

Definição do problema

Fluxo de calor local

$$q'' = h(T_s - T_\infty)$$



U_∞, T_∞

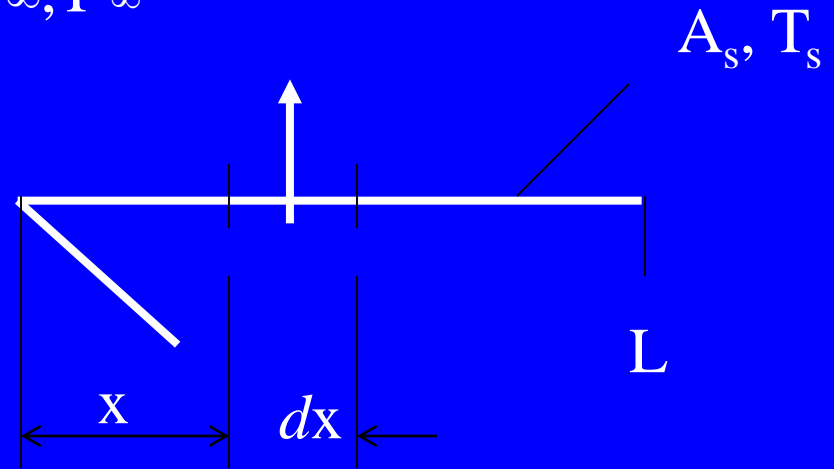
Taxa total de transferência de calor

$$q = \int_{A_s} q'' dA_s$$

Coefficiente médio de convecção

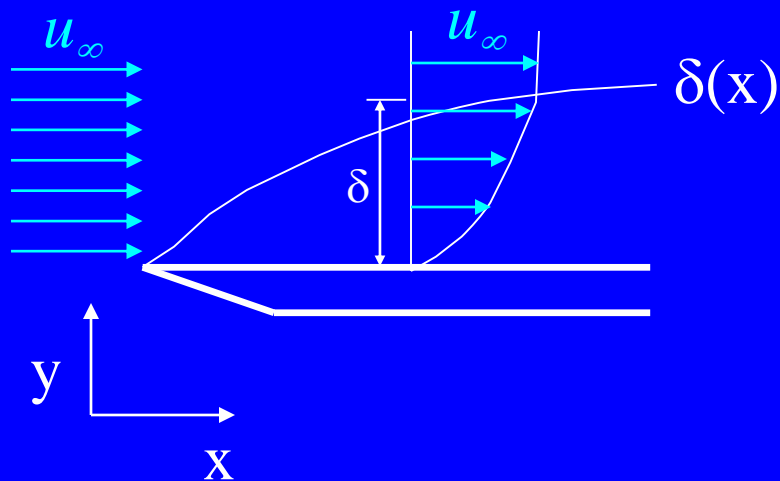
$$\bar{h} = \frac{1}{A_s} \int_{A_s} h dA_s$$

