THE VIEW FACTOR

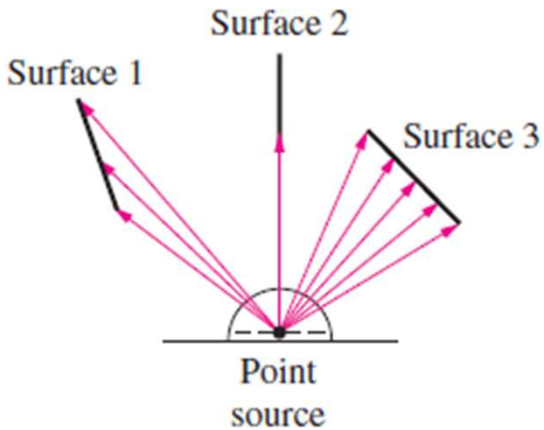


FIGURE 12–1
Radiation heat exchange between surfaces depends on the *orientation* of the surfaces relative to each other, and this dependence on orientation is accounted for by the *view factor*.

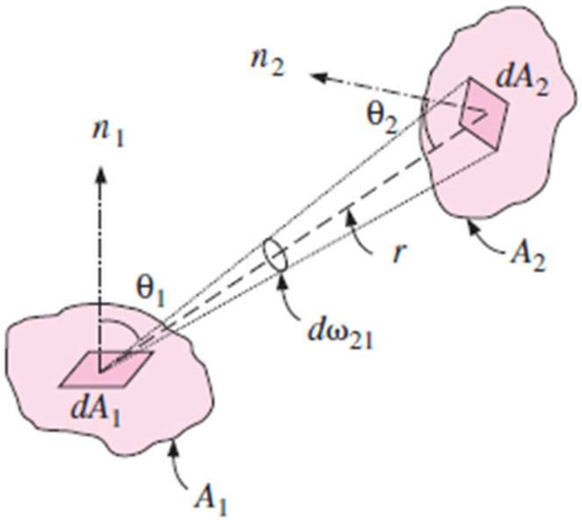


FIGURE 12–2
Geometry for the determination of the view factor between two surfaces.

$$\dot{Q}_{dA_1 \to dA_2} = I_1 \cos\theta_1 dA_1 d\omega_{21} = I_1 \cos\theta_1 dA_1 \frac{dA_2 \cos\theta_2}{r^2}$$

$$\dot{Q}_{A_1 \to A_2} = \int_{A_2}\int_{A_1} \frac{I_1 \cos\theta_1 \cos\theta_2}{r^2} dA_1\, dA_2$$

$$\dot{Q}_{dA_1} = J_1 dA_1 = \pi I_1 dA_1$$

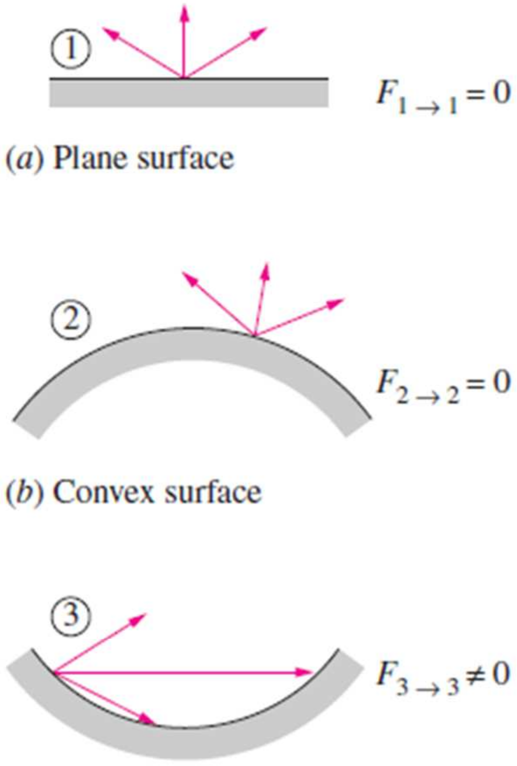
$$\dot{Q}_{A_1} = J_1 A_1 = \pi I_1 A_1$$

$$F_{12} = F_{A_1 \to A_2} = \frac{\dot{Q}_{A_1 \to A_2}}{\dot{Q}_{A_1}} = \frac{1}{A_1}\int_{A_2}\int_{A_1} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dA_1\, dA_2$$

$$F_{21} = F_{A_2 \to A_1} = \frac{\dot{Q}_{A_2 \to A_1}}{\dot{Q}_{A_2}} = \frac{1}{A_2}\int_{A_2}\int_{A_1} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dA_1 dA_2$$

$$A_1F_{12} = A_2F_{21} \quad \text{(12-10)}$$

which is known as the **reciprocity relation** for view factors. It allows the calculation of a view factor from a knowledge of the other.



