

# TRANSFERÊNCIA DE CALOR I

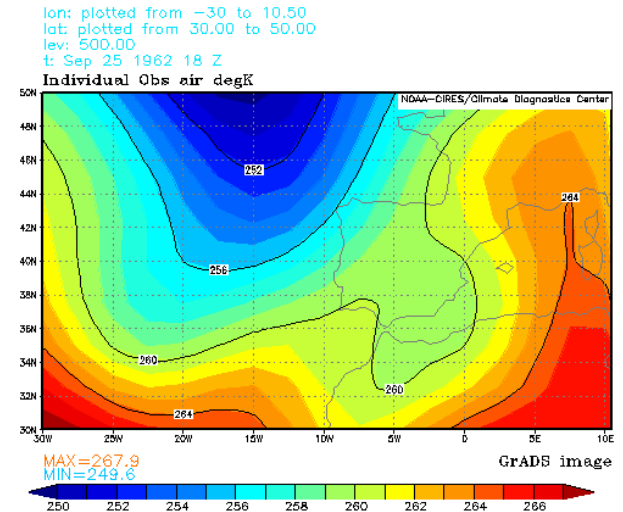
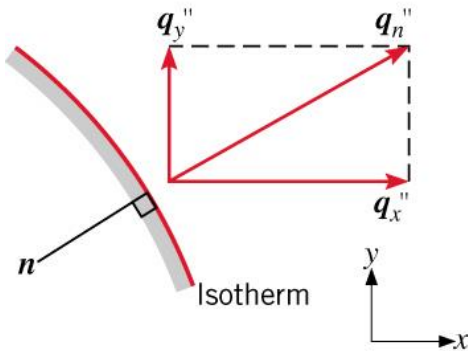
## Cap. 2: Condução

# Lei de Fourier

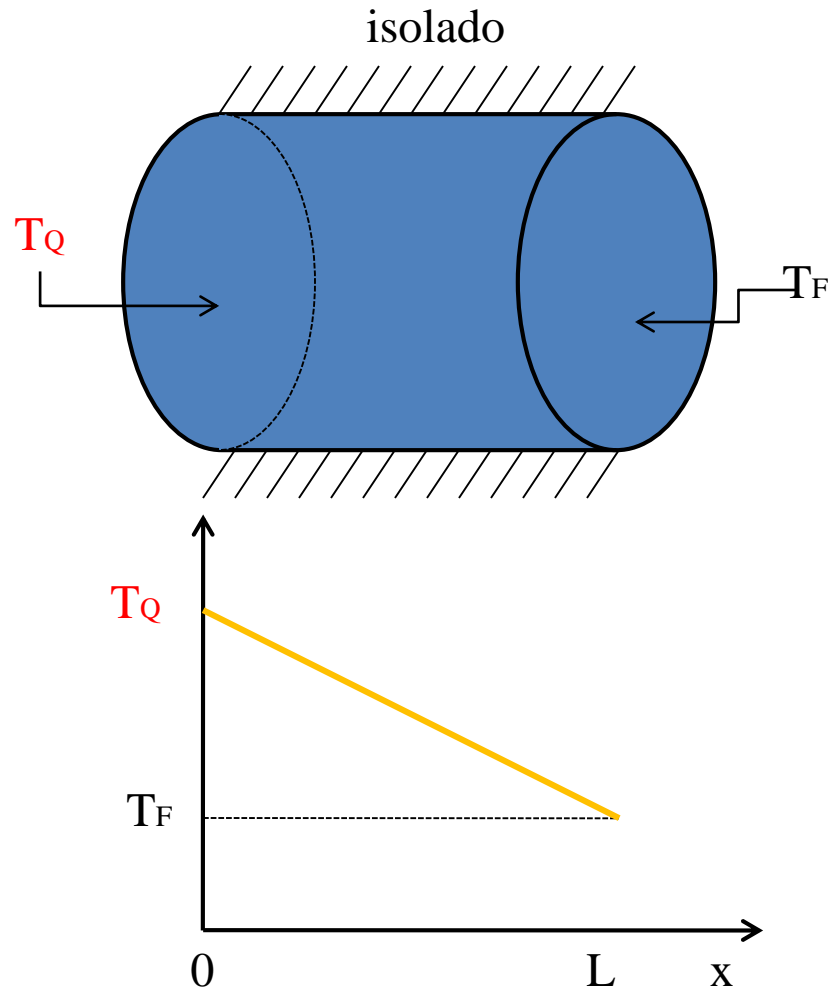
- Condutividade térmica do meio [W/mK]

$$k \equiv -\vec{q}'' / \vec{\nabla} T$$

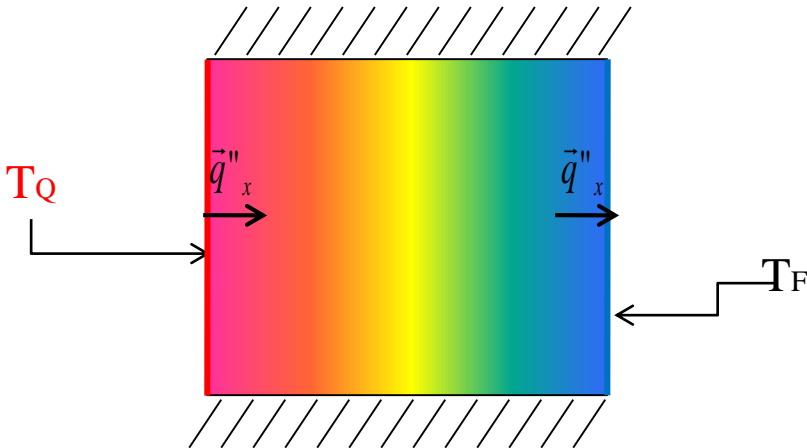
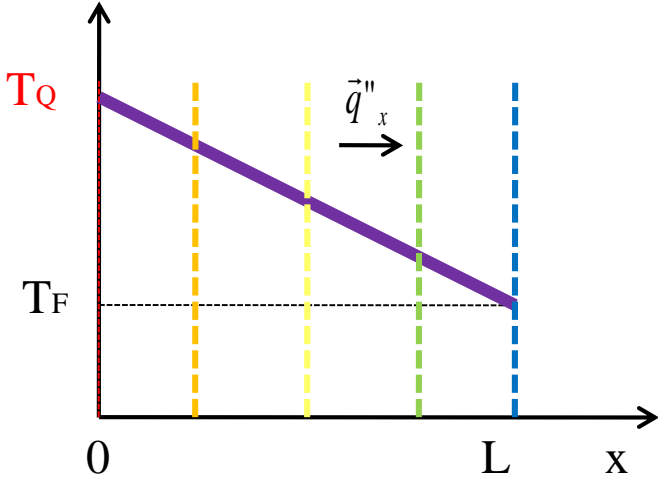
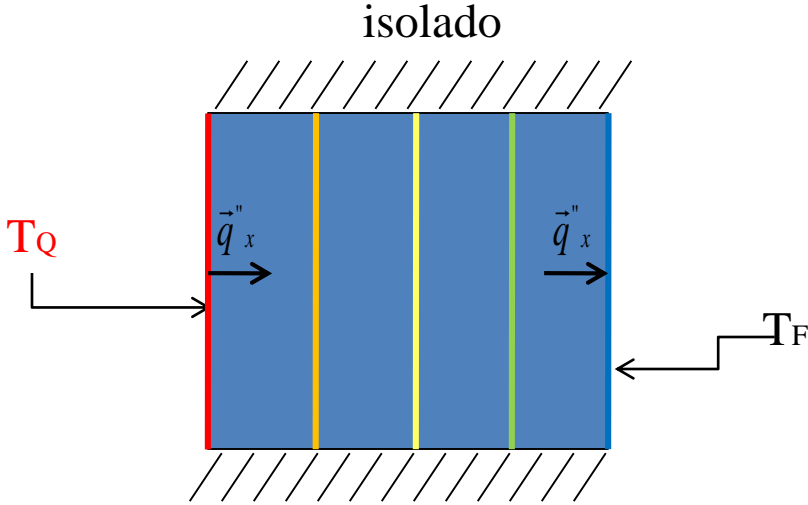
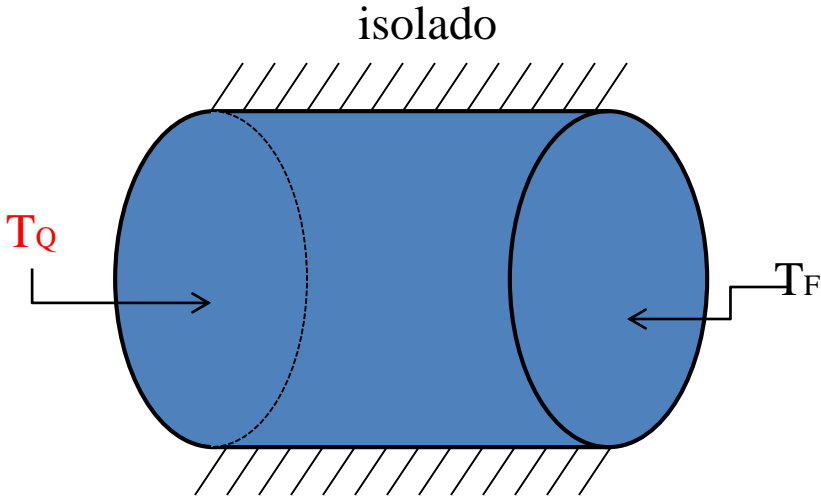
- A direção do fluxo de calor é perpendicular as linhas de temperaturas constantes,



