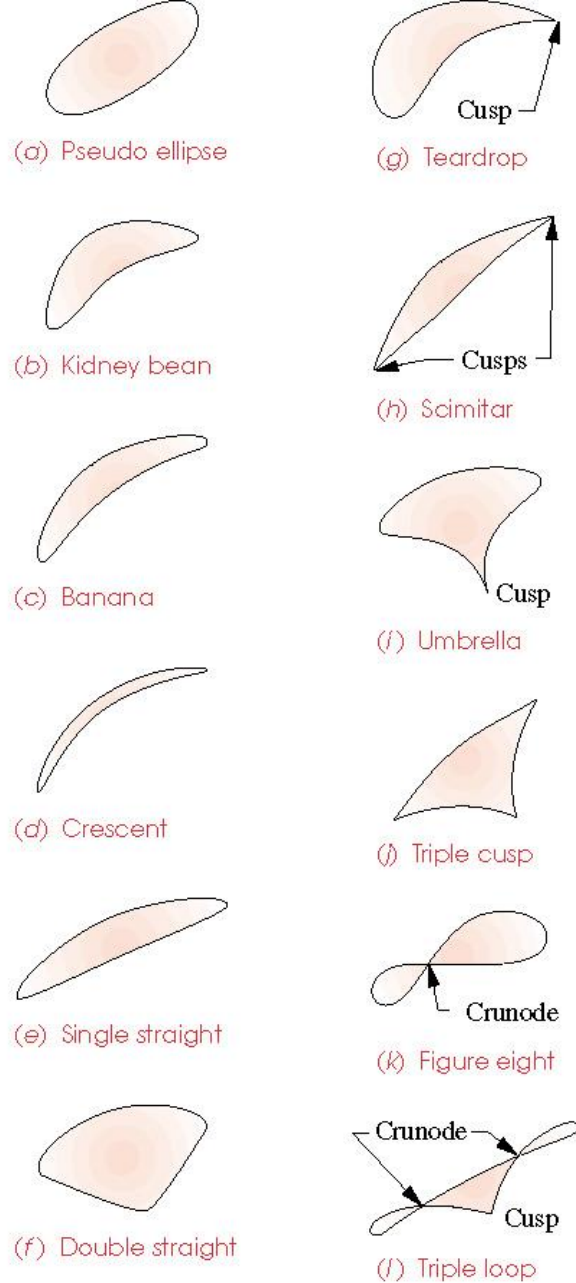
**FIGURE 3-15**

The fourbar coupler extended to include a large number of coupler points

# Desenhos de deslocamentos



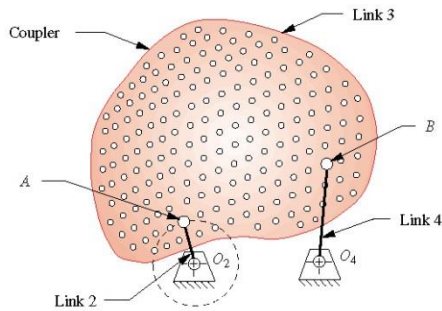
**FIGURE 3-15**  
The fourbar coupler extended to include a large number of coupler points



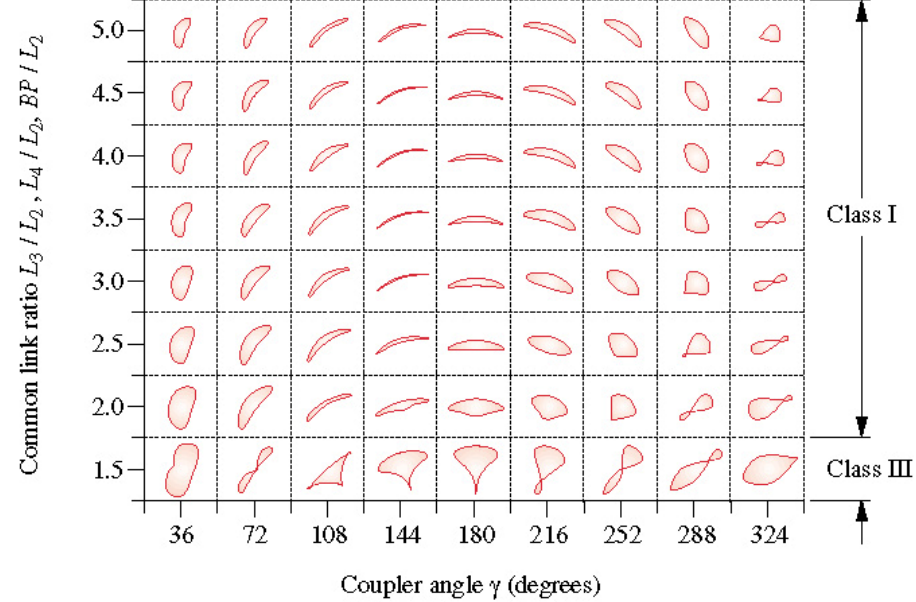
**Mecanismo FIGURE 3-16 Part 1**  
A "Cursory Catalog" of coupler curve shapes