(*c*) Concave surface

FIGURE 12–3

The view factor from a surface to itself is *zero* for *plane* or *convex* surfaces and *nonzero* for *concave* surfaces.

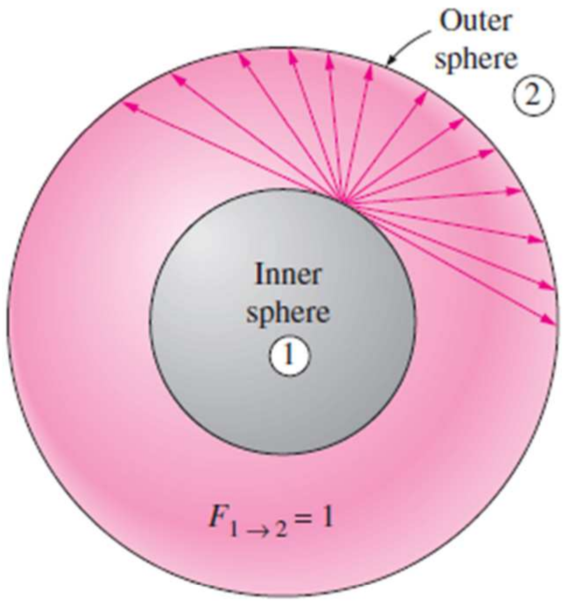


FIGURE 12–4

In a geometry that consists of two concentric spheres, the view factor $F_{1\to2} = 1$ since the entire radiation leaving the surface of the smaller sphere will be intercepted by the larger sphere.

1 The Reciprocity Relation

The view factors $F_{i \to j}$ and $F_{j \to i}$ are *not* equal to each other unless the areas of the two surfaces are. That is,

$$F_{j \to i} = F_{i \to j} \qquad \text{when} \qquad A_i = A_j$$
$$F_{j \to i} \neq F_{i \to j} \qquad \text{when} \qquad A_i \neq A_j$$

$$A_i F_{i \to j} = A_j F_{j \to i} \tag{12-11}$$

This relation is referred to as the **reciprocity relation** or the **reciprocity rule,** and it enables us to determine the counterpart of a view factor from a knowledge of the view factor itself and the areas of the two surfaces. When determining the pair of view factors $F_{i \to j}$ and $F_{j \to i}$, it makes sense to evaluate first the easier one directly and then the more difficult one by applying the reciprocity relation.

The Summation Rule

The conservation of energy principle requires that the entire radiation leaving any surface i of an enclosure be intercepted by the surfaces of the enclosure. Therefore, *the sum of the view factors from surface i of an enclosure to all surfaces of the enclosure, including to itself, must equal unity.* This is known as the **summation rule** for an enclosure and is expressed as (Fig. 12–9)

$$\sum_{j=1}^{N} F_{i \to j} = 1 \qquad \textbf{(12-12)}$$

The Summation Rule

The summation rule can be applied to each surface of an enclosure by varying i from 1 to N. Therefore, the summation rule applied to each of the N surfaces of an enclosure gives N relations for the determination of the view factors. Also, the reciprocity rule gives $\frac{1}{2}N(N-1)$ additional relations. Then the total number of view factors that need to be evaluated directly for an N-surface enclosure becomes

$$N^2 - [N + \tfrac{1}{2}N(N-1)] = \tfrac{1}{2}N(N-1)$$

For example, for a six-surface enclosure, we need to determine only $\frac{1}{2} \times 6(6-1) = 15$ of the $6^2 = 36$ view factors directly. The remaining 21 view factors can be determined from the 21 equations that are obtained by applying the reciprocity and the summation rules.