# Em um problema 1-D

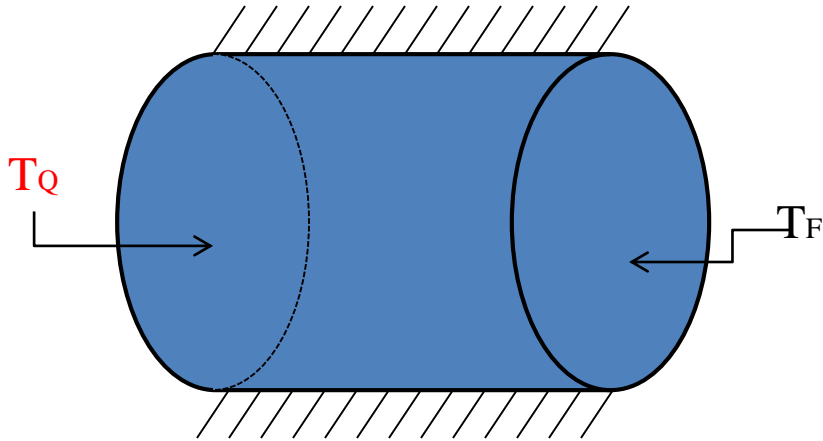


# Isotermas – 1D

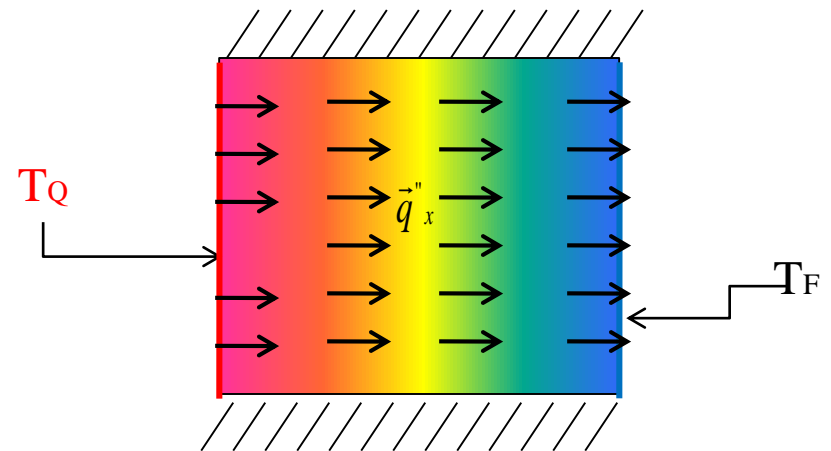
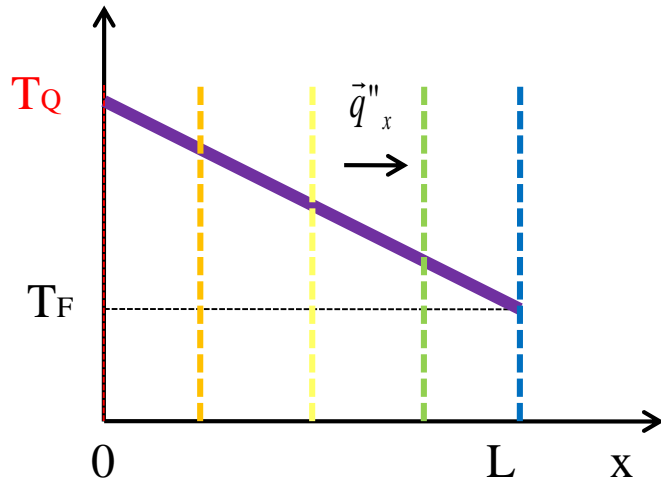
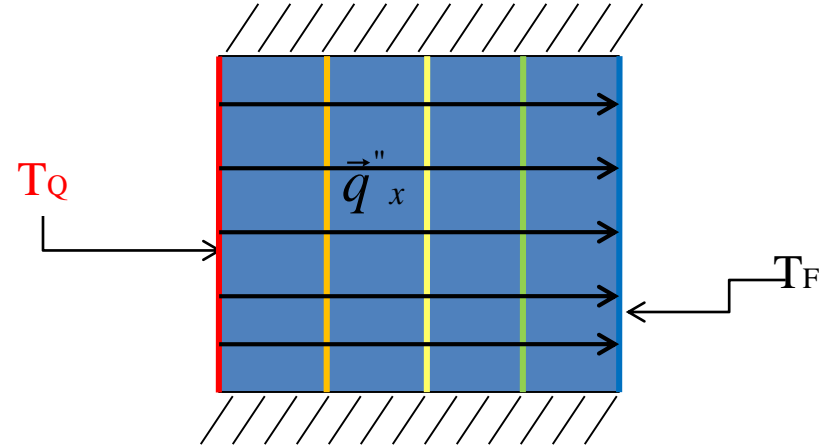


# Isofluxos – 1D

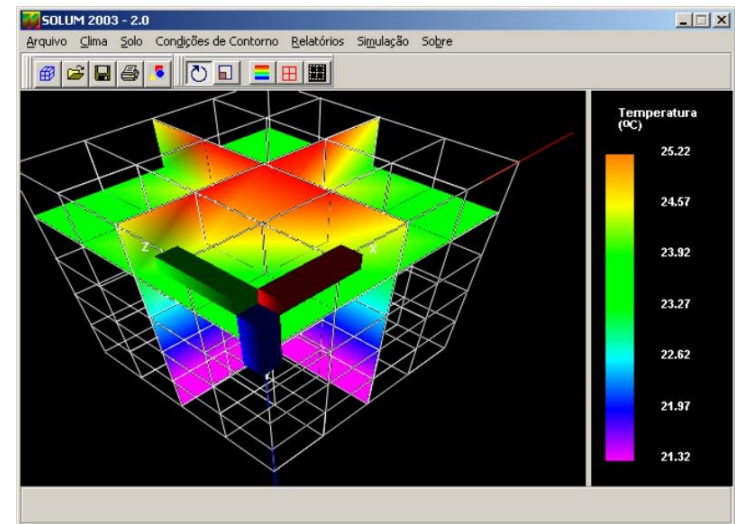
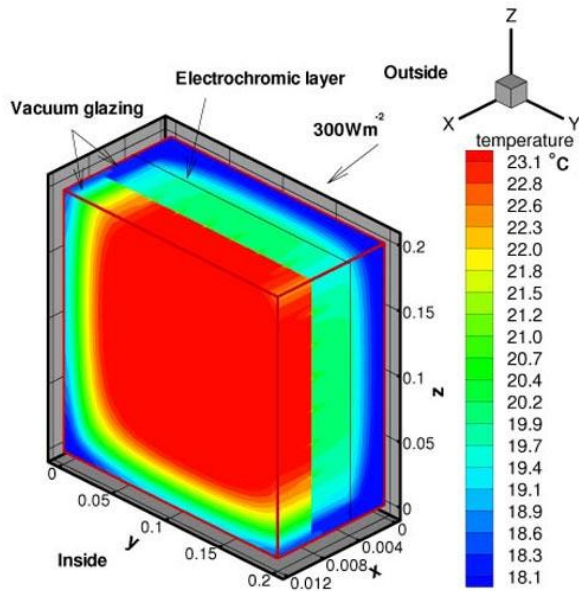
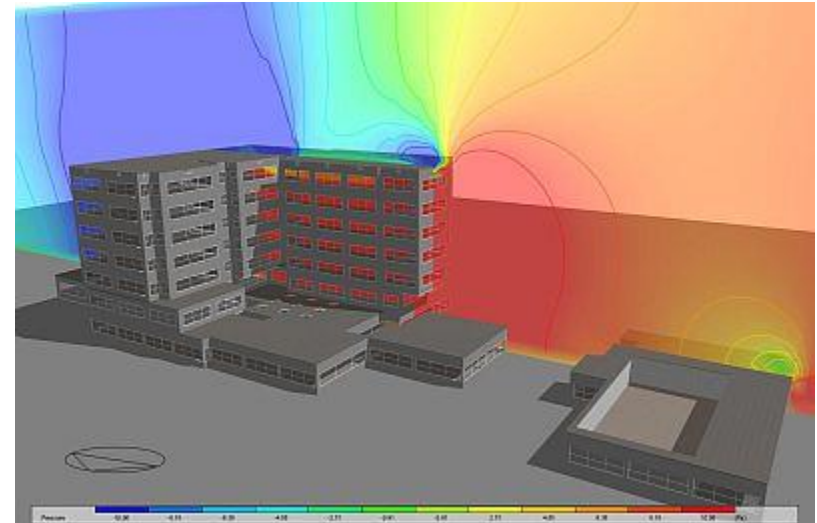
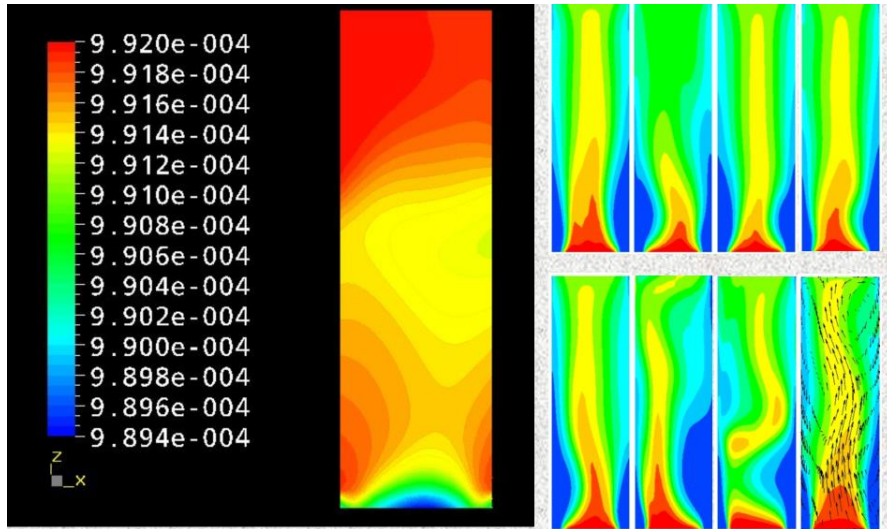
isolado