$$q = \bar{h} A_s (T_s - T_\infty)$$

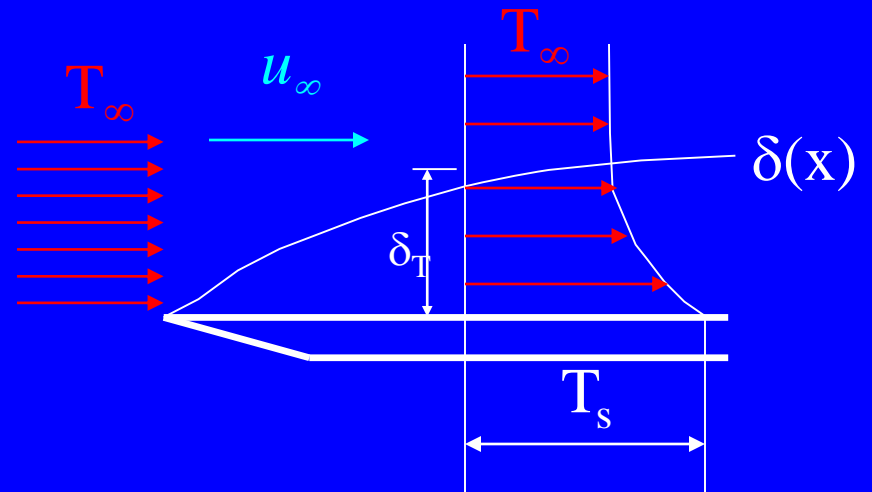


Camadas Limites da Convecção

Camada limite hidrodinâmica

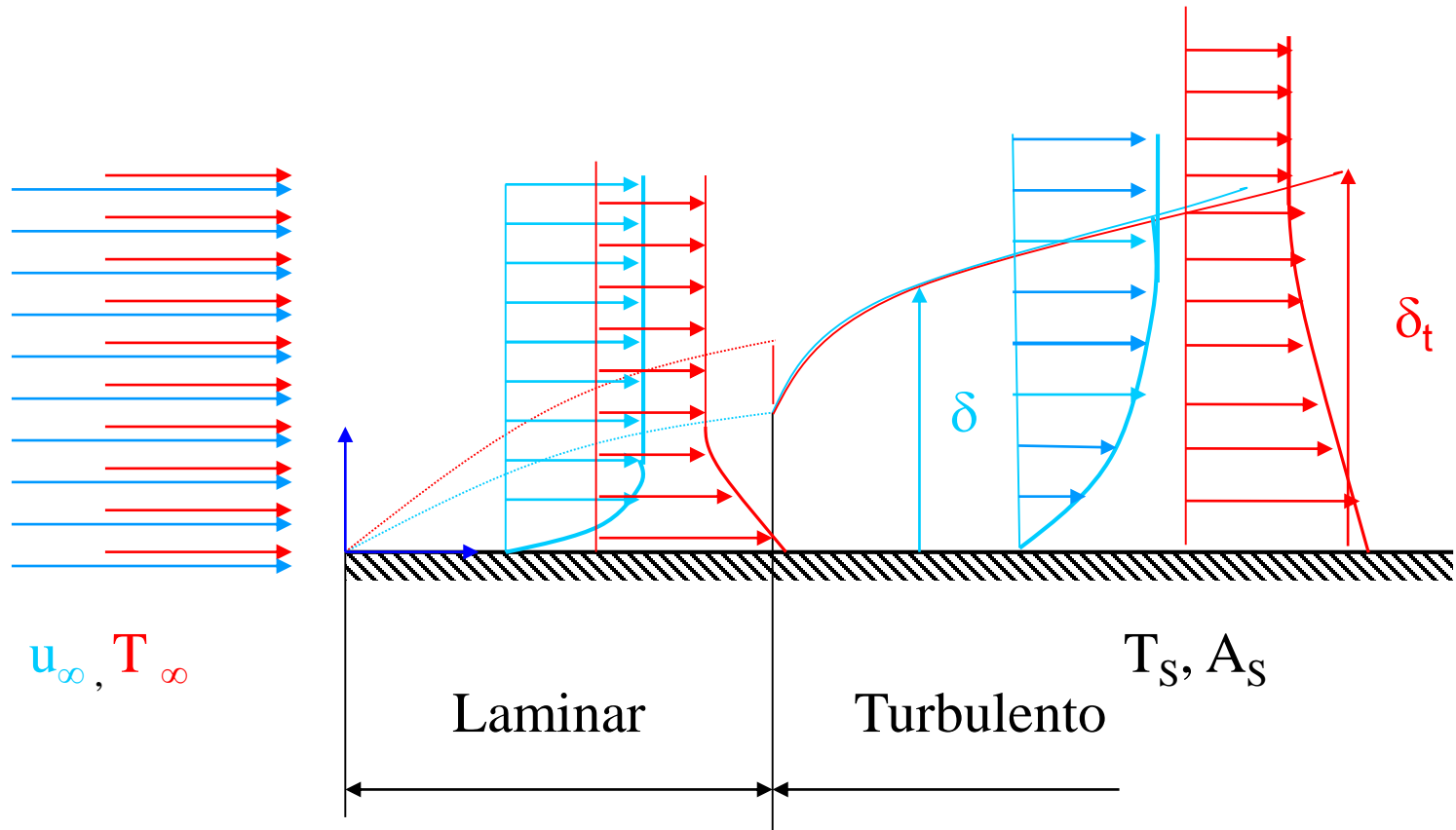


Camada limite térmica

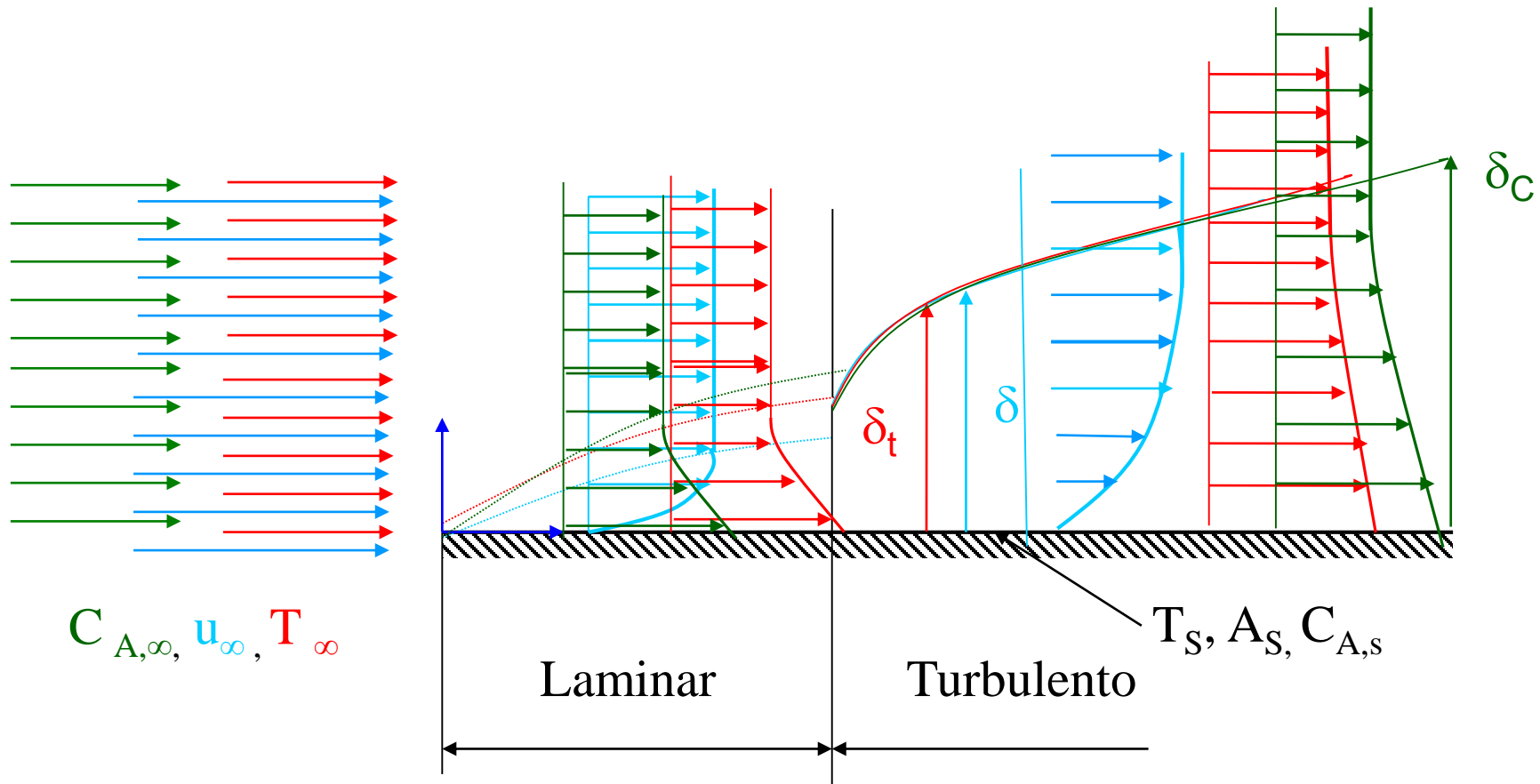


Transferência de Calor por Convecção

- Térmica



Transferência de massa convectiva



Transferência de Calor por Convecção

- Fluxo de Calor local: $q'' = h(T_S - T_\infty)$ [W/m²]
 - h : Coeficiente de convecção local ou coeficiente de película local [W/m²K]

- Fluxo de Calor total: $q = \int_{A_s} q'' dA_s$ [W]

$$q = (T_S - T_\infty) \int_{A_s} h dA_s$$

Transferência de Calor por Convecção

$$\bar{h} = \frac{1}{A_S} \int_{A_S} h dA_S$$

$$q = \bar{h} A_S (T_S - T_\infty)$$

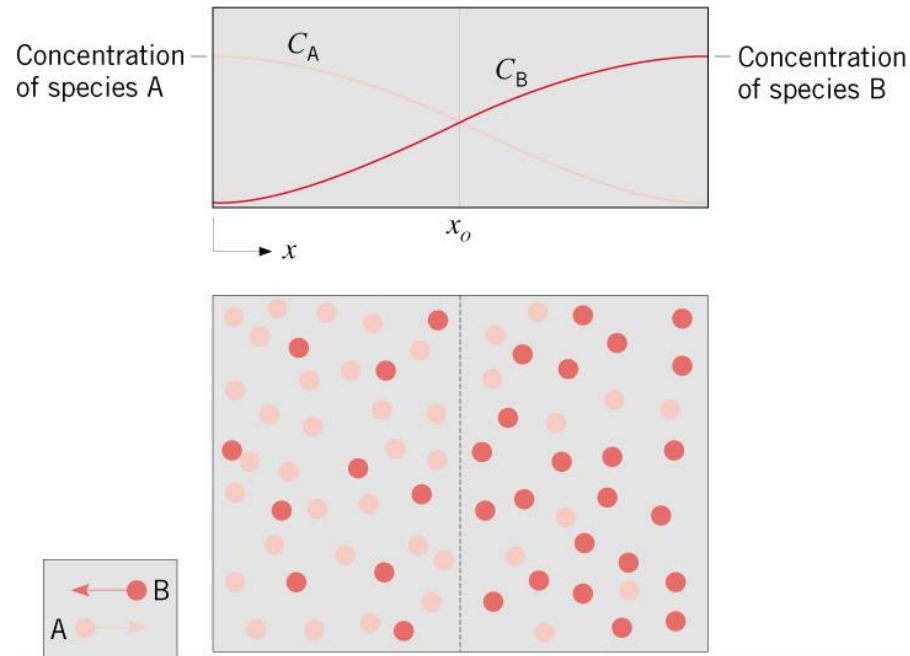
- \bar{h} : Coeficiente de convecção médio ou coeficiente de película médio
[W/m²K]

Transferência de Calor por Convecção

- Para uma placa plana

$$\bar{h} = \frac{1}{wL} \int_0^L h w dx = \frac{1}{L} \int_0^L h dx$$

Transferência de Massa: Difusão

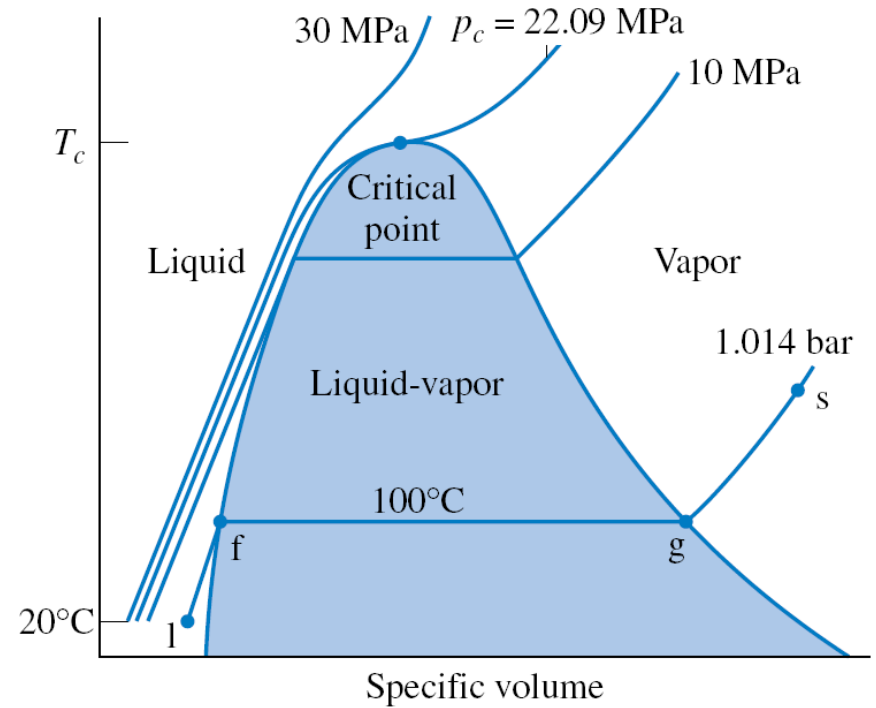
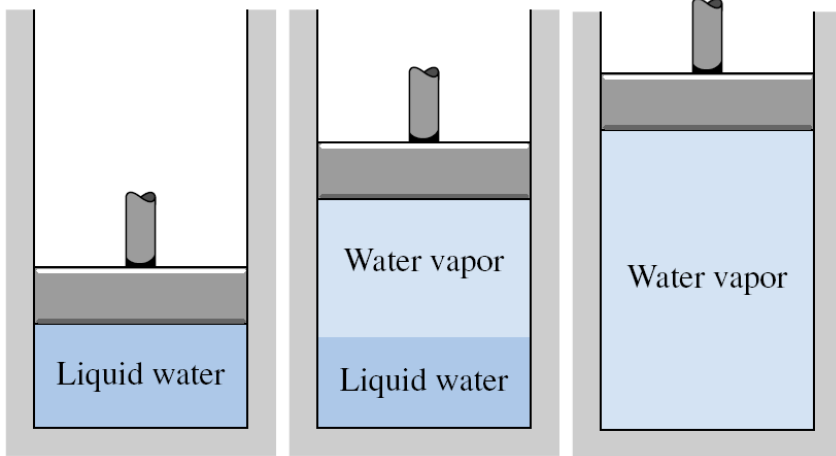


- From **Fick's law** (mass transfer analog to Fourier's law):