EXAMPLE

Determine the view factors associated with an enclosure formed by two spheres, shown in Figure 12–10.

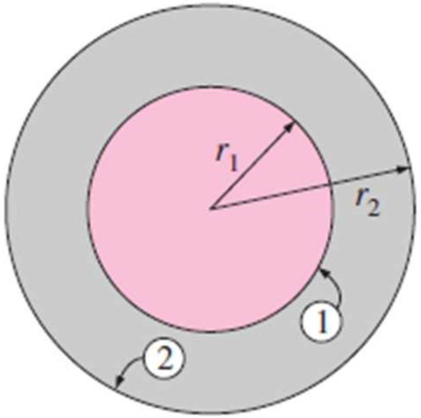


FIGURE 12–10
The geometry considered in Example 12–1.

Analysis The outer surface of the smaller sphere (surface 1) and inner surface of the larger sphere (surface 2) form a two-surface enclosure. Therefore, $N = 2$ and this enclosure involves $N^2 = 2^2 = 4$ view factors, which are F_{11}, F_{12}, F_{21}, and F_{22}. In this two-surface enclosure, we need to determine only

$$\tfrac{1}{2}N(N-1) = \tfrac{1}{2} \times 2(2-1) = 1$$

$F_{11} = 0$, since no radiation leaving surface 1 strikes itself

$F_{12} = 1$, since all radiation leaving surface 1 strikes surface 2

$$A_1 F_{12} = A_2 F_{21}$$

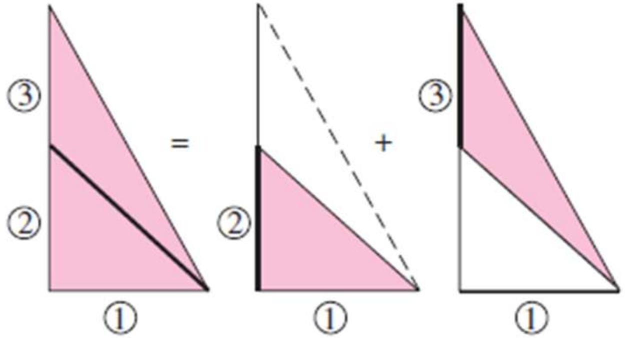
$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{4\pi r_1^2}{4\pi r_2^2} \times 1 = \left(\frac{r_1}{r_2}\right)^2$$