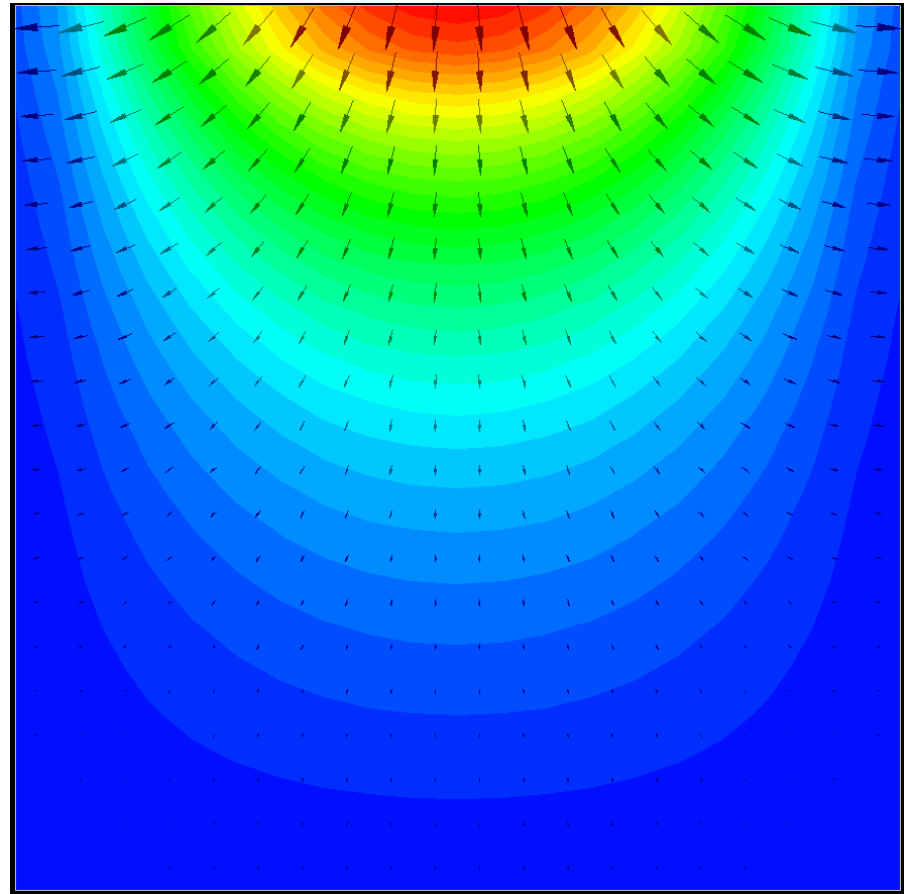
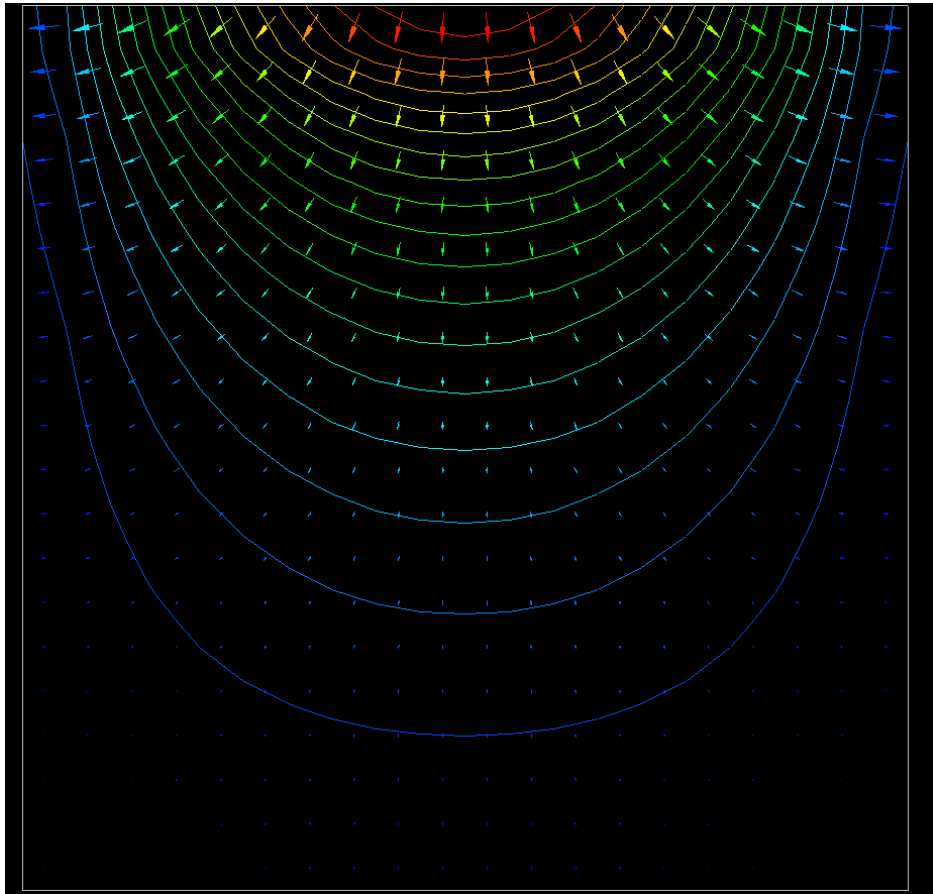
isolado