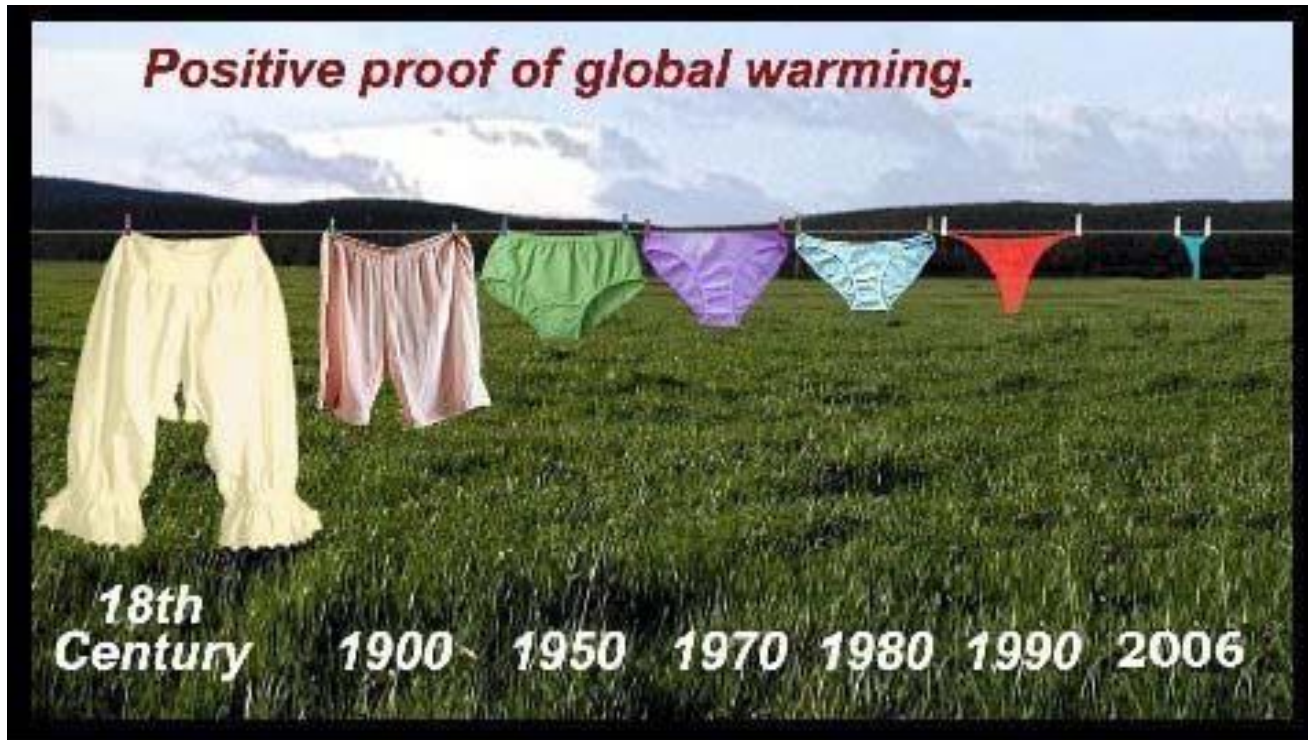
$$J_A^* = -CD_{AB} \nabla x_A$$

↳ **Binary diffusion coefficient** or **mass diffusivity** (m^2/s)

Ebulição: água



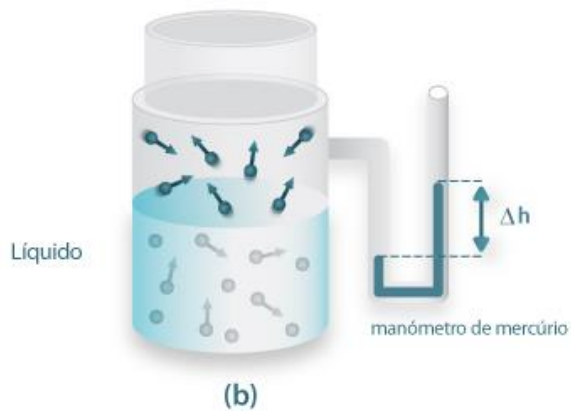
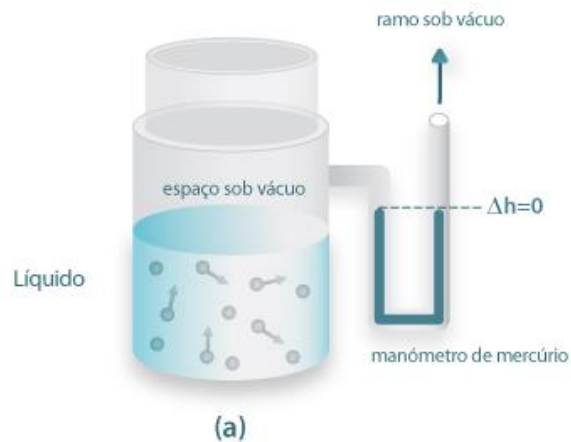
Evaporação: água



POSITIVE PROOF OF GLOBAL WARMING



Evaporação: água

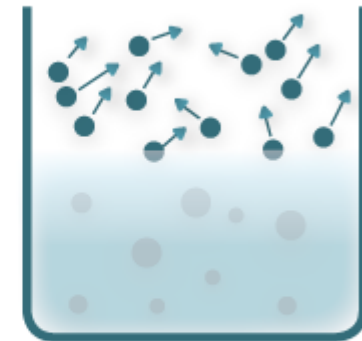


Evaporação



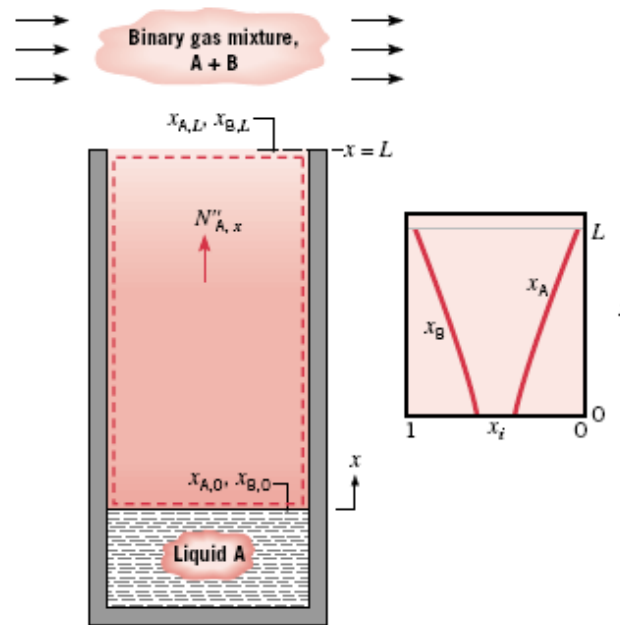
As bolhas de gás não se formam no seio do líquido, porque a pressão do gás (das moléculas na fase gasosa) é menor que a pressão atmosférica

Ebulição



As bolhas de gás formam-se e sobem no seio do líquido, porque a pressão do gás (das moléculas na fase gasosa) vence a resistência oferecida pela pressão atmosférica

Evaporation in a Column: A Nonstationary Medium



- **Species Diffusion Equation on a Molar Basis:**

$$\frac{\partial^2 C_A}{\partial x^2} + \frac{\partial^2 C_A}{\partial y^2} + \frac{\partial^2 C_A}{\partial z^2} + \frac{\square}{D_{AB}} N_A = \frac{1}{D_{AB}} \frac{\partial C_A}{\partial t}$$

- **Species Diffusion Equation on a Mass Basis:**

$$\frac{\partial^2 \rho_A}{\partial x^2} + \frac{\partial^2 \rho_A}{\partial y^2} + \frac{\partial^2 \rho_A}{\partial z^2} + \frac{\square}{D_{AB}} n_A = \frac{1}{D_{AB}} \frac{\partial \rho_A}{\partial t}$$

Transferência de Massa por Convecção

- Fluxo Molar da espécie a local: $N_A'' = h_m(C_{A,S} - C_{A,\infty})$
[kmol/sm²] = [m/s]. [kmol/m³]
- h_m : Coeficiente de transferência de massa por convecção local
[m/s]

- Taxa de transferência molar total:
[kmol/sm]

$$N_A = \int_{A_s} N_A'' dA_s$$

$$N_A = (C_{A,S} - C_{A,\infty}) \int_{A_s} h_m dA_s$$

$$N_A = \bar{h}_m A_s (C_{A,S} - C_{A,\infty})$$

Transferência de Massa por Convecção

$$\bar{h}_m = \frac{1}{A_S} \int_{A_s} h_m dA_s$$

$$N_A = \bar{h}_m A_S (C_{A,S} - C_{A,\infty})$$

- \bar{h}_m : Coeficiente de transferência de massa por convecção médio
[m/s]

Transferência de Massa por Convecção

- Para uma placa plana

$$\bar{h}_m = \frac{1}{wL} \int_0^L h_m w dx = \frac{1}{L} \int_0^L h_m dx$$

Transferência de Massa

- Fluxo de massa $C = \frac{\rho}{M_{\text{molecular A}}}$

$$N_A'' \cdot M_{\text{molecular A}} = n_A'' = h_m (\rho_{A,S} - \rho_{A,\infty}) \quad [kg / sm^2]$$

$$N_A \cdot M_{\text{molecular A}} = n_A = n_A'' A_s = h_m A_s (\rho_{A,S} - \rho_{A,\infty}) \quad [kg / s]$$

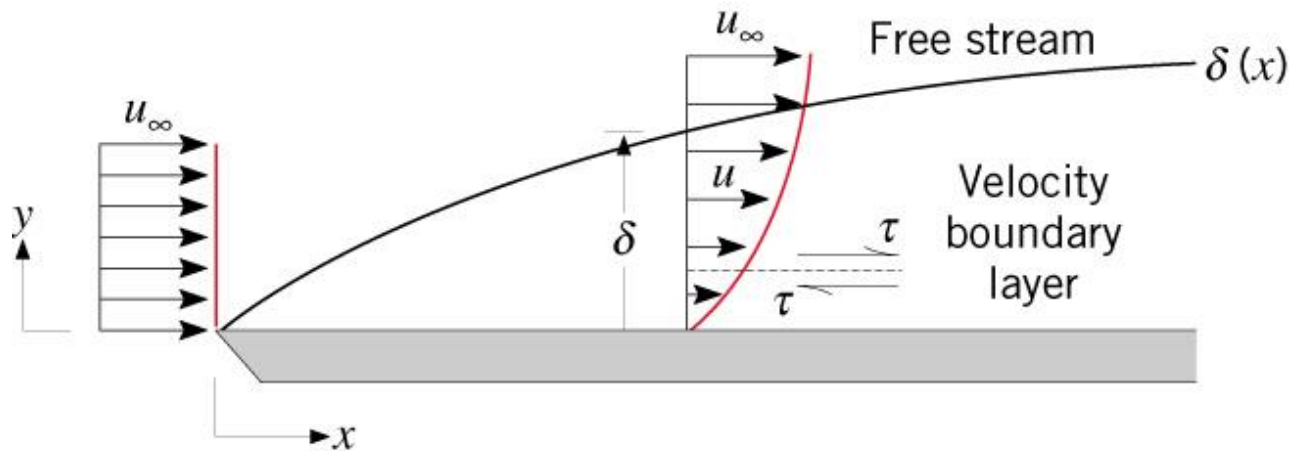
OBS: para determinar $C_{A,S}$ ou $\rho_{A,S}$ considera-se vapor saturado na temperatura T_S ou por aproximação de gás ideal:

$$C_{A,S} = \frac{\rho}{M_{\text{molecular A}}} = \frac{p_{\text{sat}}(T_S)}{\bar{R}T}$$

