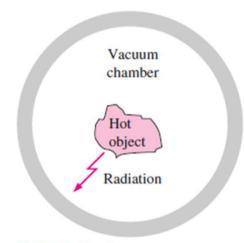
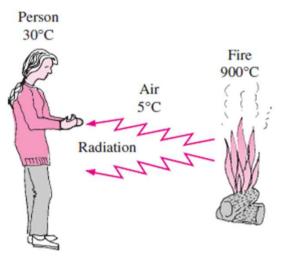


Prof. Gerson H. dos Santos



#### FIGURE 11-1

A hot object in a vacuum chamber loses heat by radiation only.



#### FIGURE 11-2

Unlike conduction and convection, heat transfer by radiation can occur between two bodies, even when they are separated by a medium colder than both of them.

#### Característica de Onda

#### Característica de Partícula

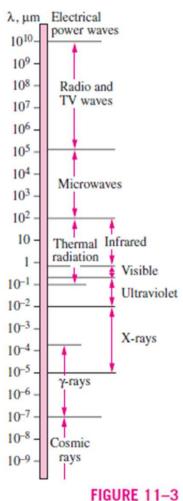
$$\lambda = \frac{c}{v}$$

$$e = hv = \frac{hc}{\lambda}$$

where  $h = 6.6256 \times 10^{-34} \text{ J} \cdot \text{s}$  is *Planck's constant*.

frequency  $\nu$  or wavelength  $\lambda$ .

c is the speed of propagation of a wave in that medium.



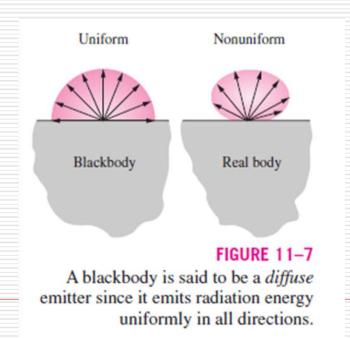
#### TABLE 11-1 The wavelength ranges of different colors Color Wavelength band Violet 0.40-0.44 µm Blue 0.44-0.49 µm 0.49-0.54 µm Green Yellow 0.54-0.60 µm 0.60-0.67 µm Orange

Red

0.63-0.76 µm

The electromagnetic wave spectrum.

A blackbody is defined as a perfect emitter and absorber of radiation. At a specified temperature and wavelength, no surface can emit more energy than a blackbody. A blackbody absorbs all incident radiation, regardless of wavelength and direction. Also, a blackbody emits radiation energy uniformly in all directions per unit area normal to direction of emission. (Fig. 11–7). That is, a blackbody is a diffuse emitter. The term diffuse means "independent of direction."



The radiation energy emitted by a blackbody per unit time and per unit surface area was determined experimentally by Joseph Stefan in 1879 and expressed as

$$E_b(T) = \sigma T^4$$
 (W/m<sup>2</sup>) (11-3)

where  $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$  is the *Stefan–Boltzmann constant* and *T* is the absolute temperature of the surface in K. This relation was theoretically verified in 1884 by Ludwig Boltzmann. Equation 11-3 is known as the **Stefan–Boltzmann law** and  $E_b$  is called the **blackbody emissive power.** 

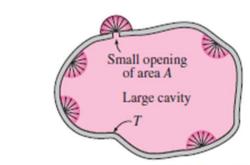


FIGURE 11-8

A large isothermal cavity at temperature T with a small opening of area A closely resembles a blackbody of surface area A at the same temperature.

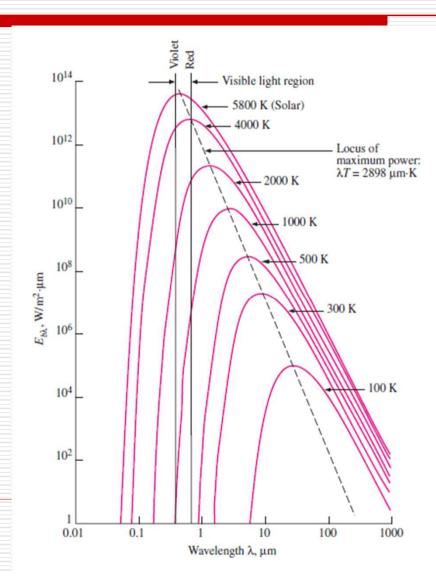
The relation for the spectral blackbody emissive power  $E_{b\lambda}$  was developed by Max Planck in 1901 in conjunction with his famous quantum theory. This relation is known as **Planck**'s law and is expressed as

$$E_{b\lambda}(\lambda, T) = \frac{C_1}{\lambda^5 [\exp(C_2/\lambda T) - 1]} \qquad (W/m^2 \cdot \mu m)$$
 (11-4)

where

$$C_1 = 2\pi h c_0^2 = 3.742 \times 10^8 \,\text{W} \cdot \mu \text{m}^4/\text{m}^2$$
  
 $C_2 = h c_0 / k = 1.439 \times 10^4 \,\mu \text{m} \cdot \text{K}$ 

Also, T is the absolute temperature of the surface,  $\lambda$  is the wavelength of the radiation emitted, and  $k = 1.38065 \times 10^{-23}$  J/K is Boltzmann's constant. This relation is valid for a surface in a vacuum or a gas. For other mediums, it needs to be modified by replacing  $C_1$  by  $C_1/n^2$ , where n is the index of refraction of the medium. Note that the term spectral indicates dependence on wavelength.