**FIGURE 3-16 Part 2**  
A "Cursory Catalog" of coupler curve shapes

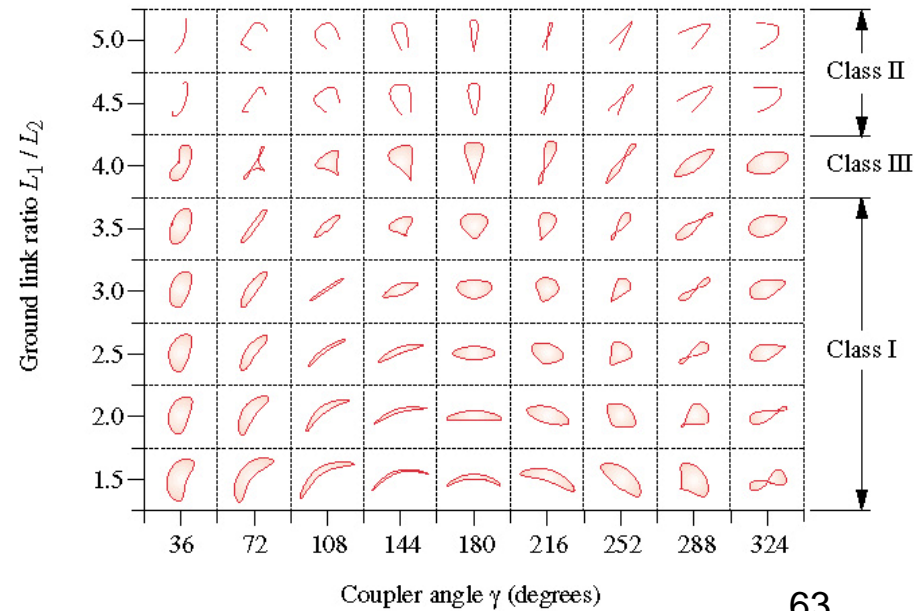
# Desenhos de deslocamentos



**FIGURE 3-15**  
The fourbar coupler extended to include a large number of coupler points

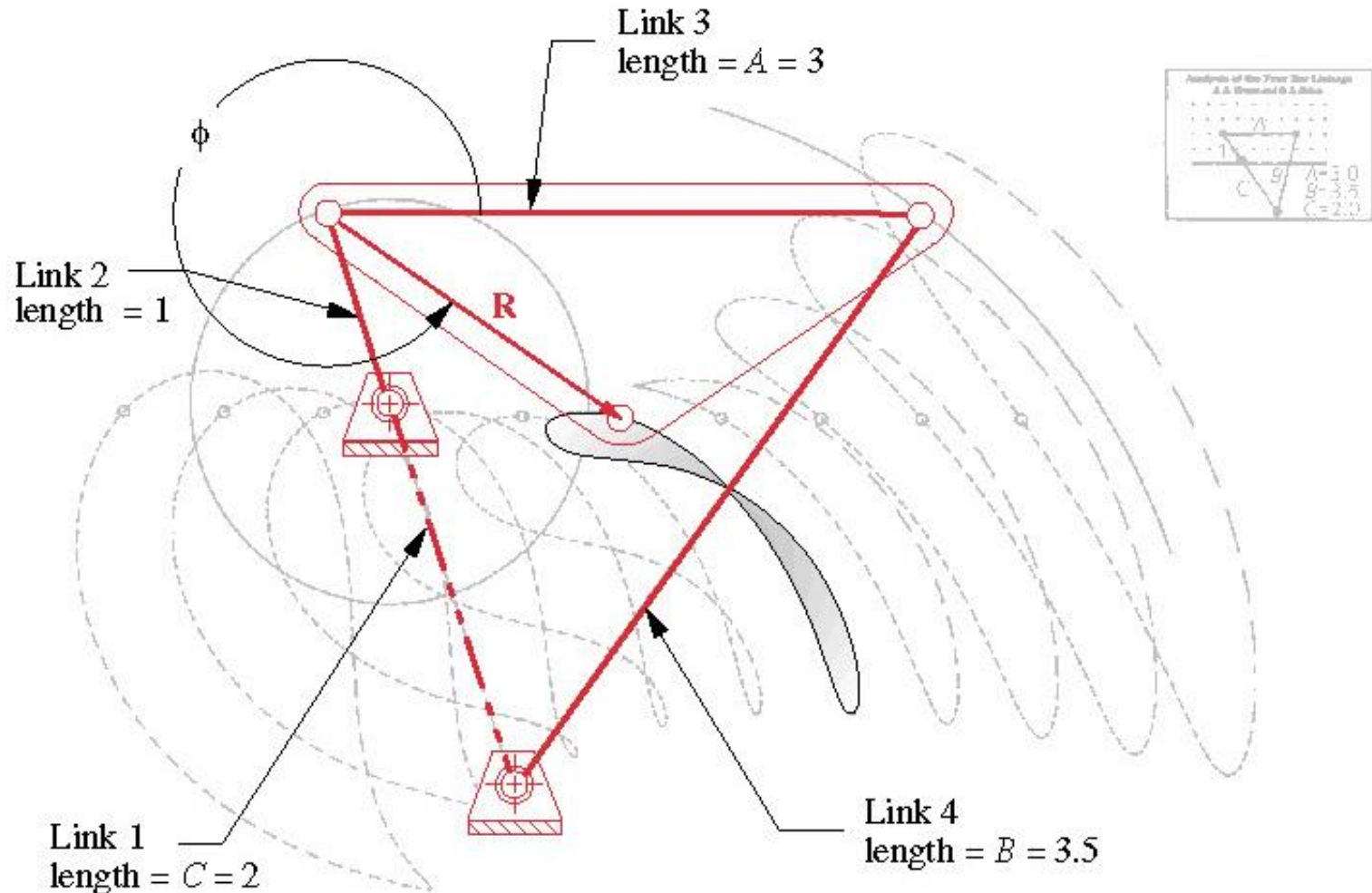


(a) Variation of coupler curve shape with common link ratio and coupler angle for a ground link ratio  $L_1 / L_2 = 2.0$



(b) Variation of coupler curve shape with ground link ratio and coupler angle for a common link ratio  $L_3 / L_2 = L_4 / L_2 = BP / L_2 = 2.5$