Ex 6.2 – Naftalina (sublimação) - Entregar

Camadas Limites

- Camada Limite Hidrodinâmica



$$\delta \rightarrow \frac{u(y)}{u_\infty} = 0.99$$

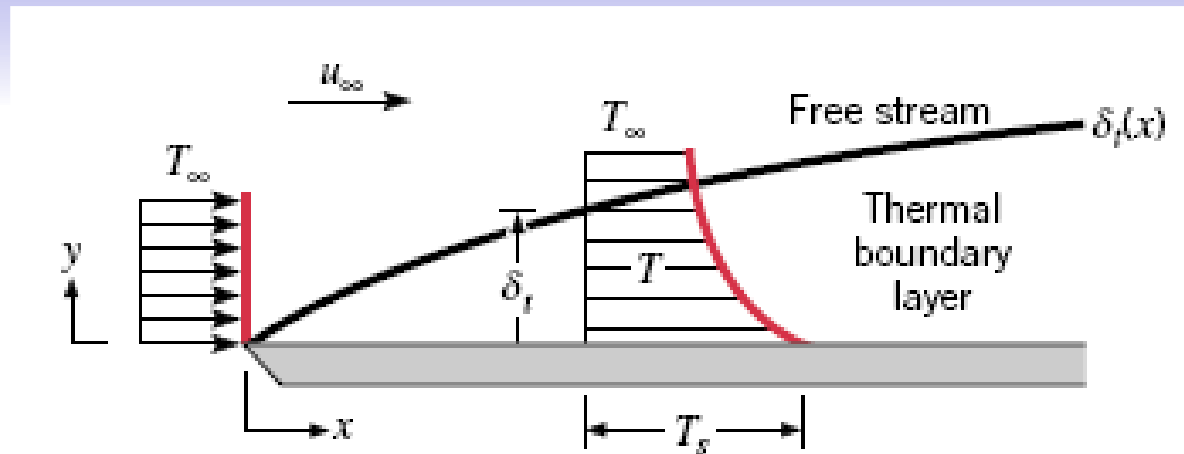
$$\tau_s = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0}$$

$$F_D = \int_{A_s} \tau_s dA_s$$

Coeficiente de atrito:

$$C_f = \frac{\tau_s}{\rho u_\infty^2 / 2}$$

- Camada Limite Térmica



$$\delta_t \rightarrow \frac{T_s - T(y)}{T_s - T_\infty} = 0.99$$

$$q_s'' = -k_f \left. \frac{\partial T}{\partial y} \right|_{y=0}$$

$$h \equiv \frac{-k_f \partial T / \partial y|_{y=0}}{T_s - T_\infty}$$

- Camada Limite de Concentração

$$\delta_t \rightarrow \delta_c$$

$$q \rightarrow N$$

$$h \rightarrow h_m$$

$$N_A'' = -D_{AB} \left. \frac{\partial C_A}{\partial y} \right|_{y=0}$$

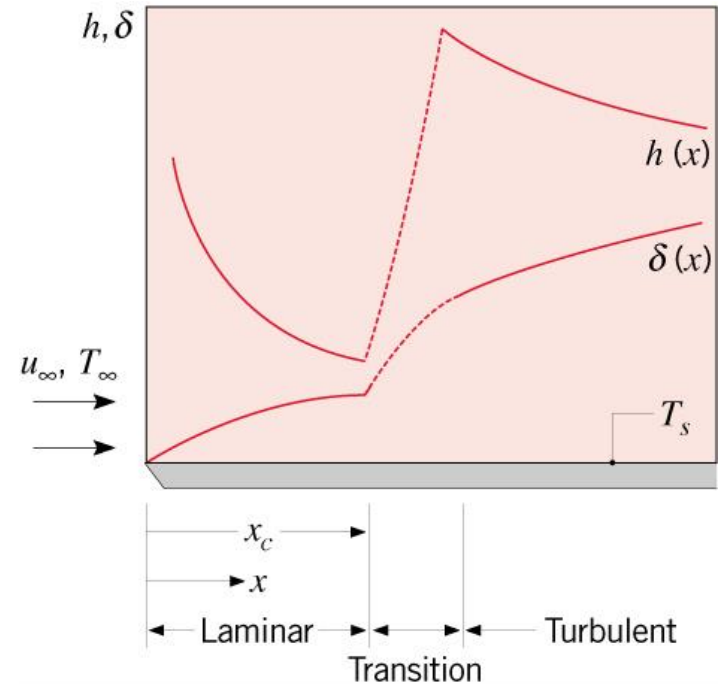
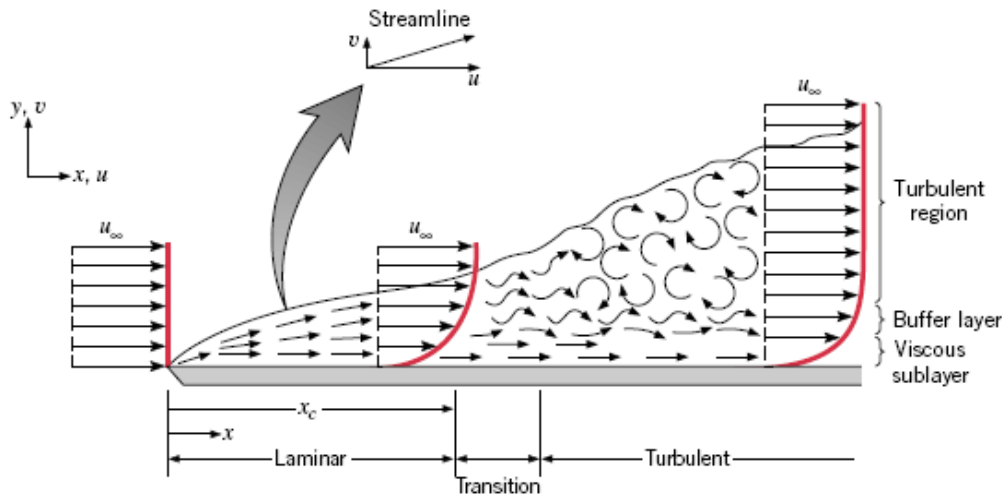
$$= h_m (C_{A,S} - C_{A,\infty})$$

$$h_m = \frac{-D_{AB} \partial C_A / \partial y|_{y=0}}{C_{A,S} - C_{A,\infty}} = \frac{-D_{AB} \partial \rho_A / \partial y|_{y=0}}{\rho_{A,S} - \rho_{A,\infty}}$$

- Coeficiente de Difusão Binária
para um gás ideal

$$D_{AB} \propto p^{-1} T^{3/2}$$

Laminar/Turbulento



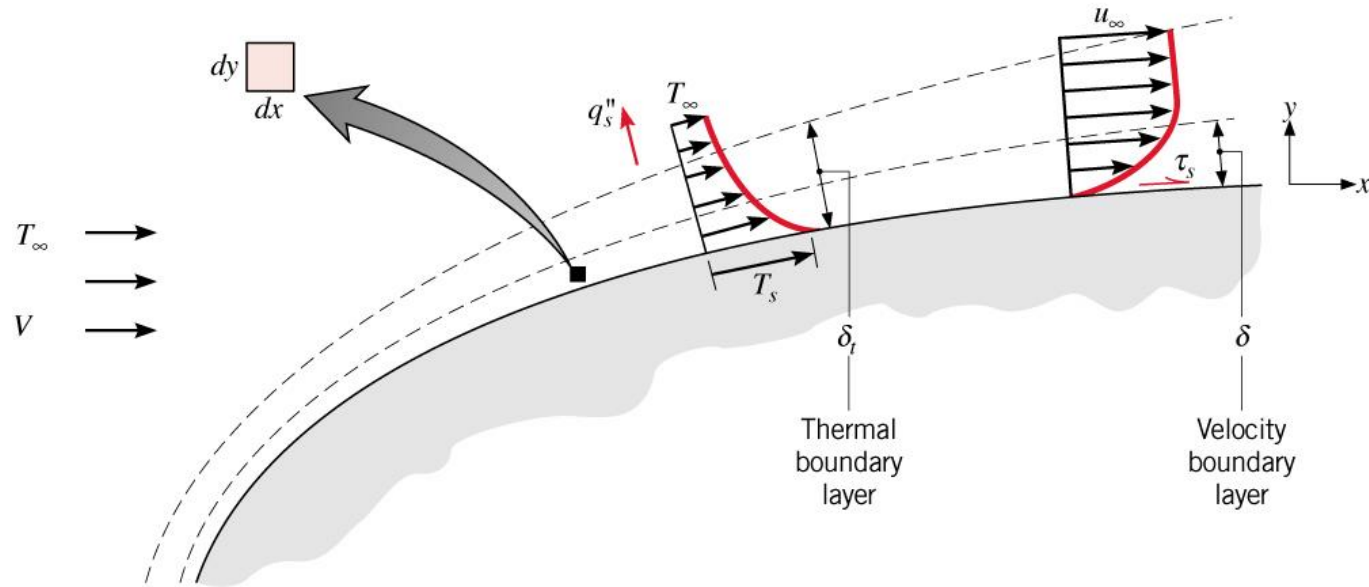
$$Re_{x,c} \equiv \frac{\rho u_{\infty} x_c}{\mu} \rightarrow \text{critical Reynolds number}$$