$$F_{21} + F_{22} = 1$$

$$F_{22} = 1 - F_{21} = 1 - \left(\frac{r_1}{r_2}\right)^2$$

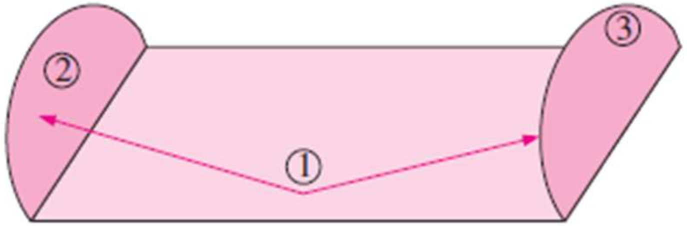
The Superposition Rule

$$F_{1 \to (2,3)} = F_{1 \to 2} + F_{1 \to 3}$$



$F_{1 \to (2,3)} = F_{1 \to 2} + F_{1 \to 3}$

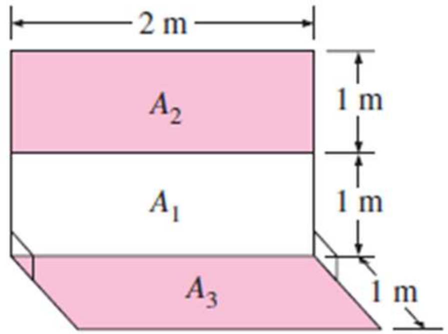
The Symmetry Rule

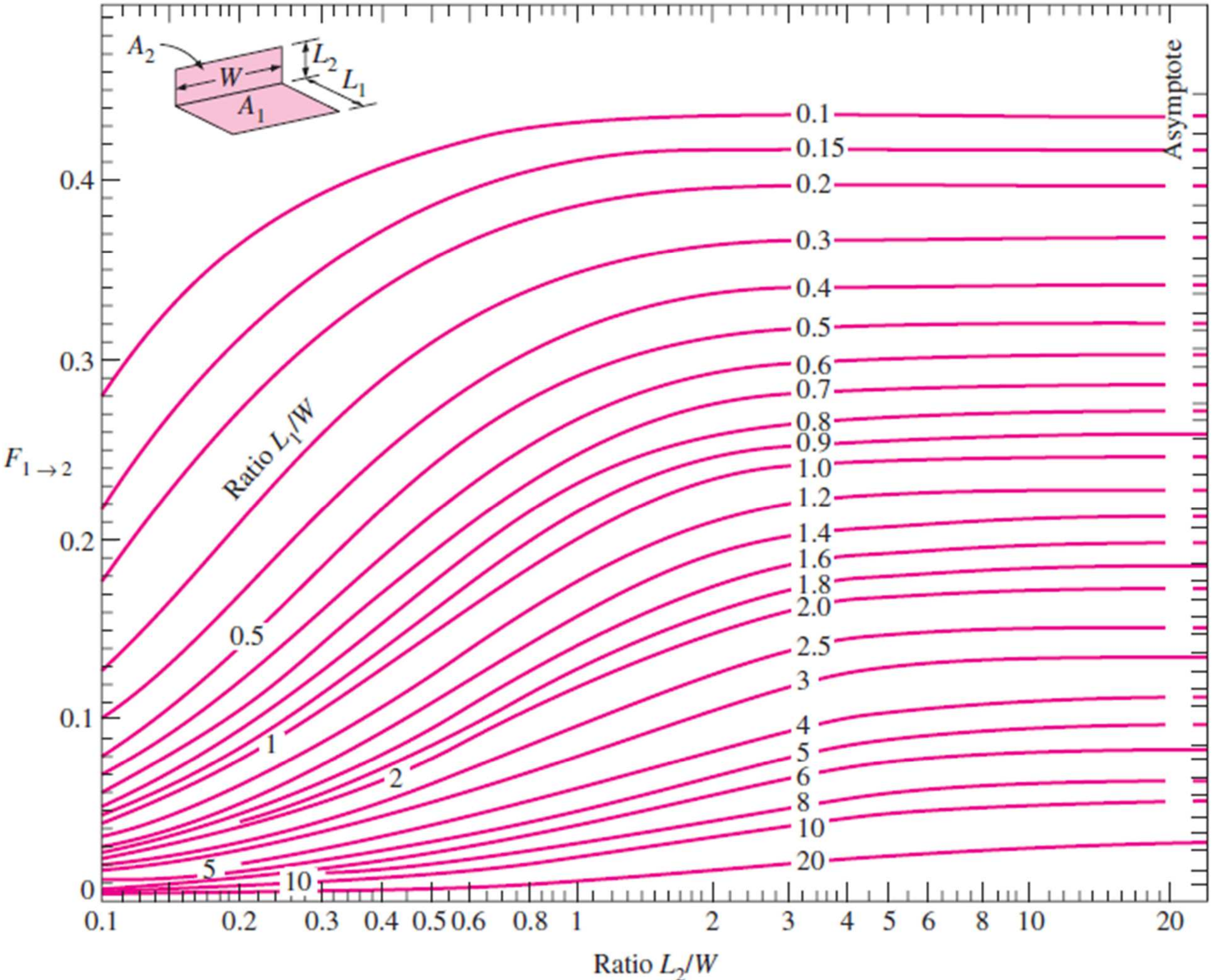


$F_{1 \to 2} = F_{1 \to 3}$
(Also, $F_{2 \to 1} = F_{3 \to 1}$)

PROBLEMS

12–8 Determine the view factors F_{13} and F_{23} between the rectangular surfaces shown in Figure P12–8.