# Desenhos de deslocamentos



(b) Creating the linkage from the information in the atlas

**FIGURE 3-17**

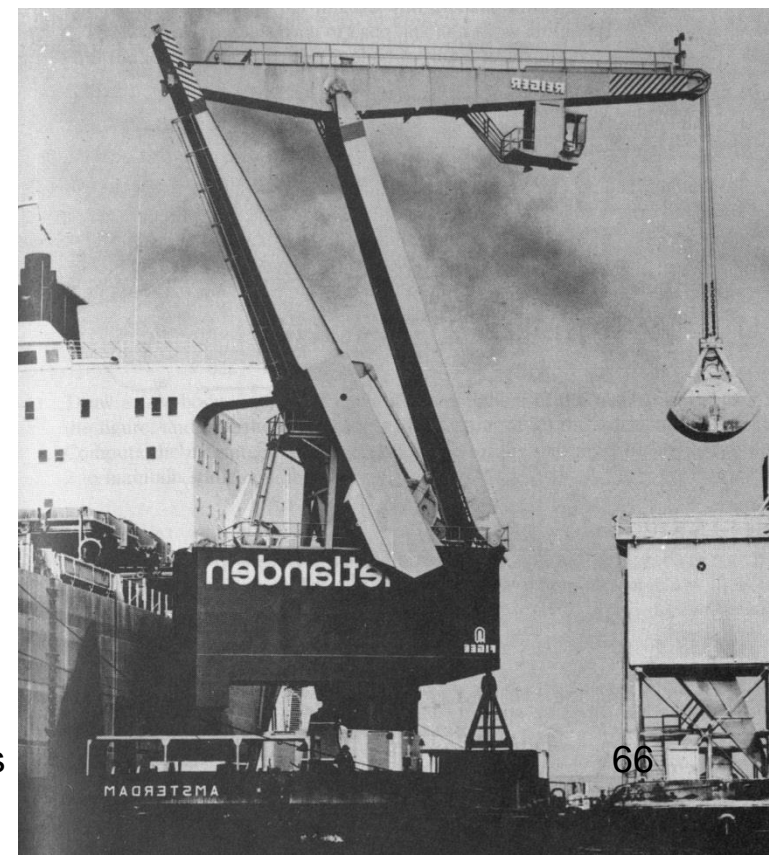
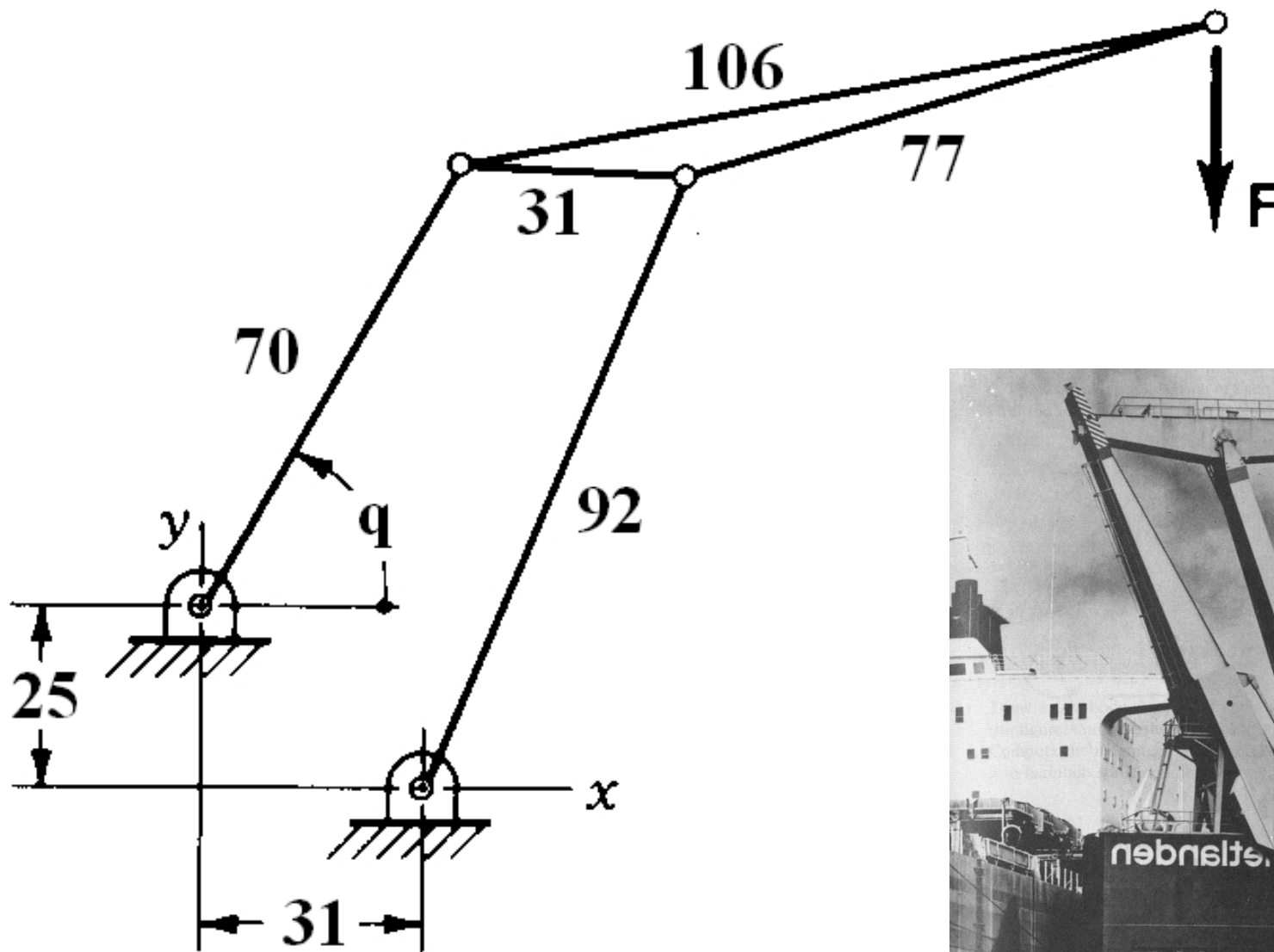
Selecting a coupler curve and constructing the linkage from the Hrones and Nelson atlas



# Desenhos de deslocamentos

[Arquivo: quatrobarras.dv](#)

# Exercício



[Arquivo: guindaste.dv](#)

Mecanismo de 4 Barras