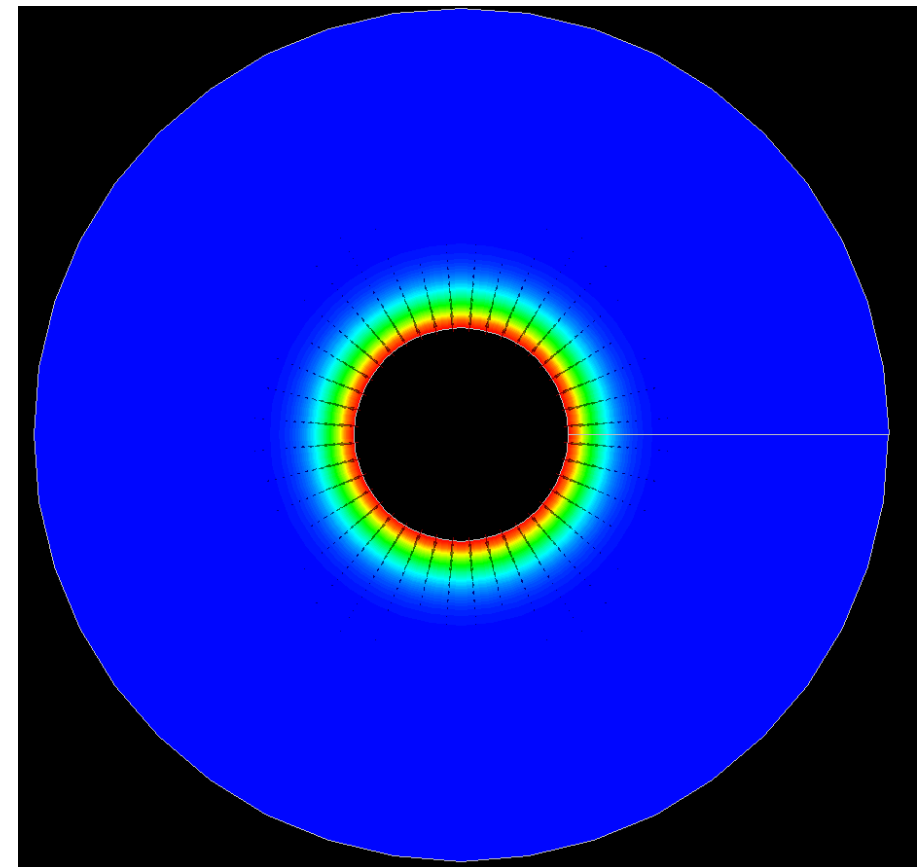
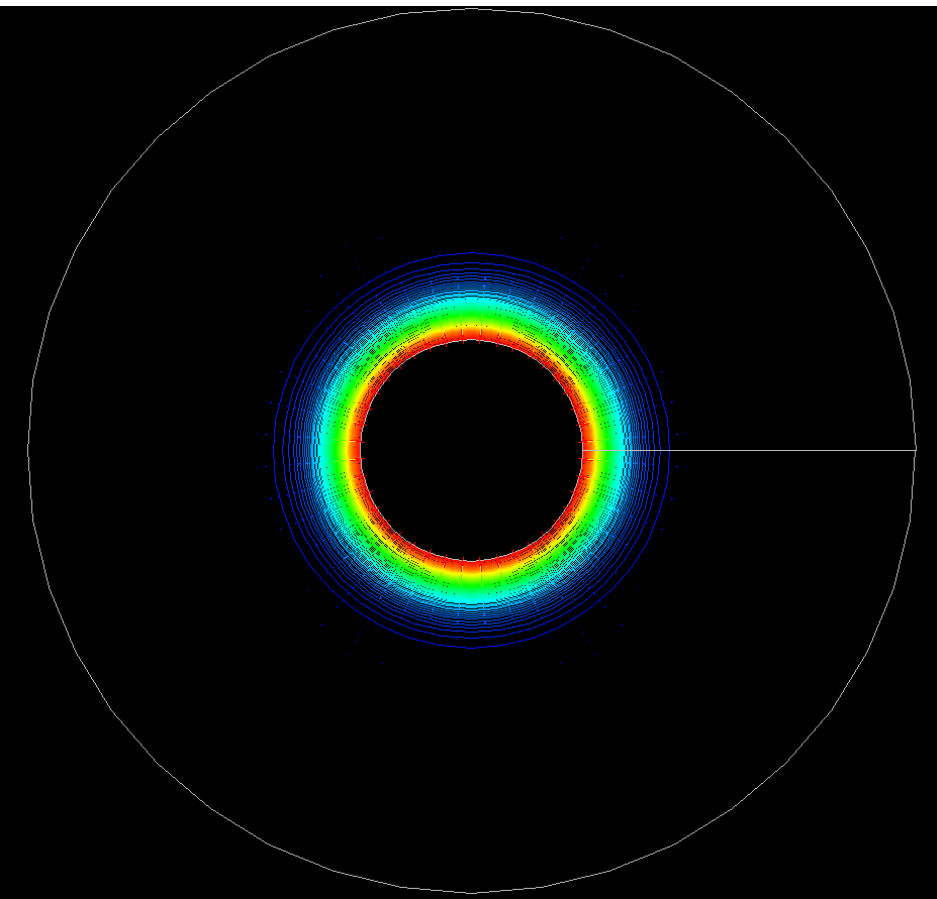
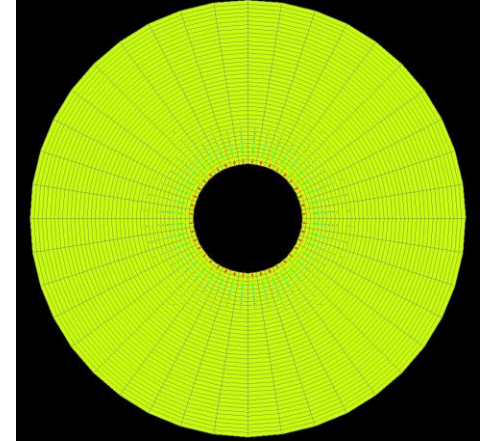
# Bidimensional ou tridimensional



# Bidimensional Cartesiano



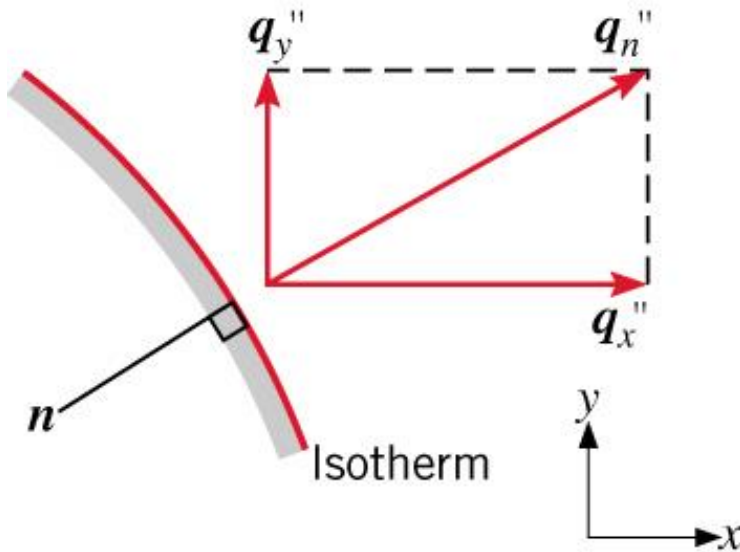
# Coordenadas cilíndricas 1-D





# Linhas de Fluxo

$$\vec{q}'' = -k \vec{\nabla} T$$



$$\begin{aligned} \vec{q}'' &= -k \vec{\nabla} T = -k \left( \frac{\partial T}{\partial x} \hat{i} + \frac{\partial T}{\partial y} \hat{j} + \frac{\partial T}{\partial z} \hat{k} \right) \\ &= q_x'' \hat{i} + q_y'' \hat{j} + q_z'' \hat{k} \end{aligned}$$

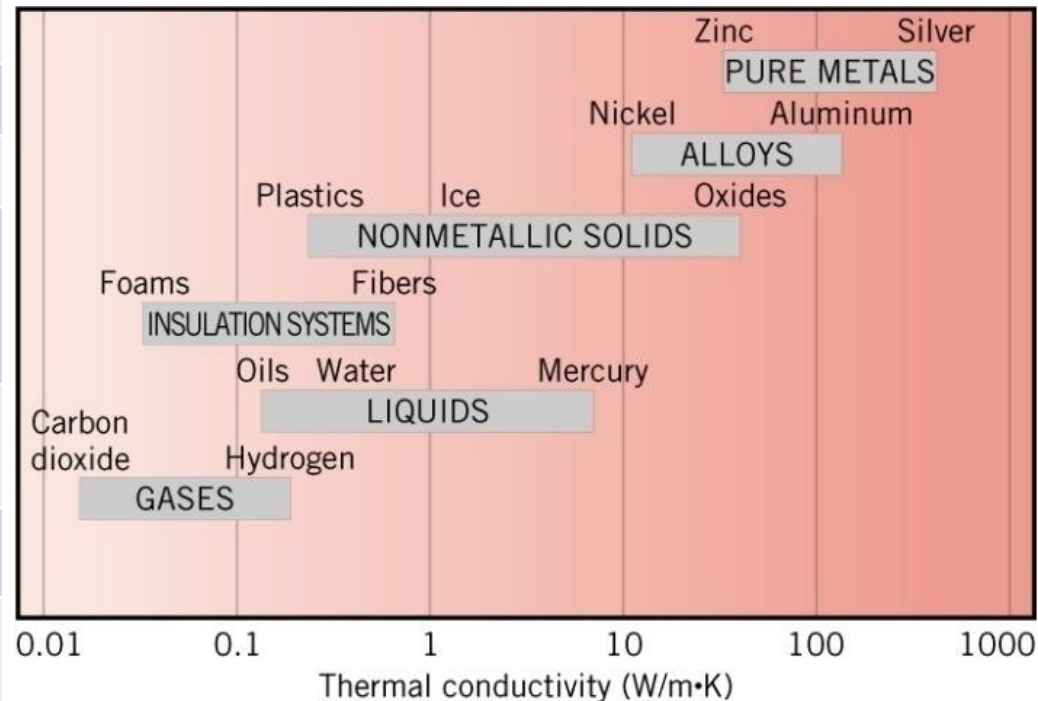
$$q_x'' = -k_x \frac{\partial T}{\partial x}; \quad q_y'' = -k_y \frac{\partial T}{\partial y}; \quad q_z'' = -k_z \frac{\partial T}{\partial z}$$