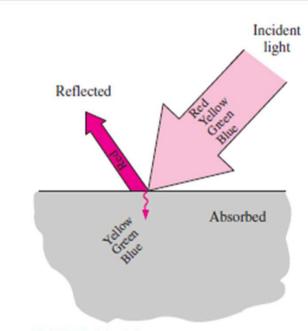


#### FIGURE 11-9

The variation of the blackbody emissive power with wavelength for several temperatures.

As the temperature increases, the peak of the curve in Figure 11–9 shifts toward shorter wavelengths. The wavelength at which the peak occurs for a specified temperature is given by Wien's displacement law as

$$(\lambda T)_{\text{max power}} = 2897.8 \ \mu\text{m} \cdot \text{K} \tag{11-5}$$

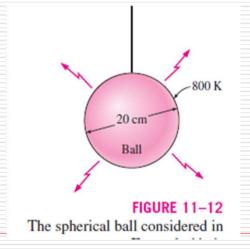


#### FIGURE 11-10

A surface that reflects red while absorbing the remaining parts of the incident light appears red to the eye.

#### Exemplo:

Consider a 20-cm-diameter spherical ball at 800 K suspended in air as shown in Figure 11–12. Assuming the ball closely approximates a blackbody, determine (a) the total blackbody emissive power, (b) the total amount of radiation emitted by the ball in 5 min, and (c) the spectral blackbody emissive power at a wavelength of 3  $\mu$ m.



#### Solução:

Analysis (a) The total blackbody emissive power is determined from the Stefan-Boltzmann law to be

$$E_b = \sigma T^4 = (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(800 \text{ K})^4 = 23.2 \times 10^3 \text{ W/m}^2 = 23.2 \text{ kW/m}^2$$

(b) The total amount of radiation energy emitted from the entire ball in 5 min is determined by multiplying the blackbody emissive power obtained above by the total surface area of the ball and the given time interval:

$$A_s = \pi D^2 = \pi (0.2 \text{ m})^2 = 0.1257 \text{ m}^2$$

$$\Delta t = (5 \text{ min}) \left( \frac{60 \text{ s}}{1 \text{ min}} \right) = 300 \text{ s}$$

$$Q_{\text{rad}} = E_b A_s \, \Delta t = (23.2 \text{ kW/m}^2)(0.1257 \text{ m}^2)(300 \text{ s}) \left( \frac{1 \text{ kJ}}{1000 \text{ W} \cdot \text{s}} \right)$$

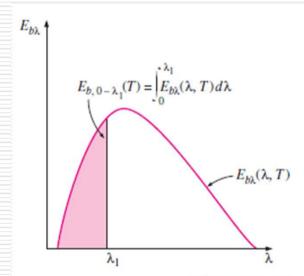
$$= 876 \text{ kJ}$$

(c) The spectral blackbody emissive power at a wavelength of 3  $\mu m$  is determined from Planck's distribution law to be

$$E_{b\lambda} = \frac{C_1}{\lambda^5 \left[ \exp\left(\frac{C_2}{\lambda T}\right) - 1 \right]} = \frac{3.743 \times 10^8 \text{ W} \cdot \mu \text{m}^4/\text{m}^2}{(3 \text{ } \mu \text{m})^5 \left[ \exp\left(\frac{1.4387 \times 10^4 \text{ } \mu \text{m} \cdot \text{K}}{(3 \text{ } \mu \text{m})(800 \text{ K})}\right) - 1 \right]}$$
$$= 3848 \text{ W/m}^2 \cdot \mu \text{m}$$

The radiation energy emitted by a blackbody per unit area over a wavelength band from  $\lambda = 0$  to  $\lambda$  is determined from (Fig. 11–13)

$$E_{b,0-\lambda}(T) = \int_0^{\lambda} E_{b\lambda}(\lambda, T) d\lambda \qquad (W/m^2)$$
 (11-7)



#### FIGURE 11-13

On an  $E_{b\lambda}$ - $\lambda$  chart, the area under the curve to the left of the  $\lambda = \lambda_1$  line represents the radiation energy emitted by a blackbody in the wavelength range 0- $\lambda_1$  for the given temperature.

Blackbody radiation fu	nctions $f_{\lambda}$		
λ <i>T</i> ,		λ <i>T</i> ,	
μm · K	$f_{\lambda}$	μm·K	$f_{\lambda}$
200	0.000000	6200	0.754140
400	0.000000	6400	0.769234
600	0.000000	6600	0.783199
800	0.000016	6800	0.796129
1000	0.000321	7000	0.808109
1200	0.002134	7200	0.819217
1400	0.002134	7400	0.829527
1600	0.019718	7600	0.839102
1800	0.039341	7800	0.848005
2000	0.066728	8000	0.856288
2200	0.100888	8500	0.874608
2400	0.140256	9000	0.890029
2600	0.183120	9500	0.903085
2800	0.227897	10,000	0.914199
3000	0.273232	10,500	0.923710
3200	0.318102	11,000	0.931890
3400	0.361735	11,500	0.939959
3600	0.403607	12,000	0.945098
3800	0.443382	13,000	0.955139
4000	0.480877	14,000	0.962898
4200	0.516014	15,000	0.969981
4400	0.548796	16,000	0.973814
4600	0.579280	18,000	0.980860
4800	0.607559	20,000	0.985602
5000	0.633747	25,000	0.992215
5200	0.658970	30,000	0.995340
5400	0.680360	40,000	0.997967
5600	0.701046	50,000	0.998953
5800	0.720158	75,000	0.999713
6000	0.737818	100,000	0.999905
0000	0.707010	100,000	0.555500

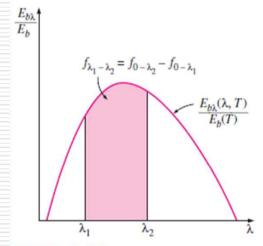


FIGURE 11-14

Graphical representation of the fraction of radiation emitted in the wavelength band from  $\lambda_1$  to  $\lambda_2$ .

$$f_{\lambda}(T) = \frac{\int_{0}^{\lambda} E_{b\lambda}(\lambda, T) d\lambda}{\sigma T^{4}}$$
 (11-8)

The function  $f_{\lambda}$  represents the fraction of radiation emitted from a blackbody at temperature T in the wavelength band from  $\lambda = 0$  to  $\lambda$ . The values of  $f_{\lambda}$  are listed in Table 11–2 as a function of  $\lambda T$ , where  $\lambda$  is in  $\mu$ m and T is in K.