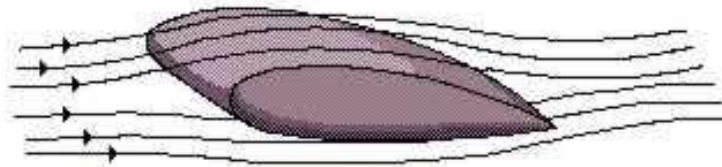
$x_c \rightarrow$ location at which transition to turbulence begins

$$10^5 < Re_{x,c} < 3 \times 10^6$$

Adotado: $Re_{x,c} = 5 \times 10^5$

Resolução Formal das Equações

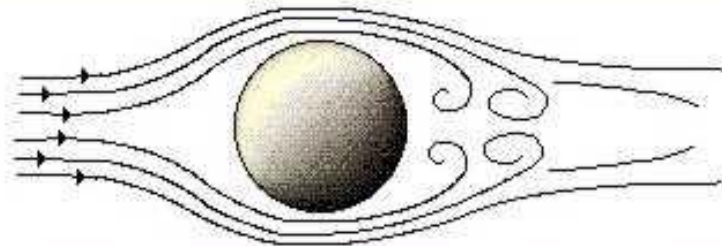




Forma de Asa

Arrasto mínimo

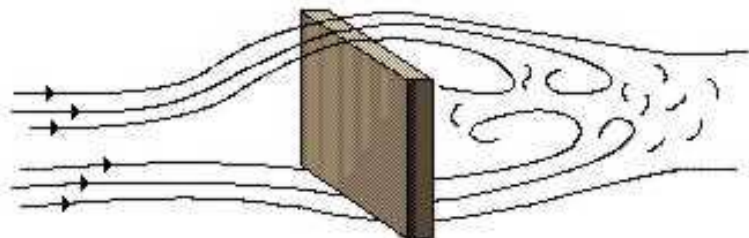
Não produz redemoinhos



Esfera

Arrasto médio

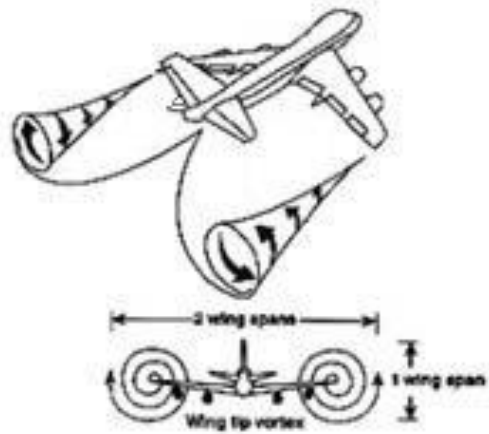
Poucos redemoinhos



Plano

Grande arrasto

Muitos redemoinhos

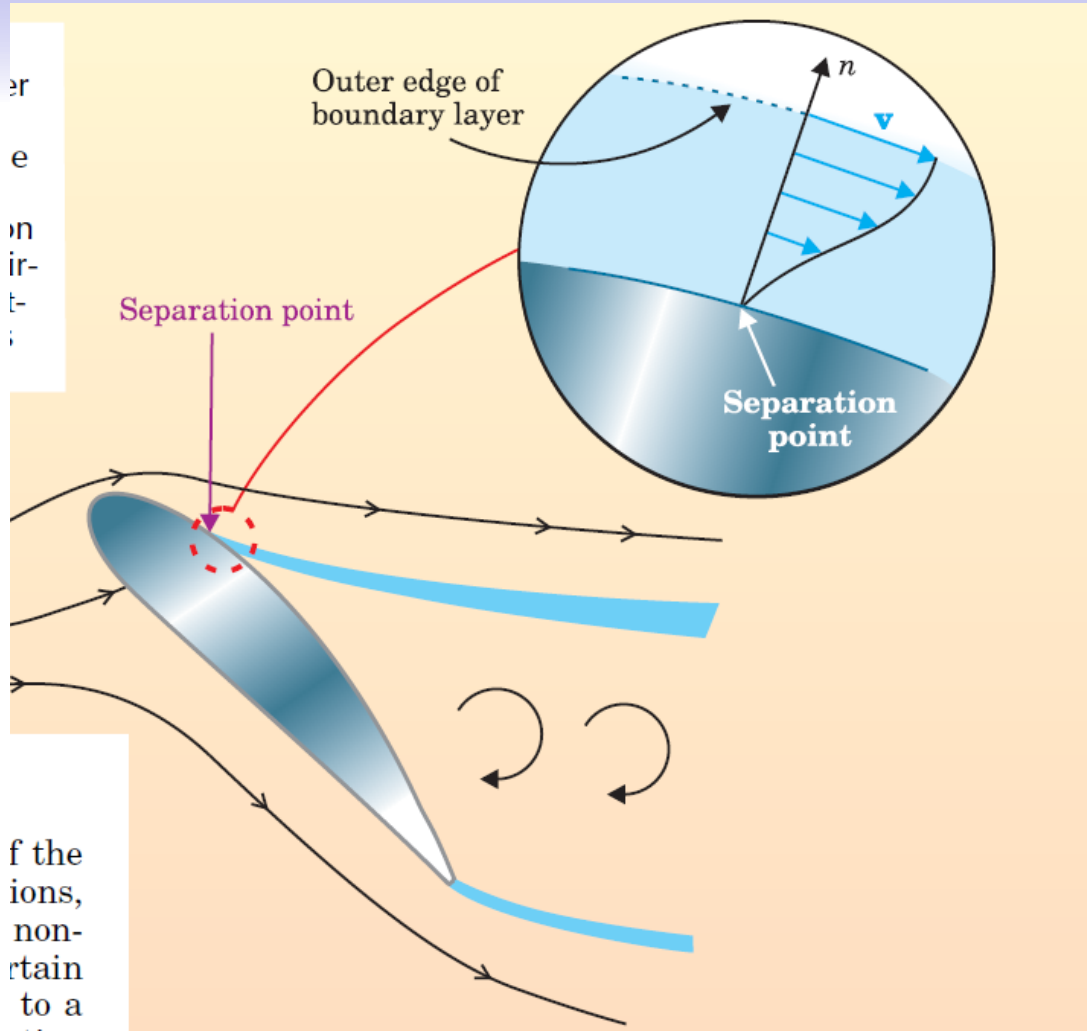


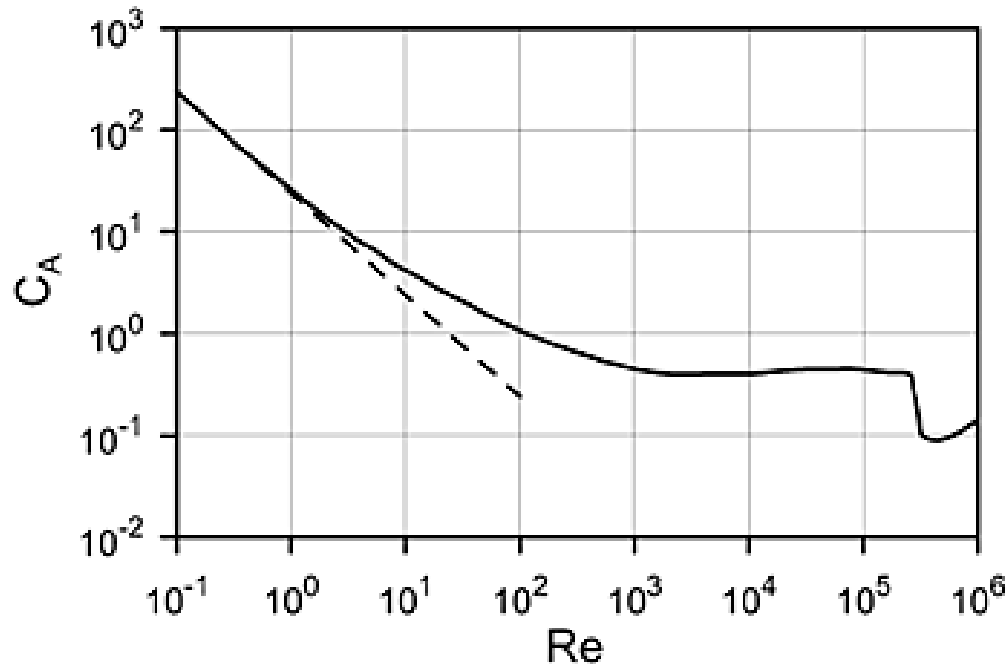
NASA Wake Vortex Study at Wallops Island
NASA Langley Research Center