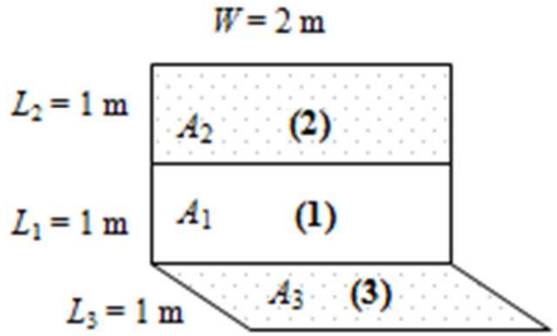
A_2
W
L_2
L_1
A_1
Asymptote
Ratio L_1/W
0.1
0.15
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.2
1.4
1.6
1.8
2.0
2.5
3
4
5
6
8
10
20
0.5
1
2
5
10
$F_{1 \to 2}$
0
0.1
0.2
0.3
0.4
0.1 0.2 0.3 0.4 0.5 0.6 0.8 1 2 3 4 5 6 8 10 20
Ratio L_2/W

12-8 The view factors between the rectangular surfaces shown in the figure are to be determined.

Assumptions The surfaces are diffuse emitters and reflectors.

Analysis From Fig. 12-6,

$$\left.\begin{aligned}\frac{L_3}{W}=\frac{1}{2}=0.5\\ \frac{L1}{W}=\frac{1}{2}=0.5\end{aligned}\right\}F_{31}=0.24$$



and

$$\left.\begin{aligned}\frac{L_3}{W}=\frac{1}{2}=0.5\\ \frac{L_1+L_2}{W}=\frac{2}{2}=1\end{aligned}\right\}F_{3\to(1+2)}=0.29$$

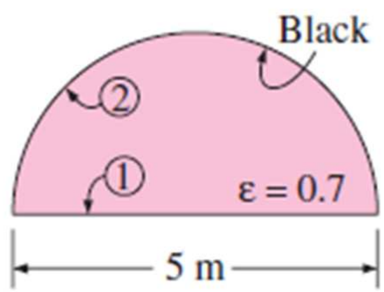
We note that $A_1 = A_3$. Then the reciprocity and superposition rules gives

$$A_1F_{13}=A_3F_{31}\longrightarrow F_{13}=F_{31}=\mathbf{0.24}$$

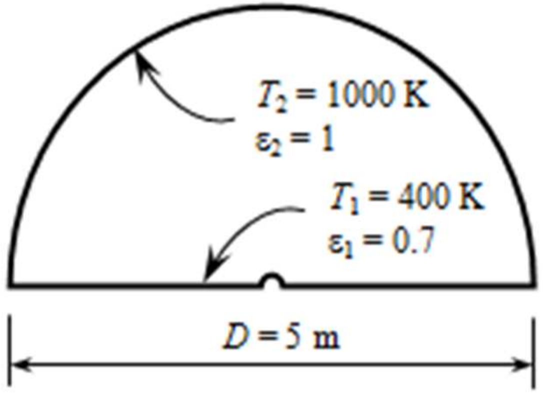
$$F_{3\to(1+2)}=F_{31}+F_{32}\longrightarrow\quad 0.29=0.24+F_{32}\longrightarrow F_{32}=0.05$$

Finally, $A_2=A_3\longrightarrow F_{23}=F_{32}=\mathbf{0.05}$

12–29 Consider a hemispherical furnace of diameter $D = 5$ m with a flat base. The dome of the furnace is black, and the base has an emissivity of 0.7. The base and the dome of the furnace are maintained at uniform temperatures of 400 and 1000 K, respectively. Determine the net rate of radiation heat transfer from the dome to the base surface during steady operation.



Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.



Analysis The view factor is first determined from

$$F_{11} = 0 \quad \text{(flat surface)}$$

$$F_{11} + F_{12} = 1 \rightarrow F_{12} = 1 \quad \text{(summation rule)}$$

Noting that the dome is black, net rate of radiation heat transfer from dome to the base surface can be determined from

$$\dot{Q}_{21} = -\dot{Q}_{12} = -\varepsilon A_1 F_{12} \sigma (T_1^{\,4} - T_2^{\,4})$$

$$= -(0.7)[\pi (5\,\text{m})^2/4\,](1)(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)[(400\ \text{K})^4 - (1000\ \text{K})^4]$$

$$= 7.594 \times 10^5\ \text{W}$$

$$= \mathbf{759.4\ kW}$$

The positive sign indicates that the net heat transfer is from the dome to the base surface, as expected.