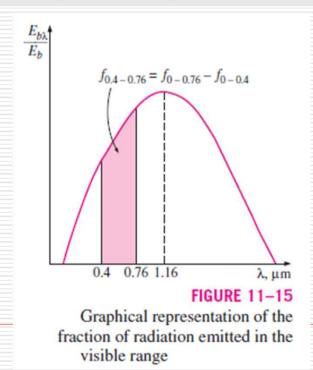
The fraction of radiation energy emitted by a blackbody at temperature T over a finite wavelength band from  $\lambda = \lambda_1$  to  $\lambda = \lambda_2$  is determined from (Fig. 11–14)

$$f_{\lambda_1 - \lambda_2}(T) = f_{\lambda_2}(T) - f_{\lambda_1}(T)$$
 (11-9)

where  $f_{\lambda_1}(T)$  and  $f_{\lambda_2}(T)$  are blackbody radiation functions corresponding to  $\lambda_1 T$  and  $\lambda_2 T$ , respectively.

#### Exercício

The temperature of the filament of an incandescent lightbulb is 2500 K. Assuming the filament to be a blackbody, determine the fraction of the radiant energy emitted by the filament that falls in the visible range. Also, determine the wavelength at which the emission of radiation from the filament peaks.



#### Solução:

$$\lambda_1 T = (0.40 \ \mu\text{m})(2500 \ \text{K}) = 1000 \ \mu\text{m} \cdot \text{K} \longrightarrow f_{\lambda_1} = 0.000321$$
  
 $\lambda_2 T = (0.76 \ \mu\text{m})(2500 \ \text{K}) = 1900 \ \mu\text{m} \cdot \text{K} \longrightarrow f_{\lambda_2} = 0.053035$ 

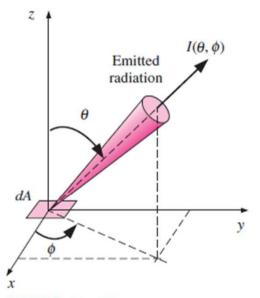
That is, 0.03 percent of the radiation is emitted at wavelengths less than 0.4  $\mu$ m and 5.3 percent at wavelengths less than 0.76  $\mu$ m. Then the fraction of radiation emitted between these two wavelengths is (Fig. 11–15)

$$f_{\lambda_1-\lambda_2} = f_{\lambda_2} - f_{\lambda_1} = 0.053035 - 0.000321 = 0.0527135$$

Therefore, only about 5 percent of the radiation emitted by the filament of the lightbulb falls in the visible range. The remaining 95 percent of the radiation appears in the infrared region in the form of radiant heat or "invisible light," as it used to be called. This is certainly not a very efficient way of converting electrical energy to light and explains why fluorescent tubes are a wiser choice for lighting.

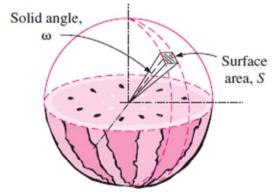
The wavelength at which the emission of radiation from the filament peaks is easily determined from Wien's displacement law to be

$$(\lambda T)_{\text{max power}} = 2897.8 \ \mu\text{m} \cdot \text{K} \rightarrow \lambda_{\text{max power}} = \frac{2897.8 \ \mu\text{m} \cdot \text{K}}{2500 \ \text{K}} = 1.16 \ \mu\text{m}$$



#### FIGURE 11-16

Radiation intensity is used to describe the variation of radiation energy with direction. If all surfaces emitted radiation uniformly in all directions, the *emissive* power would be sufficient to quantify radiation, and we would not need to deal with intensity. The radiation emitted by a blackbody per unit normal area is the same in all directions, and thus there is no directional dependence. But this is not the case for real surfaces. Before we define intensity, we need to quantify the size of an opening in space.

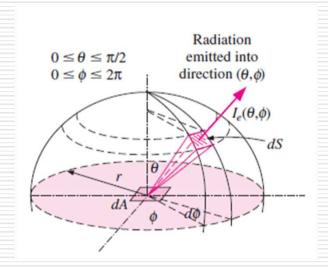


A slice of watermelon of solid angle  $\omega$  FIGURE 11–17

Describing the size of a slice of pizza by a plain angle, and the size of a watermelon slice by a solid angle.