5/4/1990

Image # EL-1996-00130

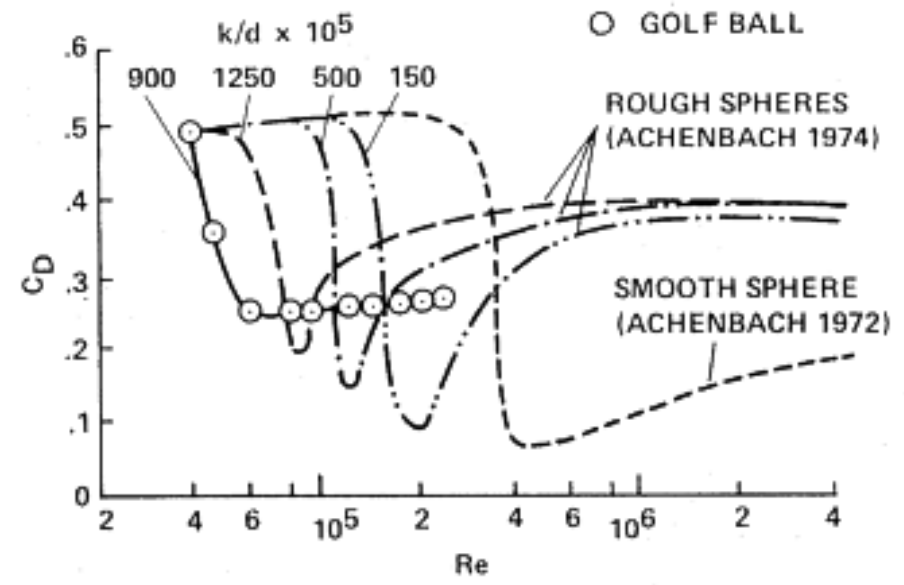
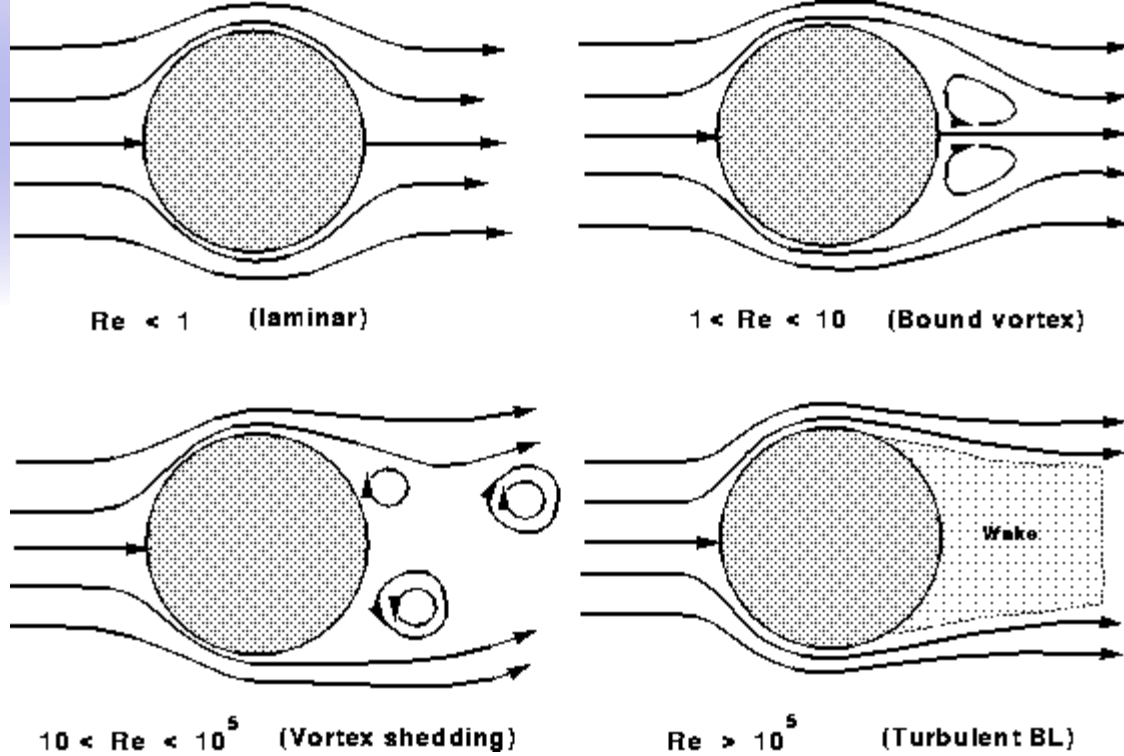


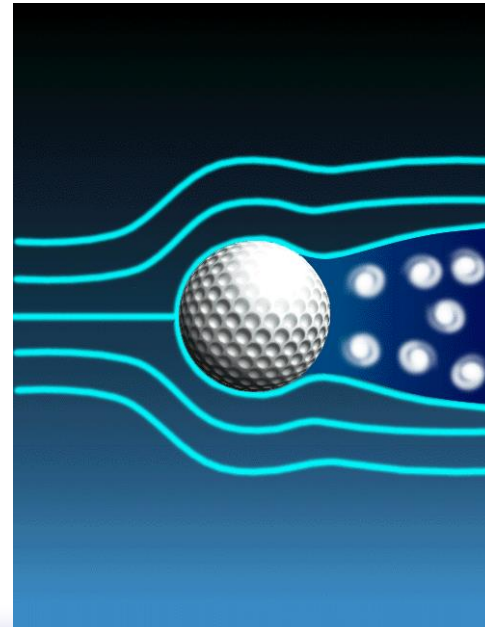
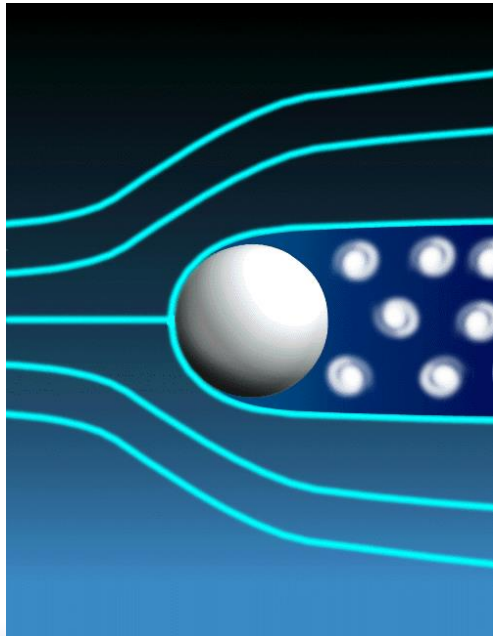
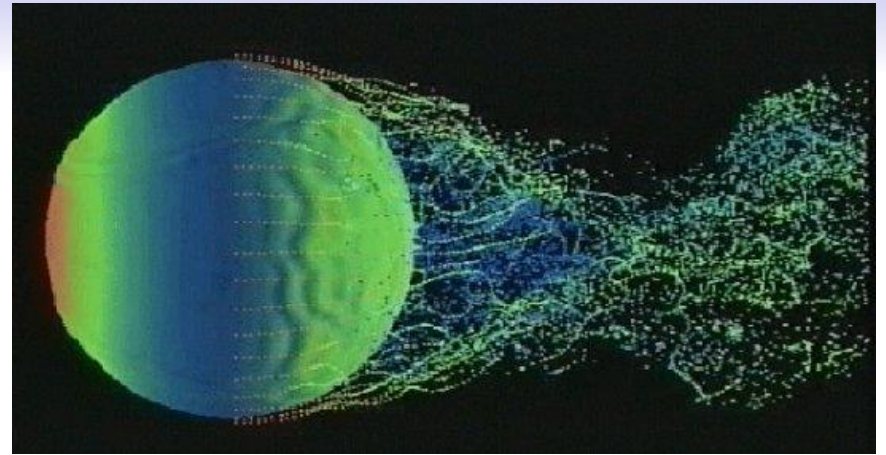
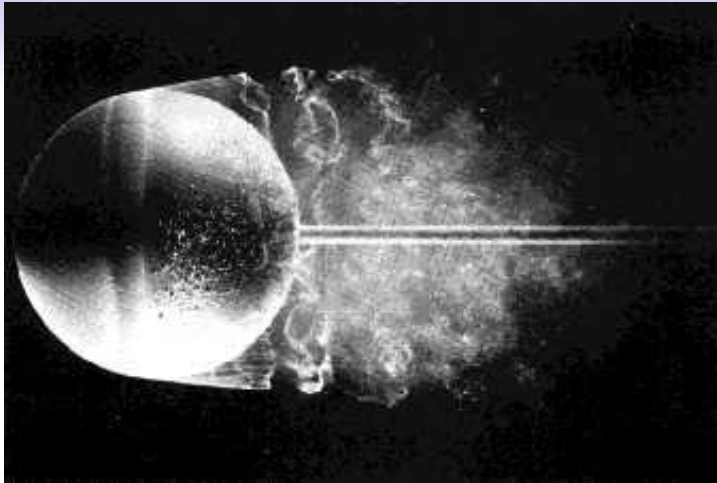




$$F_A = \frac{1}{2} C_A \rho A V^2$$

Figura 1 - Coeficiente de arrasto de uma esfera lisa, em função do número de Reynolds. A linha cheia é o resultado de medidas realizadas em túneis de vento. A linha tracejada corresponde à fórmula de Stokes (força de arrasto proporcional a V)





The Magnus Effect.