Se  $k_x = k_y = k_z = \text{isotrópico}$ , senão anisotrópico (ex. grafite)  
 Além disso  $k = k(x, y, z, t, T)$

# Propriedades Termofísicas

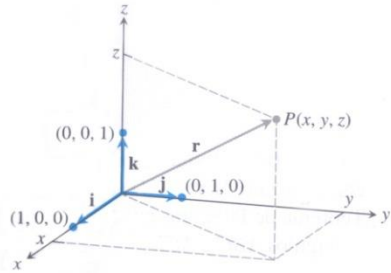
## Condutividade Térmica

material	k[W/(m.K)
Cobre Puro	401
Alumínio Puro	237
Aço Carbono	~50
Ouro	317
Prata	429
Diamante	2300
Grafite	1950 └ 5,7
Argamassa de cimento	0,72
Fibra de vidro	0,063
Ar parado	0,00934



# Flujo de calor

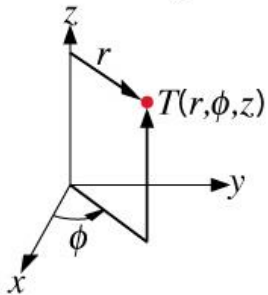
•Coordenadas Cartesianas:



$$\vec{q}'' = \underbrace{-k \frac{\partial T}{\partial x} \vec{i}}_{q''_x} - \underbrace{k \frac{\partial T}{\partial y} \vec{j}}_{q''_y} - \underbrace{k \frac{\partial T}{\partial z} \vec{k}}_{q''_z}$$

(2.3)

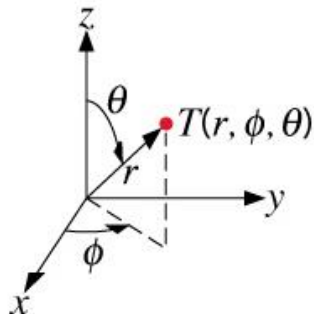
•Coordenadas cilíndricas:



$$\vec{q}'' = \underbrace{-k \frac{\partial T}{\partial r} \vec{i}}_{q''_r} - \underbrace{k \frac{\partial T}{r \partial \phi} \vec{j}}_{q''_\phi} - \underbrace{k \frac{\partial T}{\partial z} \vec{k}}_{q''_z}$$

(2.22)

•Coordenadas esféricas:



$$\vec{q}'' = \underbrace{-k \frac{\partial T}{\partial r} \vec{i}}_{q''_r} - \underbrace{k \frac{\partial T}{r \partial \theta} \vec{j}}_{q''_\theta} - \underbrace{k \frac{\partial T}{r \sin \theta \partial \phi} \vec{k}}_{q''_\phi}$$

(2.25)

- Fluxo de calor:

- Cilindro

$$q_r = A_r q_r'' = 2\pi r L q_r''$$

- ou,

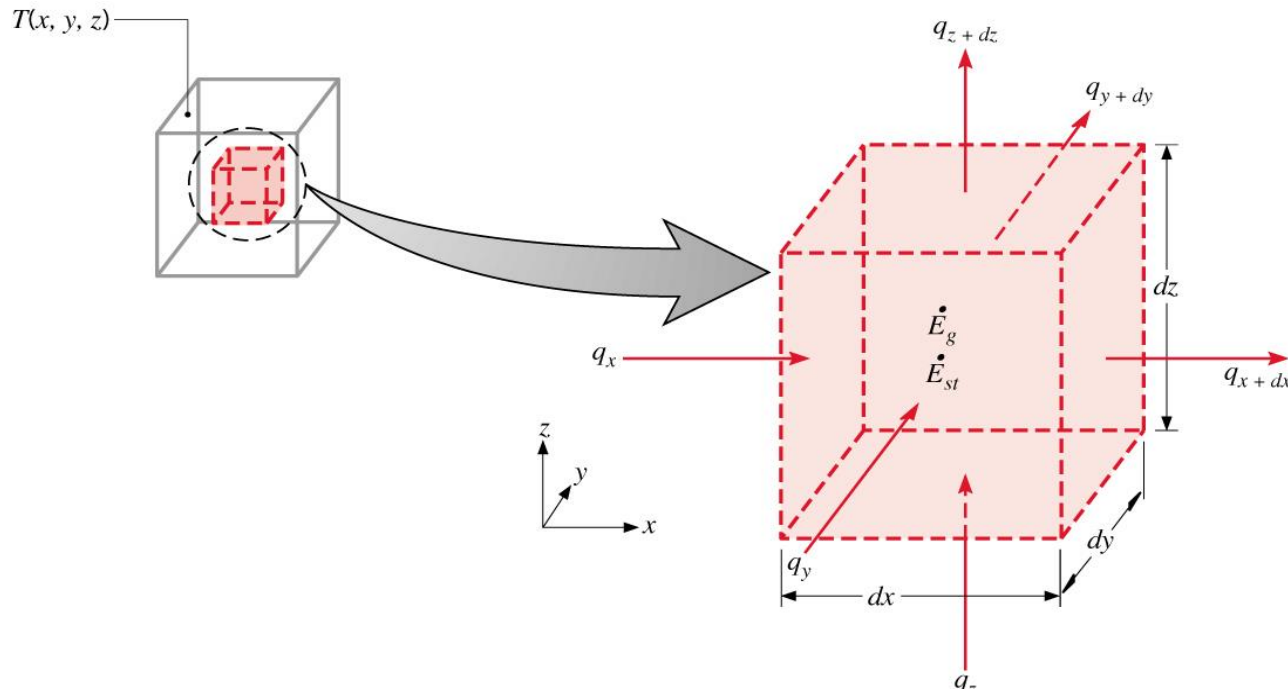
$$q_r' = A_r' q_r'' = 2\pi r q_r''$$

- Esfera

$$q_r = A_r q_r'' = 4\pi r^2 q_r''$$

# Equação da Difusão de Calor

- Conservação de Energia em um Volume de Controle infinitesimal
- Coordenadas Cartesianas:



$$\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\vec{q}'' = -k \frac{\partial T}{\partial r} \vec{i} - k \frac{\partial T}{r \partial \phi} \vec{j} - k \frac{\partial T}{\partial z} \vec{k}$$