The differential solid angle  $d\omega$  subtended by a differential area dS on a sphere of radius r can be expressed as

$$d\omega = \frac{dS}{r^2} = \sin\theta d\theta d\phi \tag{11-11}$$



The **radiation intensity** for emitted radiation  $I_e(\theta, \phi)$  is defined as the rate at which radiation energy  $d\dot{Q}_e$  is emitted in the  $(\theta, \phi)$  direction per unit area normal to this direction and per unit solid angle about this direction. That is,

$$I_e(\theta, \phi) = \frac{d\dot{Q}_e}{dA\cos\theta \cdot d\omega} = \frac{d\dot{Q}_e}{dA\cos\theta\sin\theta d\theta d\phi}$$
 (W/m<sup>2</sup> · sr) (11-13)

The radiation flux for emitted radiation is the emissive power E (the rate at which radiation energy is emitted per unit area of the emitting surface), which can be expressed in differential form as

$$dE = \frac{d\dot{Q}_e}{dA} = I_e(\theta, \phi) \cos \theta \sin \theta d\theta d\phi$$
 (11-14)

$$E = \int_{\text{hemisphere}} dE = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I_e(\theta, \phi) \cos \theta \sin \theta d\theta d\phi \qquad (W/m^2)$$
 (11-15)

The intensity of radiation emitted by a surface, in general, varies with direction (especially with the zenith angle  $\theta$ ). But many surfaces in practice can be approximated as being diffuse. For a *diffusely emitting* surface, the intensity of the emitted radiation is independent of direction and thus  $I_e$  = constant.

Noting that  $\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \cos \theta \sin \theta d\theta d\phi = \pi$ , the emissive power relation in Eq. 11-15 reduces in this case to

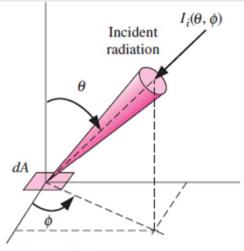
Diffusely emitting surface: 
$$E = \pi I_e$$
 (W/m<sup>2</sup>) (11-16)

For a blackbody, which is a diffuse emitter, Eq. 11-16 can be expressed as

Blackbody: 
$$E_b = \pi I_b$$
 (11-17)

where  $E_b = \sigma T^4$  is the blackbody emissive power. Therefore, the intensity of the radiation emitted by a blackbody at absolute temperature T is

Blackbody: 
$$I_b(T) = \frac{E_b(T)}{\pi} = \frac{\sigma T^4}{\pi} \qquad (\text{W/m}^2 \cdot \text{sr})$$
 (11-18)



#### FIGURE 11-20

Radiation incident on a surface in the direction  $(\theta, \phi)$ .

#### **Incident Radiation**

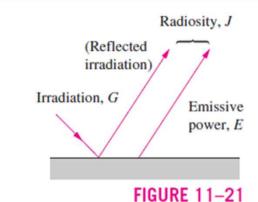
All surfaces emit radiation, but they also receive radiation emitted or reflected by other surfaces. The intensity of incident radiation  $I_i(\theta, \phi)$  is defined as the rate at which radiation energy dG is incident from the  $(\theta, \phi)$  direction per unit area of the receiving surface normal to this direction and per unit solid angle about this direction (Fig. 11–20). Here  $\theta$  is the angle between the direction of incident radiation and the normal of the surface.

The radiation flux incident on a surface from *all directions* is called **irradiation** *G*, and is expressed as

$$G = \int_{\phi=0}^{2\pi} dG = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I_i(\theta, \phi) \cos \theta \sin \theta d\theta d\phi \qquad (W/m^2)$$
 (11-19)

Diffusely incident radiation:

$$G = \pi I_i$$
 (W/m<sup>2</sup>)



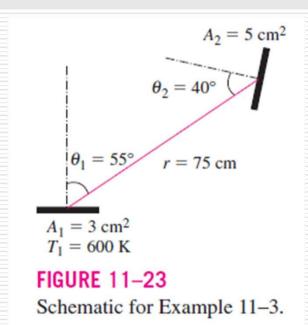
The three kinds of radiation flux (in W/m<sup>2</sup>): emissive power, irradiation, and radiosity.

The spectral radiation intensity  $I_{\lambda}(\lambda, \theta, \phi)$ , for example, is simply the total radiation intensity  $I(\theta, \phi)$  per unit wavelength interval about  $\lambda$ . The spectral intensity for emitted radiation  $I_{\lambda, e}(\lambda, \theta, \phi)$  can be defined as the rate at which radiation energy  $d\dot{Q}_e$  is emitted at the wavelength  $\lambda$  in the  $(\theta, \phi)$  direction per unit area normal to this direction, per unit solid angle about this direction, and it can be expressed as

$$I_{\lambda, e}(\lambda, \theta, \phi) = \frac{d\dot{Q}_e}{dA \cos \theta \cdot d\omega \cdot d\lambda} \qquad (W/m^2 \cdot \text{sr} \cdot \mu \text{m})$$
 (11-23)

#### Exemplo:

A small surface of area  $A_1 = 3$  cm<sup>2</sup> emits radiation as a blackbody at  $T_1 = 600$  K. Part of the radiation emitted by  $A_1$  strikes another small surface of area  $A_2 = 5$  cm<sup>2</sup> oriented as shown in Figure 11–23. Determine the solid angle subtended by  $A_2$  when viewed from  $A_1$ , and the rate at which radiation emitted by  $A_1$  that strikes  $A_2$ .



#### Solução:

$$\omega_{2-1} \cong \frac{A_{n,2}}{r^2} = \frac{A_2 \cos \theta_2}{r^2} = \frac{(5 \text{ cm}^2) \cos 40^\circ}{(75 \text{ cm})^2} = 6.81 \times 10^{-4} \text{ sr}$$

$$I_1 = \frac{E_h(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi} = \frac{(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(600 \text{ K})^4}{\pi} = 2339 \text{ W/m}^2 \cdot \text{sr}$$

$$\dot{Q}_{1-2} = I_1 (A_1 \cos \theta_1) \omega_{2-1}$$
= (2339 W/m<sup>2</sup> · sr)(3 × 10<sup>-4</sup> cos 55° m<sup>2</sup>)(6.81 × 10<sup>-4</sup> sr)
= 2.74 × 10<sup>-4</sup> W