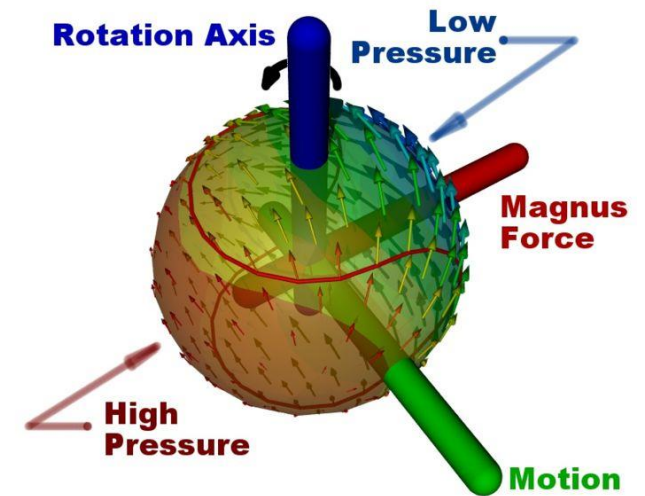
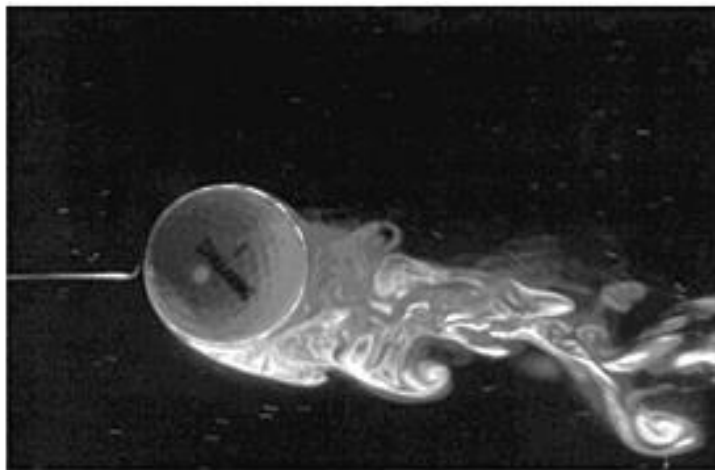
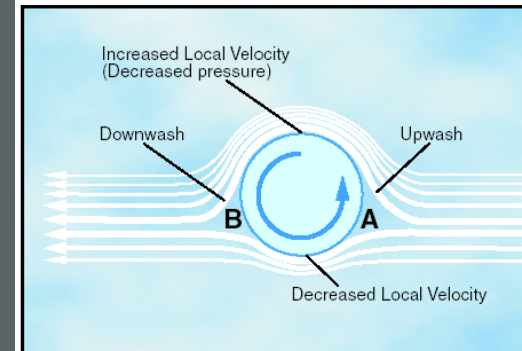
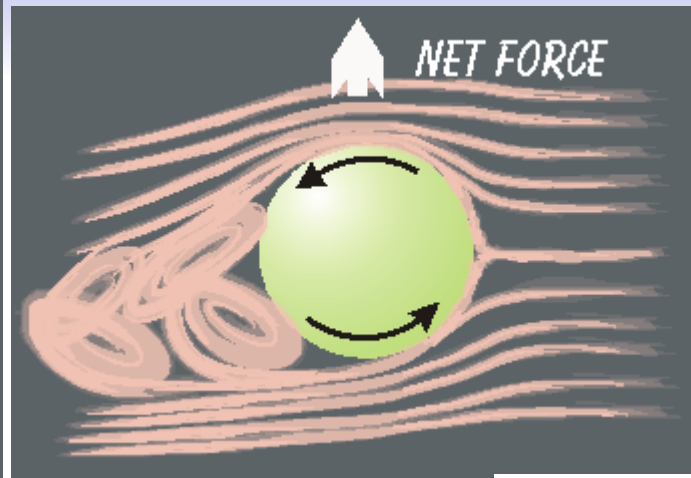
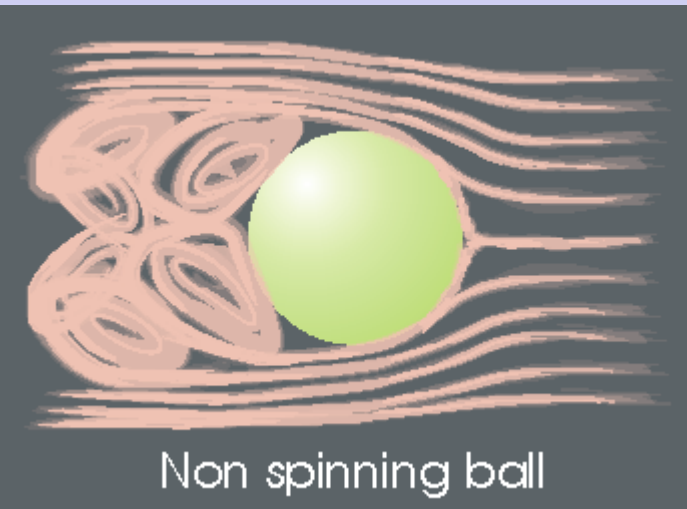
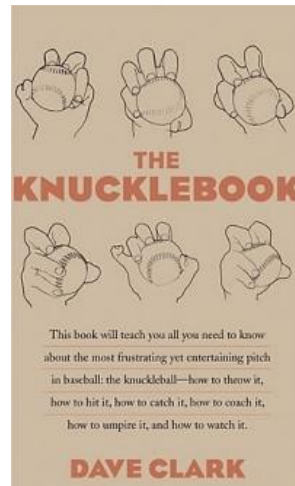


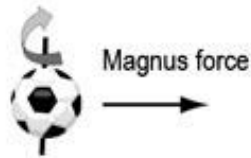
Figura 5 - Separação da camada limite em uma bola girando no sentido horário [17].

- Video: Daisuke Matsuzaka throws a gyroball



As Bert Blyleven, Three-Finger Mordecai Brown, and Joan Joyce (of the Stratford, nee Raybestos, Brakettes, maybe the best of this elite company) could tell you, a curveball is more than just the physics behind the Magnus

Pure sidespin: ball moves strongly left to right



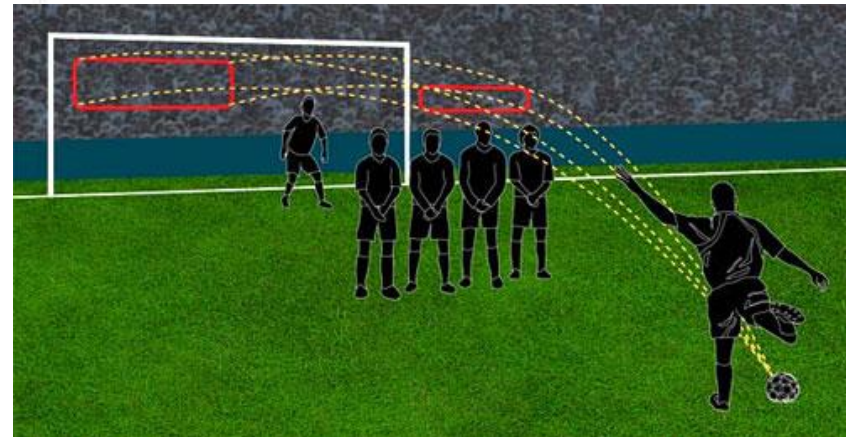
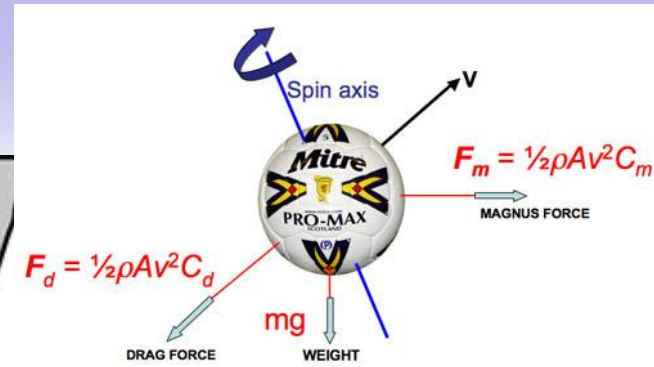
Partial sidespin: ball rises less steeply, moves left to right



Pure backspin: ball rises steeply



Formation of spin: foot contact at side of ball for maximum sidespin



Resolução Formal das Equações

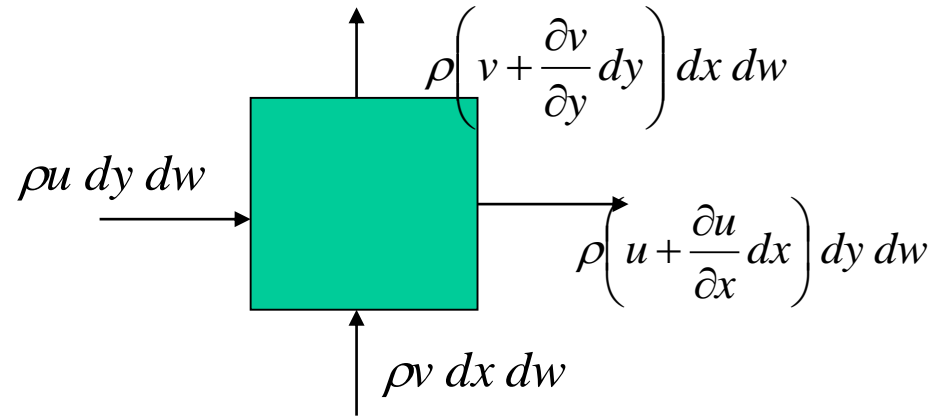
- Conservação da massa
- Energia
- 2º Lei de Newton
- Espécie Química
- 2º Lei da Termodinâmica

Para um escoamento 2-D estacionário,
incompressível e com propriedades constantes

- Conservação da Massa

$$0 = \frac{\partial}{\partial t} \int_{VC} \rho dV + \int_{SC} \rho \vec{V} \cdot d\vec{A}$$

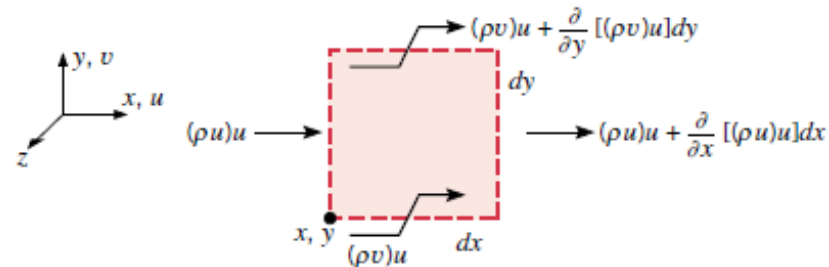
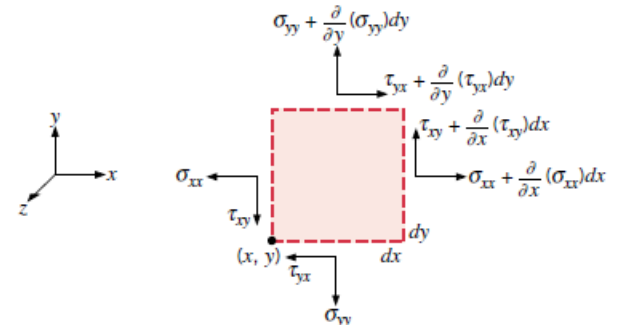
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$