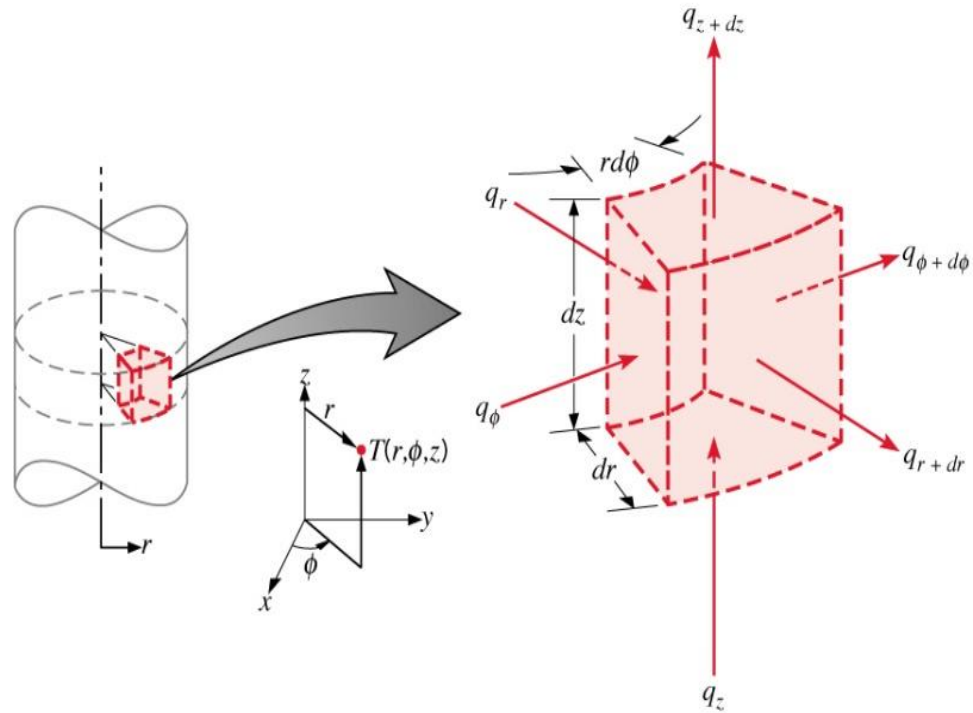
$q_r''$

$q_\phi''$

$q_z''$

(2.22)

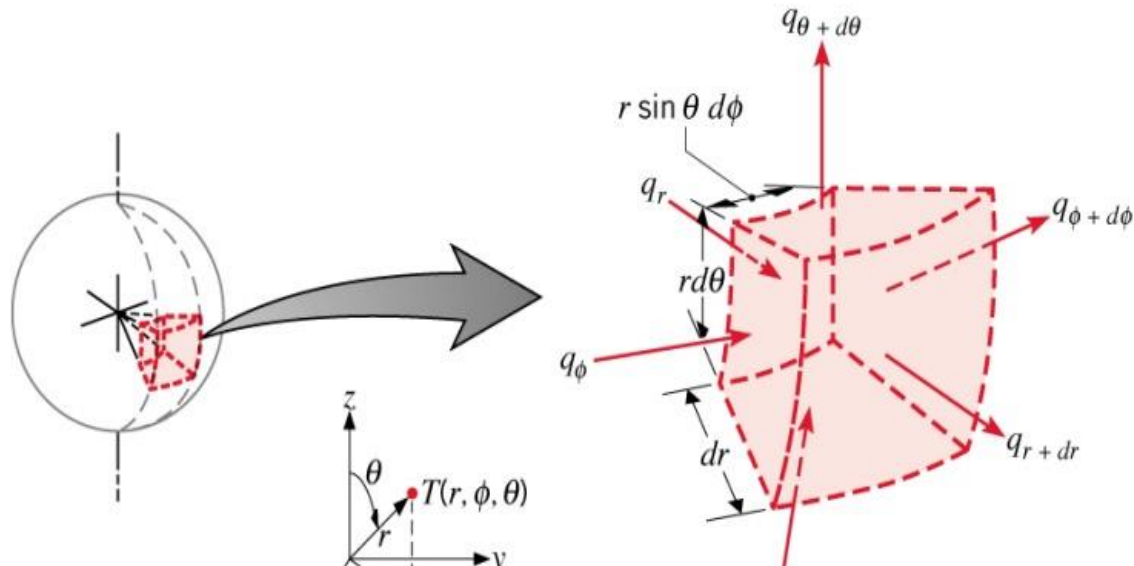
- Coordenadas cilíndricas:



$$\frac{1}{r} \frac{\partial}{\partial r} \left( kr \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \phi} \left( k \frac{\partial T}{\partial \phi} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\vec{q}'' = -k \frac{\partial T}{\partial r} \vec{i} - k \frac{\partial T}{r \partial \theta} \vec{j} - k \frac{\partial T}{r \sin \theta \partial \phi} \vec{k}$$

•Coordenadas esféricas:



$$\underbrace{\frac{1}{r^2} \frac{\partial}{\partial r} \left( kr^2 \frac{\partial T}{\partial r} \right)}_{q_r''} + \underbrace{\frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} \left( k \frac{\partial T}{\partial \phi} \right)}_{q_\phi''} + \underbrace{\frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( k \sin \theta \frac{\partial T}{\partial \theta} \right)}_{q_\theta''} + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

# ~~Dedução da Equação da Difusão~~

## ~~Data:~~

~~Descrição: Dedução da Equação da Difusão em coordenadas cilíndricas e esféricas.~~

~~Trabalho: Dedução da equação da difusão em coordenadas cilíndricas e esféricas.~~

~~OBS: Trabalho em dupla.~~

~~-Este trabalho pode ser entregue feito a mão em sala de aula até a data programada.~~

~~Aceitar entregas até a data:~~



Equação da Difusão de calor em coordenadas cartesianas, tridimensional, transiente

$$\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

- Simplificação em coordenadas cartesianas, unidimensional, transiente, sem geração e propriedades constantes

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

$$\alpha = \frac{k}{\rho c} \left[ \frac{m^2}{s} \right]$$

Difusividade térmica

Tabelas de propriedades:

Sólidos: Tab. A.1 – A.3

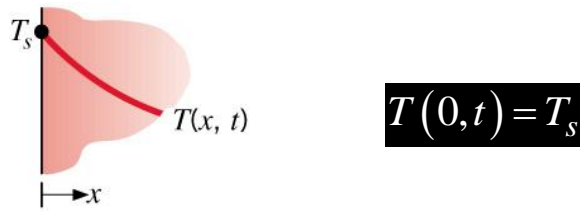
Gases: Tab. A.4

Líquidos: Tab. A.5 – A.7

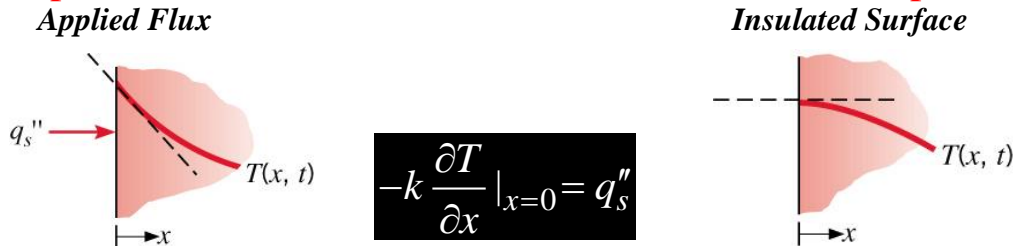
# Condições de contorno e condições iniciais

- Em regime transiente é necessário conhecer uma condição inicial:
- Como a EDL ou EDP é de segunda ordem é necessário conhecer duas condições de contorno em cada direção:

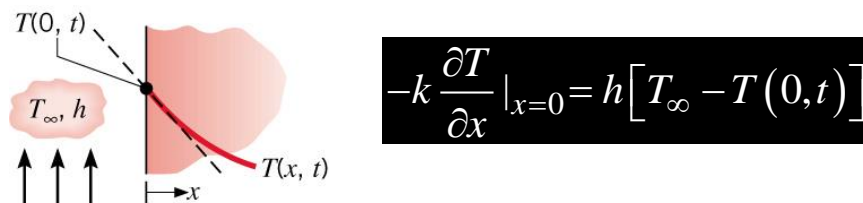
1ª Espécie (Dirichlet) - Temperatura constante na superfície:



2ª Espécie (Newmann) - Fluxo de calor constante na superfície:



3ª Espécie - Convecção na superfície



# Micro e Nanoescala

- Condução pode ser tratada como um transporte de energia (**electrons ou phonon**).