#### **Emissivity**

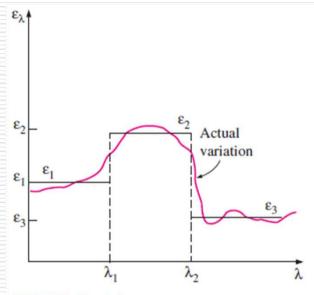
The emissivity of a surface represents the ratio of the radiation emitted by the surface at a given temperature to the radiation emitted by a blackbody at the same temperature. The emissivity of a surface is denoted by  $\varepsilon$ , and it varies between zero and one,  $0 \le \varepsilon \le 1$ . Emissivity is a measure of how closely a surface approximates a blackbody, for which  $\varepsilon = 1$ .

$$\varepsilon_{\lambda,\,\theta}(\lambda,\,\theta,\,\phi,\,T) = \frac{I_{\lambda,\,e}(\lambda,\,\theta,\,\phi,\,T)}{I_{b\lambda}(\lambda,\,T)}$$

$$\varepsilon_{\theta}(\theta, \phi, T) = \frac{I_{e}(\theta, \phi, T)}{I_{b}(T)}$$

Finally, the total hemispherical emissivity is defined in terms of the radiation energy emitted over all wavelengths in all directions as

$$\varepsilon(T) = \frac{E(T)}{E_b(T)} \tag{11-33}$$



#### FIGURE 11-24

Approximating the actual variation of emissivity with wavelength by a step function.

$$\varepsilon(T) = \frac{E(T)}{E_b(T)} = \frac{\int_0^\infty \varepsilon_\lambda(\lambda, T) E_{b\lambda}(\lambda, T) d\lambda}{\sigma T^4}$$
(11-34)

$$\varepsilon(T) = \frac{\varepsilon_1 \int_0^{\lambda_1} E_{b\lambda} d\lambda}{E_b} + \frac{\varepsilon_2 \int_{\lambda_1}^{\lambda_2} E_{b\lambda} d\lambda}{E_b} + \frac{\varepsilon_3 \int_{\lambda_2}^{\infty} E_{b\lambda} d\lambda}{E_b}$$
$$= \varepsilon_1 f_{0-\lambda_1}(T) + \varepsilon_2 f_{\lambda_1-\lambda_2}(T) + \varepsilon_3 f_{\lambda_2-\infty}(T)$$

#### Real surface:

 $\varepsilon_{\Omega} \neq constant$ 

 $\varepsilon_{\lambda} \neq constant$ 

#### Diffuse surface:

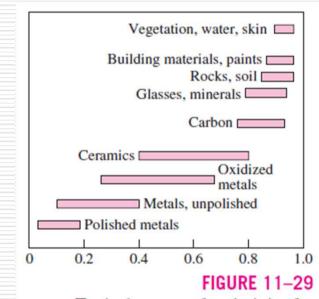
 $\varepsilon_{\Theta} = constant$ 

#### Gray surface:

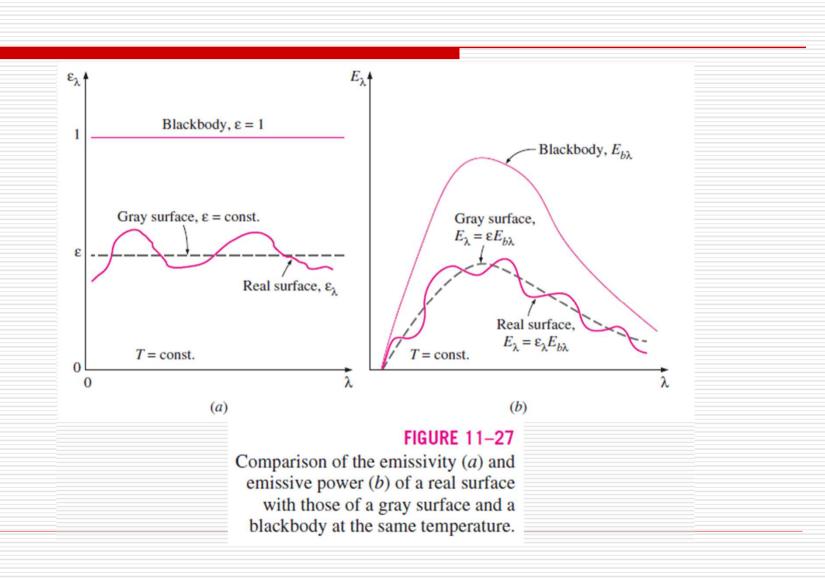
 $\varepsilon_{\lambda} = constant$ 

#### Diffuse, gray surface:

 $\varepsilon = \varepsilon_{\lambda} = \varepsilon_{\theta} = constant$ 



Typical ranges of emissivity for various materials.

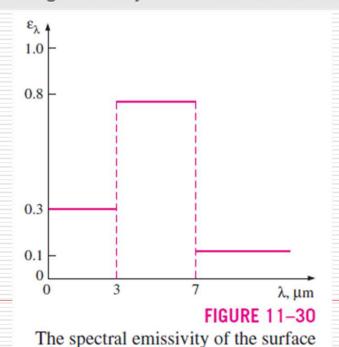


#### Exemplo:

The spectral emissivity function of an opaque surface at 800 K is approximated as (Fig. 11-30)

$$\varepsilon_{\lambda} = \begin{cases} \varepsilon_1 = 0.3, & 0 \leq \lambda < 3 \; \mu m \\ \varepsilon_2 = 0.8, & 3 \; \mu m \leq \lambda < 7 \; \mu m \\ \varepsilon_3 = 0.1, & 7 \; \mu m \leq \lambda < \infty \end{cases}$$

Determine the average emissivity of the surface and its emissive power.