RADIATION HEAT TRANSFER: BLACK SURFACES

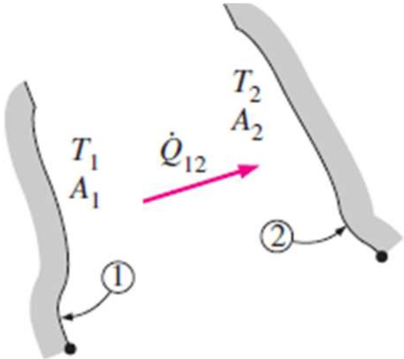


FIGURE 12–18

Two general black surfaces maintained at uniform temperatures T_1 and T_2.

$$\dot{Q}_{1\to 2} = \begin{pmatrix}\text{Radiation leaving}\\ \text{the entire surface 1}\\ \text{that strikes surface 2}\end{pmatrix} - \begin{pmatrix}\text{Radiation leaving}\\ \text{the entire surface 2}\\ \text{that strikes surface 1}\end{pmatrix}$$

$$= A_1 E_{b1} F_{1\to 2} - A_2 E_{b2} F_{2\to 1} \qquad \text{(W)}$$

Applying the reciprocity relation $A_1F_{1\to 2} = A_2F_{2\to 1}$ yields

$$\dot{Q}_{1\to 2} = A_1 F_{1\to 2}\,\sigma(T_1^4 - T_2^4) \qquad \text{(W)}$$

$$\dot{Q}_i = \sum_{j=1}^{N} \dot{Q}_{i\to j} = \sum_{j=1}^{N} A_i F_{i\to j}\sigma(T_i^4 - T_j^4) \qquad \text{(W)}$$

RADIATION HEAT TRANSFER: DIFFUSE, GRAY SURFACES

Radiosity

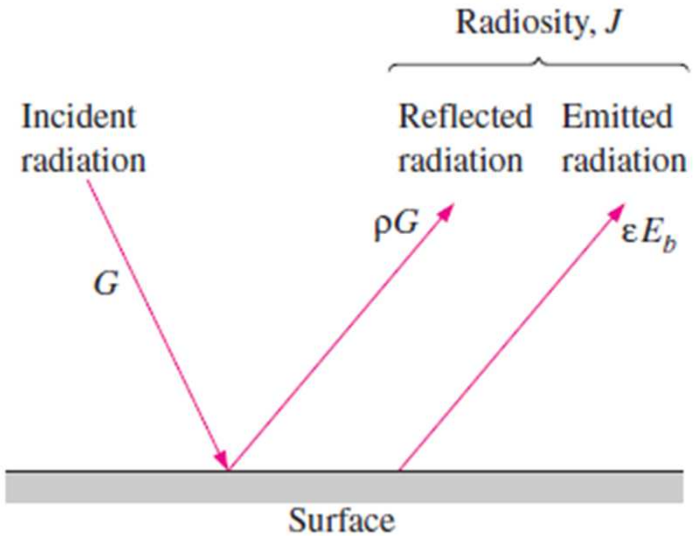


FIGURE 12–20
Radiosity represents the sum of the radiation energy emitted and reflected by a surface.

For a surface i that is *gray* and *opaque* ($\varepsilon_i = \alpha_i$ and $\alpha_i + \rho_i = 1$), the radiosity can be expressed as

$$J_i = \begin{pmatrix}\text{Radiation emitted} \\ \text{by surface } i\end{pmatrix} + \begin{pmatrix}\text{Radiation reflected} \\ \text{by surface } i\end{pmatrix}$$

$$= \varepsilon_i E_{bi} + \rho_i G_i$$

$$= \varepsilon_i E_{bi} + (1 - \varepsilon_i) G_i \qquad (\text{W/m}^2) \qquad \textbf{(12-21)}$$

For a surface that can be approximated as a *blackbody* ($\varepsilon_i = 1$), the radiosity relation reduces to

$$J_i = E_{bi} = \sigma T_i^4 \qquad \text{(blackbody)} \qquad \textbf{(12-22)}$$