- Para um sólido:

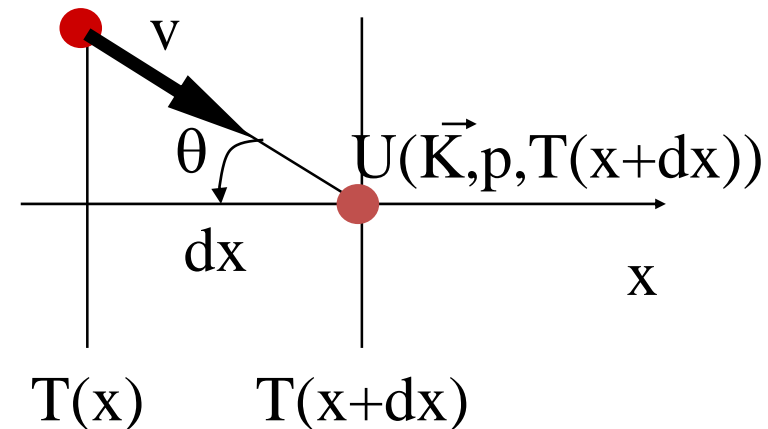
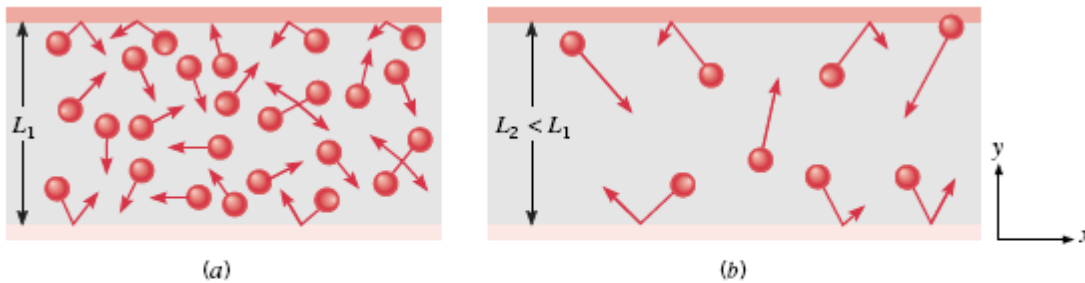
$$k = \frac{1}{3} C \bar{c} \lambda_{mfp} \quad (2.7)$$

Energia transportada por unidade de volume

Livre caminho médio – distância média percorrida no transporte da energia antes da colisão

Velocidade do transporte.

- Os pacotes de energia colidem também com as fronteiras, afetando a propagação.



**FIGURE 2.6** Electron or phonon trajectories in (a) a relatively thick film and (b) a relatively thin film with boundary effects.

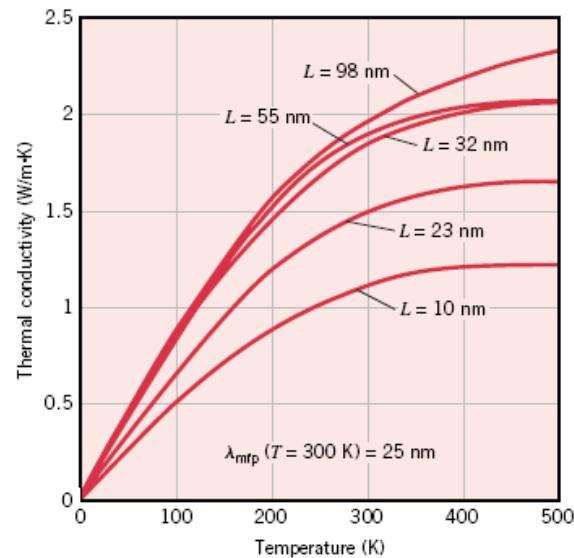
$$k_x(K, p) = C(K, p) v^2(K, p) \tau(K, p) \cos^2(\theta_{K,x})$$

For  $L/\lambda_{mfp} < 1$ ,

$$k_x/k = 1 - 2\lambda_{mfp}/(3\pi L) \quad (2.9a)$$

$$k_y/k = 1 - \lambda_{mfp}/(3L) \quad (2.9b)$$

➤ Condição de contorno com grão em contato com sólidos



Measured thermal conductivity of a ceramic material vs. grain size, L.

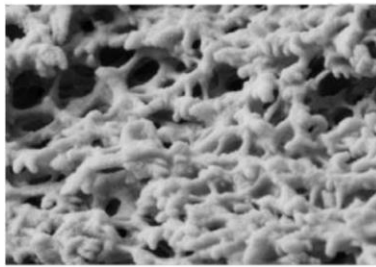
- A Lei de Fourier não descreve adequadamente problemas em nanoescala pois não considera o tempo de deslocamento da energia;

# Equação de Não-Fourier

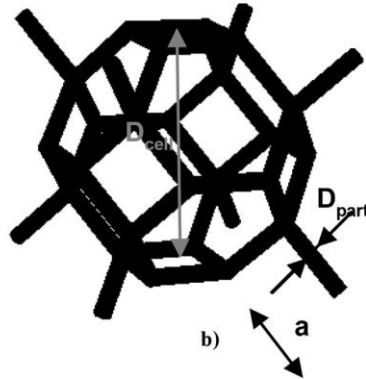
$$\nabla^2 T = \frac{1}{c^2} \frac{\partial^2 T}{\partial t^2} + \frac{1}{D} \frac{\partial T}{\partial t}$$

# Heat transfer in cellulose-based aerogels: Analytical modelling and measurements

D. Baillis, R. Coquard, L.M. Moura  
Energy 84 (2015) 732-744



a)



b)

Illustration of the representative structure retained: a) SEM pictures of cellulose aerogels (elaborated by Mines ParisTech); b) tetrakaedahedron open-cell with circular cylindrical struts.

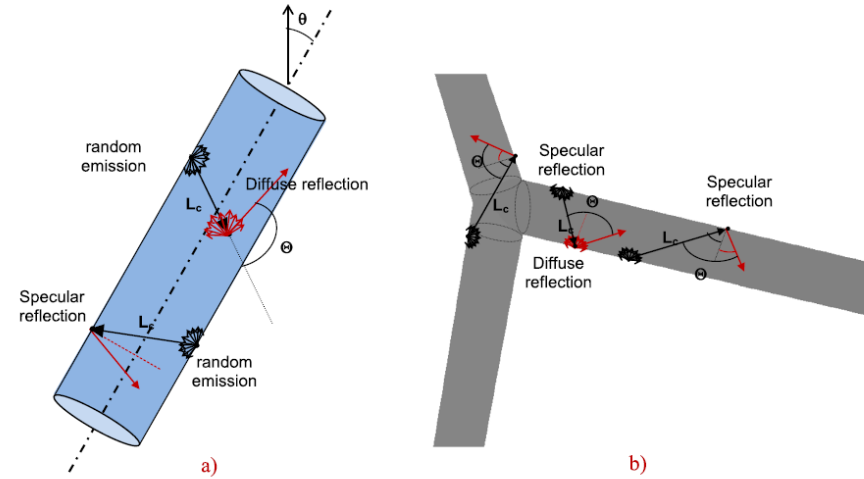


Fig. 2. Illustration of the phonon tracking inside an infinite circular cylinder (a) and a cylindrical cell struts (b).

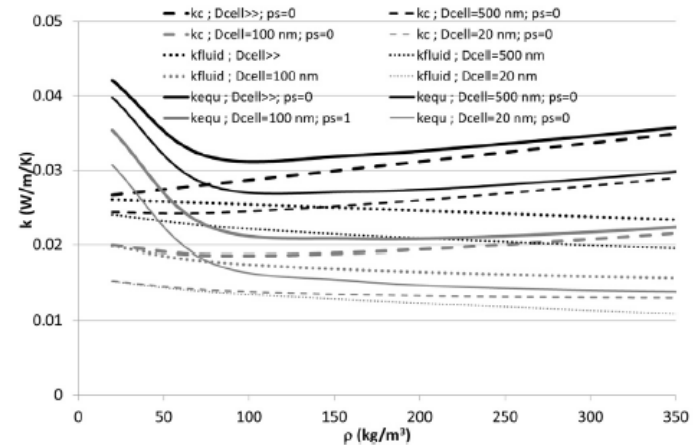
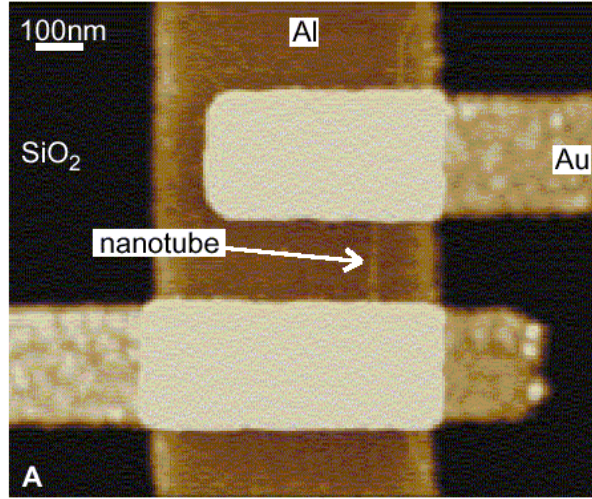


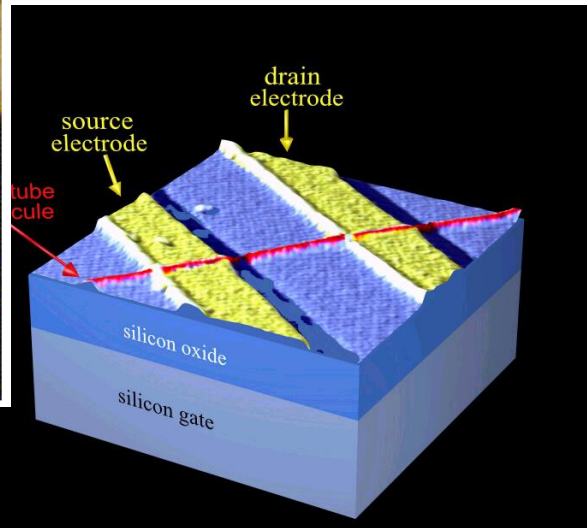
Fig. 8. Evolution of the equivalent conductivity and of the solid conduction, fluid conduction and radiative contributions with the density for  $p_{psd} = 0.5$ .