#### Solução:

$$\begin{split} \varepsilon(T) &= \frac{\varepsilon_1 \int_0^{\lambda_1} E_{b\lambda} \, d\lambda}{\sigma T^4} + \frac{\varepsilon_2 \int_{\lambda_1}^{\lambda_2} E_{b\lambda} \, d\lambda}{\sigma T^4} + \frac{\varepsilon_3 \int_{\lambda_2}^{\infty} E_{b\lambda} \, d\lambda}{\sigma T^4} \\ &= \varepsilon_1 f_{0-\lambda_1}(T) + \varepsilon_2 f_{\lambda_1-\lambda_2}(T) + \varepsilon_3 f_{\lambda_2-\infty}(T) \\ &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (f_{\lambda_2} - f_{\lambda_1}) + \varepsilon_3 (1 - f_{\lambda_2}) \end{split}$$

$$\lambda_1 T = (3 \ \mu\text{m})(800 \ \text{K}) = 2400 \ \mu\text{m} \cdot \text{K} \rightarrow f_{\lambda_1} = 0.140256$$
  
 $\lambda_2 T = (7 \ \mu\text{m})(800 \ \text{K}) = 5600 \ \mu\text{m} \cdot \text{K} \rightarrow f_{\lambda_2} = 0.701046$ 

Note that  $f_{0-\lambda_1}=f_{\lambda_1}-f_0=f_{\lambda_1}$ , since  $f_0=0$ , and  $f_{\lambda_2-\infty}=f_\infty-f_{\lambda_2}=1-f_{\lambda_2}$ , since  $f_\infty=1$ . Substituting,

$$\varepsilon = 0.3 \times 0.140256 + 0.8(0.701046 - 0.140256) + 0.1(1 - 0.701046)$$
  
= 0.521

That is, the surface will emit as much radiation energy at 800 K as a gray surface having a constant emissivity of  $\varepsilon = 0.521$ . The emissive power of the surface is

$$E = \varepsilon \sigma T^4 = 0.521(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(800 \text{ K})^4 = 12,100 \text{ W/m}^2$$

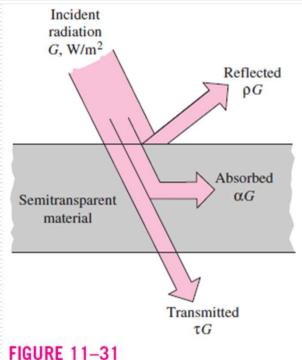
#### Absorptivity, Reflectivity, and Transmissivity

When radiation strikes a surface, part of it is absorbed, part of it is reflected, and the remaining part, if any, is transmitted, as illustrated in Figure 11–31. The fraction of irradiation absorbed by the surface is called the absorptivity  $\alpha$ , the fraction reflected by the surface is called the reflectivity  $\rho$ , and the fraction transmitted is called the transmissivity  $\tau$ . That is,

Absorptivity: 
$$\alpha = \frac{\text{Absorbed radiation}}{\text{Incident radiation}} = \frac{G_{\text{abs}}}{G}, \quad 0 \le \alpha \le 1$$
 (11-37)

Reflectivity: 
$$\rho = \frac{\text{Reflected radiation}}{\text{Incident radiation}} = \frac{G_{\text{ref}}}{G}, \qquad 0 \le \rho \le 1$$
 (11-38)

Transmissivity: 
$$\tau = \frac{\text{Transmitted radiation}}{\text{Incident radiation}} = \frac{G_{\text{tr}}}{G}, \qquad 0 \le \tau \le 1$$
 (11-39)



The absorption, reflection, and transmission of incident radiation by a semitransparent material.

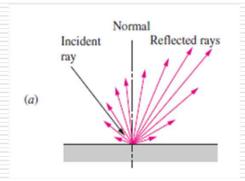
where G is the radiation energy incident on the surface, and  $G_{abs}$ ,  $G_{ref}$ , and  $G_{tr}$  are the absorbed, reflected, and transmitted portions of it, respectively. The first law of thermodynamics requires that the sum of the absorbed, reflected, and transmitted radiation energy be equal to the incident radiation. That is,

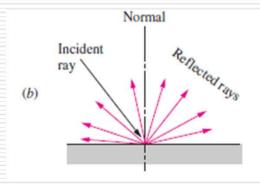
$$G_{\text{abs}} + G_{\text{ref}} + G_{\text{tr}} = G \tag{11-40}$$

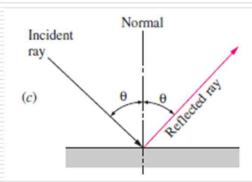
Dividing each term of this relation by G yields

$$\alpha + \rho + \tau = 1 \tag{11-41}$$

For opaque surfaces,  $\tau = 0$ , and thus

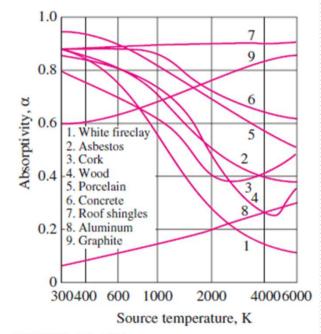






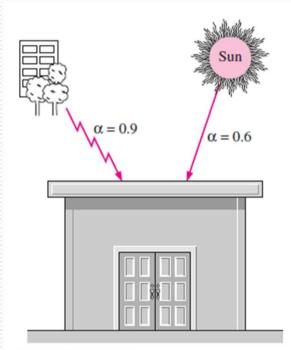
Different types of reflection from a surface: (a) actual or irregular, (b) diffuse, and (c) specular or mirrorlike.