Net Radiation Heat Transfer to or from a Surface

$$\dot{Q}_i = \begin{pmatrix}\text{Radiation leaving}\\ \text{entire surface } i\end{pmatrix} - \begin{pmatrix}\text{Radiation incident}\\ \text{on entire surface } i\end{pmatrix}$$

$$= A_i(J_i - G_i) \qquad \text{(W)} \qquad \textbf{(12-23)}$$

Solving for G_i from Eq. 12–21 and substituting into Eq. 12–23 yields

$$\dot{Q}_i = A_i\left(J_i - \frac{J_i - \varepsilon_i E_{bi}}{1 - \varepsilon_i}\right) = \frac{A_i \varepsilon_i}{1 - \varepsilon_i}(E_{bi} - J_i) \qquad \text{(W)} \qquad \textbf{(12-24)}$$

In an electrical analogy to Ohm's law, this equation can be rearranged as

$$\dot{Q}_i = \frac{E_{bi} - J_i}{R_i} \qquad \text{(W)} \qquad \textbf{(12-25)}$$

where

$$R_i = \frac{1 - \varepsilon_i}{A_i \varepsilon_i} \qquad \textbf{(12-26)}$$

Net Radiation Heat Transfer between Any Two Surfaces

$$\dot{Q}_{i \to j} = \begin{pmatrix} \text{Radiation leaving} \\ \text{the entire surface } i \\ \text{that strikes surface } j \end{pmatrix} - \begin{pmatrix} \text{Radiation leaving} \\ \text{the entire surface } j \\ \text{that strikes surface } i \end{pmatrix}$$

$$= A_i J_i F_{i \to j} - A_j J_j F_{j \to i} \qquad \text{(W)}$$

Applying the reciprocity relation $A_i F_{i \to j} = A_j F_{j \to i}$ yields

$$\dot{Q}_{i \to j} = A_i F_{i \to j} (J_i - J_j) \qquad \text{(W)}$$

Again in analogy to Ohm's law, this equation can be rearranged as

$$\dot{Q}_{i \to j} = \frac{J_i - J_j}{R_{i \to j}} \qquad \text{(W)}$$

$$R_{i \to j} = \frac{1}{A_i F_{i \to j}}$$

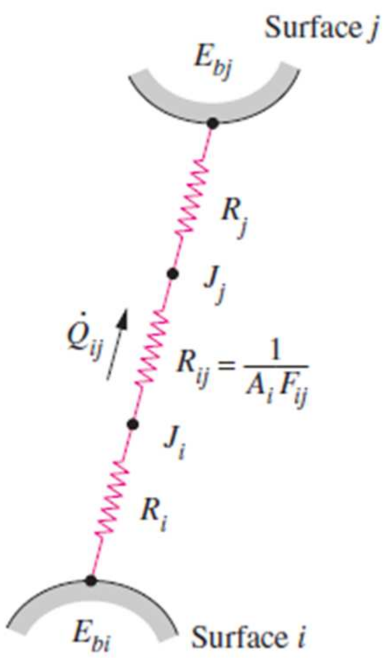


FIGURE 12–22
Electrical analogy of space resistance to radiation.

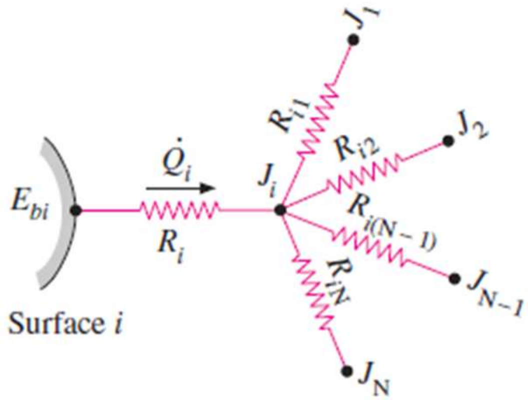


FIGURE 12–23
Network representation of net radiation heat transfer from surface i to the remaining surfaces of an N-surface enclosure.

$$\dot{Q}_i = \sum_{j=1}^{N} \dot{Q}_{i \to j} = \sum_{j=1}^{N} A_i F_{i \to j} (J_i - J_j) = \sum_{j=1}^{N} \frac{J_i - J_j}{R_{i \to j}} \qquad \text{(W)}$$

$$\frac{E_{bi} - J_i}{R_i} = \sum_{j=1}^{N} \frac{J_i - J_j}{R_{i \to j}}$$