# Kinetic theory of gas – Thermal conductivity of CNT - Chantrenne, 2009)

## Electronic nanocomponents (doped and undoped CNT's)

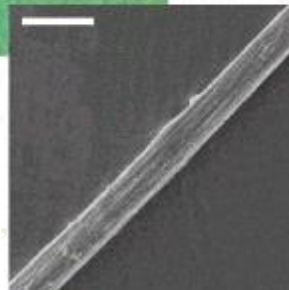


A. Bachtold, *Science*, 294, 1317, 2001  
S.J. Trans, *Nature*, 393, 49, 1998



Composite materials

Dalton et al., *Nature*, 423, 703, 2003



Connexions, actuators

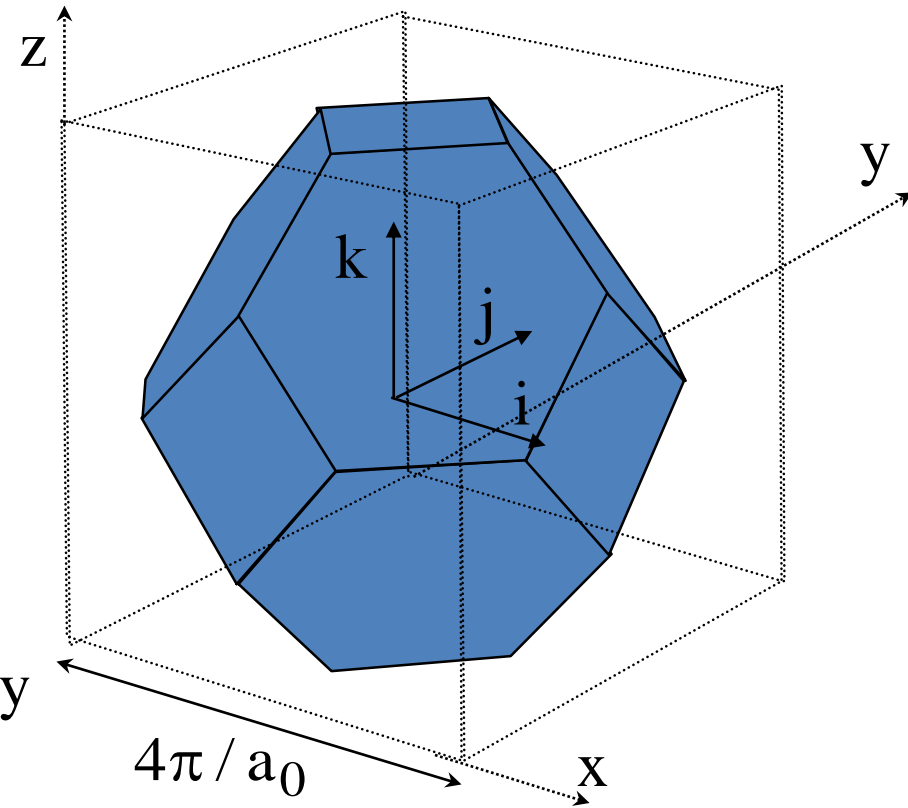
Fennimore et al., *Nature*, 424, 408, 2003  
Baughman et al., *Science*, 284,1340,1999

...

# Kinetic theory of gas – Thermal conductivity SiC nanostructures, Chantrenne, 2009

## Vibration modes in the reciprocal space

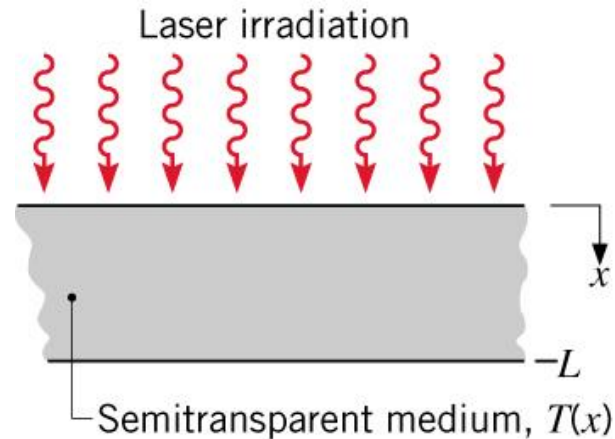
- $\mathbf{K}$  = linear combination of  $\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3$
- $\mathbf{K}$  belong to the first Brillouin zone
- Number of  $\mathbf{K}$  : number of elementary cell
- Number of polarizations  $p = 6$



$$\mathbf{b}_1 = \frac{\pi}{a_0}(\mathbf{i} + \mathbf{j} - \mathbf{k}) \quad \mathbf{b}_2 = \frac{\pi}{a_0}(-\mathbf{i} + \mathbf{j} + \mathbf{k})$$
$$\mathbf{b}_3 = \frac{\pi}{a_0}(\mathbf{i} - \mathbf{j} + \mathbf{k})$$

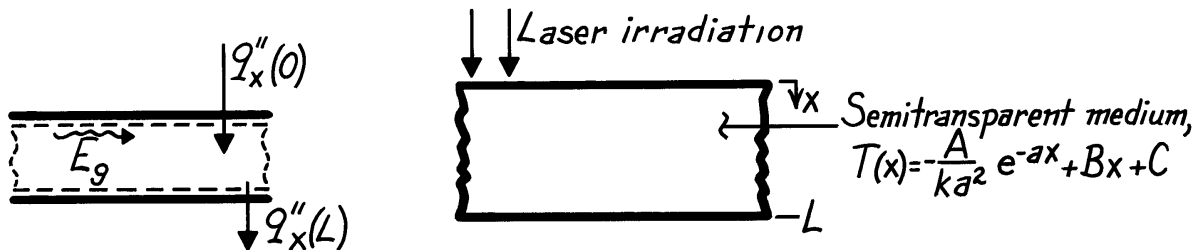


Problem 2.28 Surface heat fluxes, heat generation and total rate of radiation absorption in an irradiated semi-transparent material with a prescribed temperature distribution.



**KNOWN:** Temperature distribution in a semi-transparent medium subjected to radiative flux.

**FIND:** (a) Expressions for the heat flux at the front and rear surfaces, (b) The heat generation rate  $\dot{q}(x)$ , and (c) Expression for absorbed radiation per unit surface area.



**ANALYSIS:** (a) Knowing the temperature distribution, the surface heat fluxes are found using Fourier's law,

$$q_x'' = -k \left[ \frac{dT}{dx} \right] = -k \left[ -\frac{A}{ka^2} (-a) e^{-ax} + B \right]$$

$$\text{Front Surface, } x=0: \quad q_x''(0) = -k \left[ +\frac{A}{ka} \cdot 1 + B \right] = -\left[ \frac{A}{a} + kB \right] \quad <$$

$$\text{Rear Surface, } x=L: \quad q_x''(L) = -k \left[ +\frac{A}{ka} e^{-aL} + B \right] = -\left[ \frac{A}{a} e^{-aL} + kB \right] \quad <$$

(b) The heat diffusion equation for the medium is

$$\frac{d}{dx} \left( \frac{dT}{dx} \right) + \frac{\dot{q}}{k} = 0 \quad \text{or} \quad \dot{q} = -k \frac{d}{dx} \left( \frac{dT}{dx} \right)$$

$$\dot{q}(x) = -k \frac{d}{dx} \left[ +\frac{A}{ka} e^{-ax} + B \right] = A e^{-ax}$$

(c) Performing an energy balance on the medium,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$$

On a unit area basis

$$\dot{E}_g'' = -\dot{E}_{in}'' + \dot{E}_{out}'' = -q_x''(0) + q_x''(L) = +\frac{A}{a}(1 - e^{-aL}).$$

<

Alternatively, evaluate  $\dot{E}_g''$  by integration over the volume of the medium,

$$\dot{E}_g'' = \int_0^L \dot{q}(x) dx = \int_0^L A e^{-ax} dx = -\frac{A}{a} [e^{-ax}]_0^L = \frac{A}{a}(1 - e^{-aL}).$$