# **Propriedades Radiativas**



#### FIGURE 11-33

Variation of absorptivity with the temperature of the source of irradiation for various common materials at room temperature.



#### FIGURE 11-34

The absorptivity of a material may be quite different for radiation originating from sources at different temperatures.

# **Propriedades Radiativas**

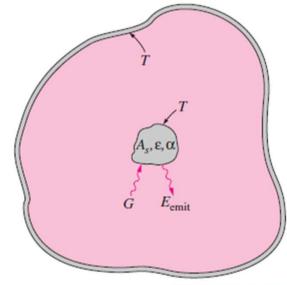


FIGURE 11-35

The small body contained in a large isothermal enclosure used in the development of Kirchhoff's law.

$$G_{abs} = \alpha G = \alpha \sigma T^4$$

$$E_{\rm emit} = \varepsilon \sigma T^4$$

$$A_s \varepsilon \sigma T^4 = A_s \alpha \sigma T^4$$

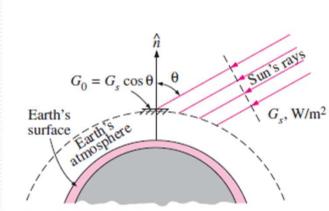
$$\varepsilon(T) = \alpha(T)$$

$$\varepsilon_{\lambda}(T) = \alpha_{\lambda}(T)$$

The *sun* is a nearly spherical body that has a diameter of  $D \approx 1.39 \times 10^9$  m and a mass of  $m \approx 2 \times 10^{30}$  kg and is located at a mean distance of  $L = 1.50 \times 10^{11}$  m from the earth. It emits radiation energy continuously at a rate of  $E_{\text{sun}} \approx 3.8 \times 10^{26}$  W. Less than a billionth of this energy (about  $1.7 \times 10^{17}$  W) strikes the earth, which is sufficient to keep the earth warm and to maintain life through the photosynthesis process. The energy of the sun is due to the continuous *fusion* reaction during which two hydrogen atoms fuse to form one atom of helium. Therefore, the sun is essentially a *nuclear reactor*, with temperatures as high as 40,000,000 K in its core region. The temperature drops to about 5800 K in the outer region of the sun, called the convective zone, as a result of the dissipation of this energy by radiation.

The solar energy reaching the earth's atmosphere is called the **total solar** irradiance  $G_s$ , whose value is

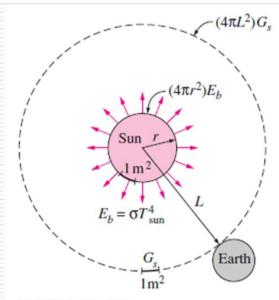
$$G_s = 1373 \text{ W/m}^2$$
 (11-49)



#### FIGURE 11-38

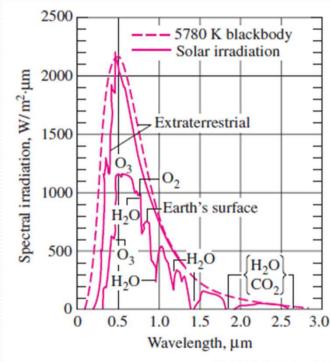
Solar radiation reaching the earth's atmosphere and the total solar irradiance.

$$(4\pi L^2)G_s = (4\pi r^2) \sigma T_{\text{sun}}^4$$



#### FIGURE 11-39

The total solar energy passing through concentric spheres remains constant, but the energy falling per unit area decreases with increasing radius.



#### FIGURE 11-40

Spectral distribution of solar radiation just outside the atmosphere, at the surface of the earth on a typical day, and comparison with blackbody radiation at 5780 K.

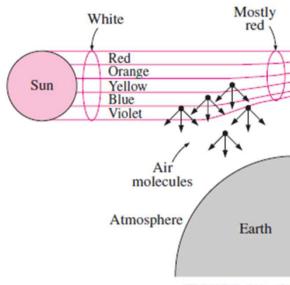
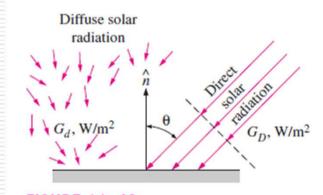


FIGURE 11-41

Air molecules scatter blue light much more than they do red light. At sunset, the light travels through a thicker layer of atmosphere, which removes much of the blue from the natural light, allowing the red to dominate.

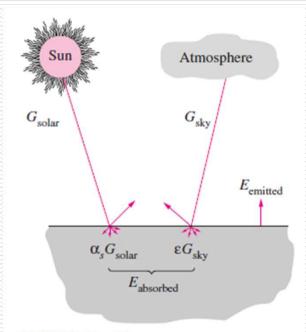
The solar energy incident on a surface on earth is considered to consist of direct and diffuse parts. The part of solar radiation that reaches the earth's surface without being scattered or absorbed by the atmosphere is called direct solar radiation  $G_D$ . The scattered radiation is assumed to reach the earth's surface uniformly from all directions and is called diffuse solar radiation  $G_d$ .



#### FIGURE 11-42

The direct and diffuse radiation incident on a horizontal surface at the earth's surface.

$$G_{\text{solar}} = G_D \cos \theta + G_d \qquad (W/\text{m}^2)$$



#### FIGURE 11-43

Radiation interactions of a surface exposed to solar and atmospheric radiation.

$$G_{\rm sky} = \sigma T_{\rm sky}^4$$
 (W/m<sup>2</sup>)

$$E_{\text{sky, absorbed}} = \alpha G_{\text{sky}} = \alpha \sigma T_{\text{sky}}^4 = \varepsilon \sigma T_{\text{sky}}^4$$

$$\begin{split} \dot{q}_{\text{net, rad}} &= \sum E_{\text{absorbed}} - \sum E_{\text{emitted}} \\ &= E_{\text{solar, absorbed}} + E_{\text{sky, absorbed}} - E_{\text{emitted}} \\ &= \alpha_s \, G_{\text{solar}} + \varepsilon \sigma T_{\text{sky}}^4 - \varepsilon \sigma T_s^4 \\ &= \alpha_s \, G_{\text{solar}} + \varepsilon \sigma (T_{\text{sky}}^4 - T_s^4) \end{split} \tag{W/m}^2 \end{split}$$

Comparison of the solar absorptivity  $\alpha_s$  of some surfaces with their emissivity  $\epsilon$  at room temperature

Surface	$\alpha_s$	ε
Aluminum		
Polished	0.09	0.03
Anodized	0.14	0.84
Foil	0.15	0.05
Copper		
Polished	0.18	0.03
Tarnished	0.65	0.75
Stainless steel		
Polished	0.37	0.60
Dull	0.50	0.21
Plated metals		
Black nickel oxide	0.92	0.08
Black chrome	0.87	0.09
Concrete	0.60	0.88
White marble	0.46	
Red brick	0.63	
Asphalt	0.90	0.90
Black paint	0.97	0.97
White paint	0.14	0.93
Snow	0.28	0.97
Human skin		
(caucasian)	0.62	0.97

#### Exemplo:

Consider a surface exposed to solar radiation. At a given time, the direct and diffuse components of solar radiation are  $G_D = 400$  and  $G_d = 300$  W/m<sup>2</sup>, and the direct radiation makes a 20° angle with the normal of the surface. The surface temperature is observed to be 320 K at that time. Assuming an effective sky temperature of 260 K, determine the net rate of radiation heat transfer for these cases (Fig. 11–45):

- (a)  $\alpha_s = 0.9$  and  $\varepsilon = 0.9$  (gray absorber surface)
- (b)  $\alpha_s = 0.1$  and  $\varepsilon = 0.1$  (gray reflector surface)
- (c)  $\alpha_s = 0.9$  and  $\varepsilon = 0.1$  (selective absorber surface)
- (d)  $\alpha_s = 0.1$  and  $\varepsilon = 0.9$  (selective reflector surface)

Solução:

$$G_{\text{solar}} = G_D \cos \theta + G_d$$
  
=  $(400 \text{ W/m}^2) \cos 20^\circ + (300 \text{ W/m}^2)$   
=  $676 \text{ W/m}^2$ 

$$\dot{q}_{\rm net, \, rad} = \alpha_s \, G_{\rm solar} + \varepsilon \sigma (T_{\rm sky}^4 - T_s^4)$$

(a)  $\alpha_s = 0.9$  and  $\varepsilon = 0.9$  (gray absorber surface):

$$\dot{q}_{\text{net, rad}} = 0.9(676 \text{ W/m}^2) + 0.9(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(260 \text{ K})^4 - (320 \text{ K})^4]$$
  
= 307 W/m<sup>2</sup>

(b)  $\alpha_s = 0.1$  and  $\varepsilon = 0.1$  (gray reflector surface):

$$\dot{q}_{\text{net, rad}} = 0.1(676 \text{ W/m}^2) + 0.1(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(260 \text{ K})^4 - (320 \text{ K})^4]$$
  
= 34 W/m<sup>2</sup>

(c)  $\alpha_s = 0.9$  and  $\varepsilon = 0.1$  (selective absorber surface):

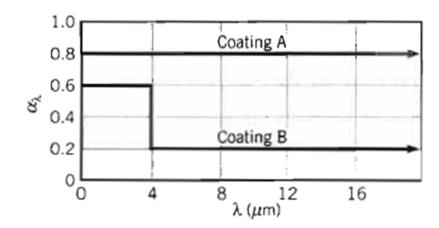
$$\dot{q}_{\text{net, rad}} = 0.9(676 \text{ W/m}^2) + 0.1(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(260 \text{ K})^4 - (320 \text{ K})^4]$$
  
= 575 W/m<sup>2</sup>

(d)  $\alpha_s = 0.1$  and  $\varepsilon = 0.9$  (selective reflector surface):

$$\dot{q}_{\text{net, rad}} = 0.1(676 \text{ W/m}^2) + 0.9(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(260 \text{ K})^4 - (320 \text{ K})^4]$$
  
=  $-234 \text{ W/m}^2$ 

#### Exercício

A contractor must select a roof covering material from the two diffuse, opaque coatings with  $\alpha_{\lambda}(\lambda)$  as shown. Which of the two coatings would result in a lower roof temperature? Which is preferred for summer use? For winter use? Sketch the spectral distribution of  $\alpha_{\lambda}$  that would be ideal for summer use. For winter use.



#### Solução:

**ASSUMPTIONS:** (1) Opaque, diffuse surface behavior, (2) Negligible convection effects and heat transfer from bottom of roof, negligible atmospheric irradiation, (3) Steady-state conditions.

$$\varepsilon \sigma T_s^4 = \alpha_S G_S$$
.

$$T_{\rm S} = \left(\frac{\alpha_{\rm S}}{\varepsilon} \frac{G_{\rm S}}{\sigma}\right)^{1/4}$$

Solar irradiation is concentrated in the spectral region  $\lambda < 4\mu m$ , while surface emission is concentrated in the region  $\lambda > 4\mu m$ . Hence, with  $\alpha_{\lambda} = \epsilon_{\lambda}$ 

Coating A:  $\alpha_S \approx 0.8$ ,  $\epsilon \approx 0.8$ 

Coating B:  $\alpha_S \approx 0.6$ ,  $\epsilon \approx 0.2$ .