- 2ª Lei de Newton

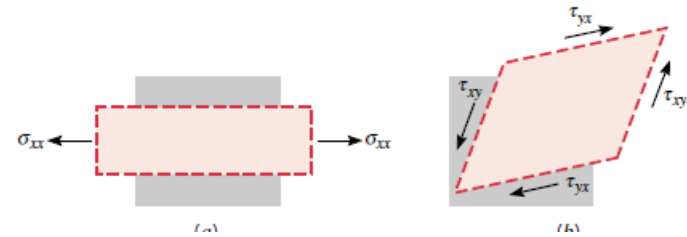
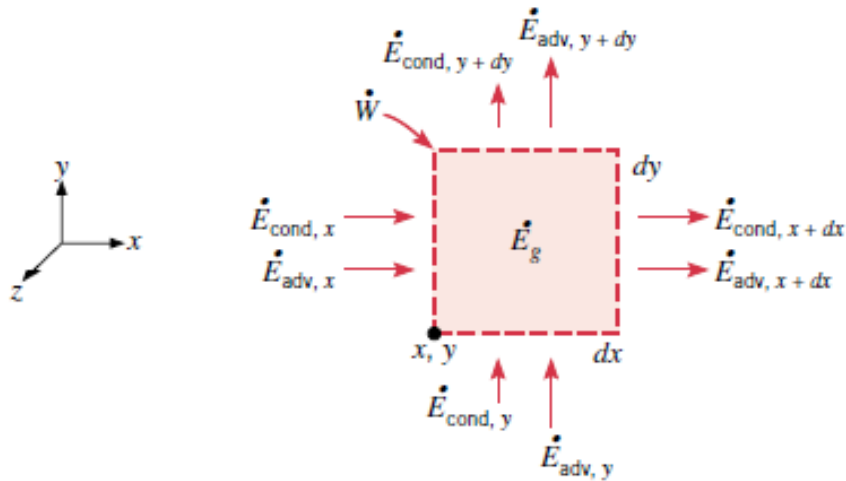
$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \rho g_x$$

$$\rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \rho g_y$$



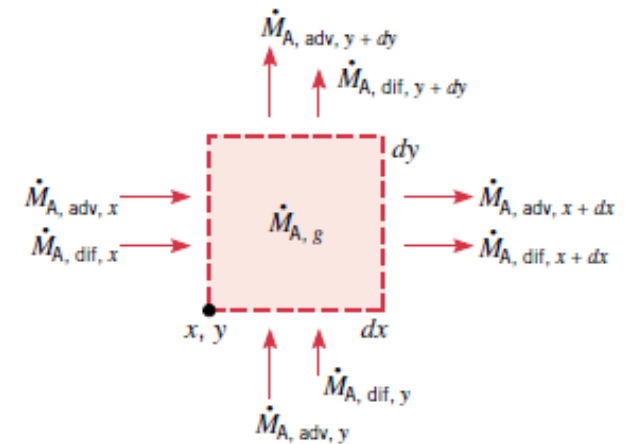
- Energia

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \mu \left\{ \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + 2 \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right] \right\} + \dot{q}$$



- Concentração

$$u \frac{\partial C_A}{\partial x} + v \frac{\partial C_A}{\partial y} = D_{AB} \left(\frac{\partial^2 C_A}{\partial x^2} + \frac{\partial^2 C_A}{\partial y^2} \right) + \dot{N}_A$$



Conjunto de Equações

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \rho g_x$$

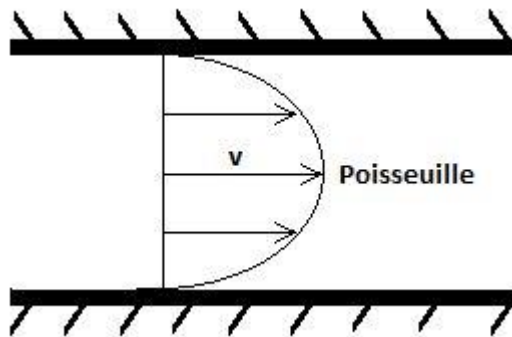
$$\rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \rho g_y$$

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \mu \left\{ \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + 2 \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right] \right\} + \dot{q}$$

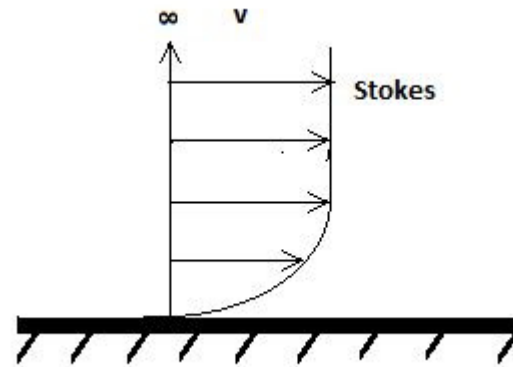
$$u \frac{\partial C_A}{\partial x} + v \frac{\partial C_A}{\partial y} = D_{AB} \left(\frac{\partial^2 C_A}{\partial x^2} + \frac{\partial^2 C_A}{\partial y^2} \right) + \dot{N}_A$$

Padrões de escoamento

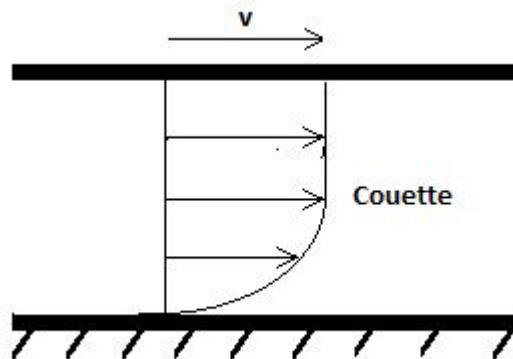
Escoamento de Poiseuille



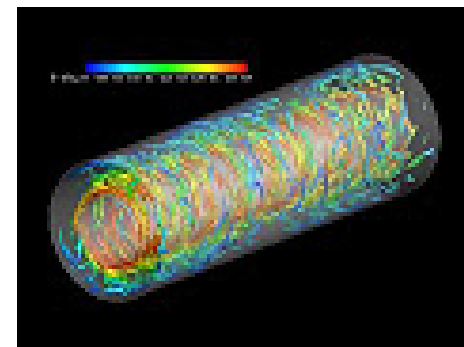
Escoamento de Stokes



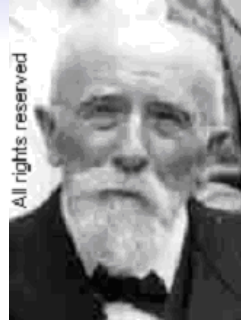
Escoamento de Couette



Escoamento de Taylor-Couette



Escoamento de Couette



Maurice Marie Alfred Couette (1858 -1943)

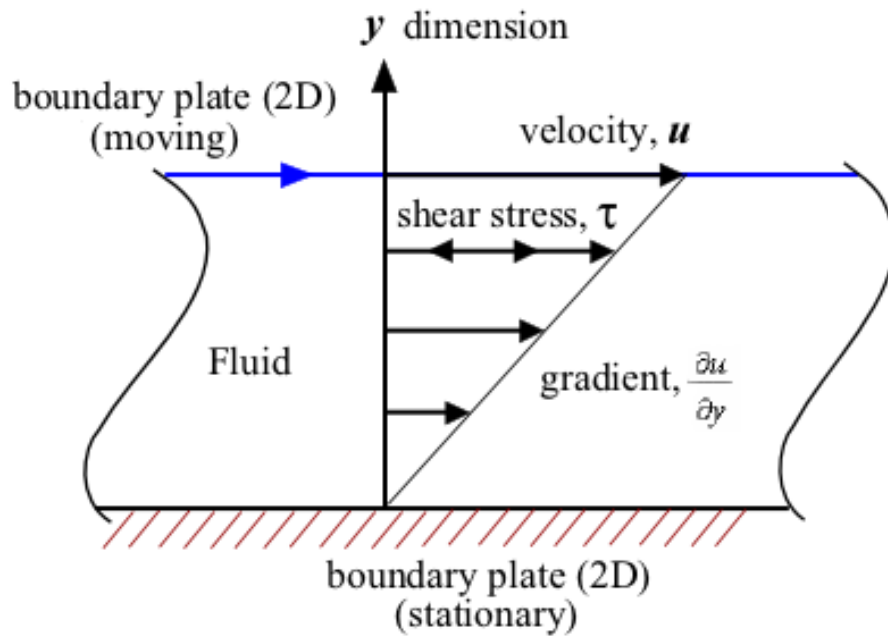
*Professor da Université Catholique de l'Ouest
Angers, France*



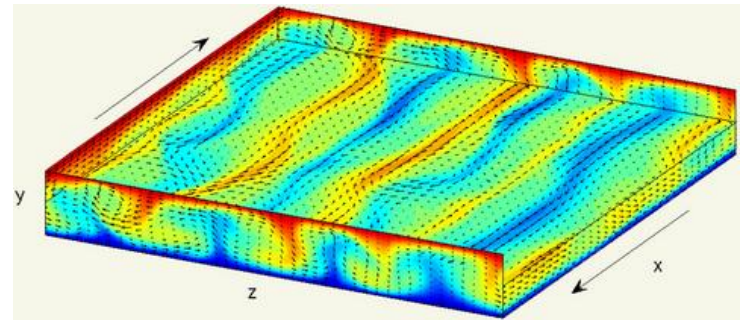
Figure 2. Maurice Couette's concentric cylinder apparatus (viscometer) (1888)

http://rheologie.ujf-grenoble.fr/Couette/gfr_Couettepiou.pdf

Escoamento de Couette

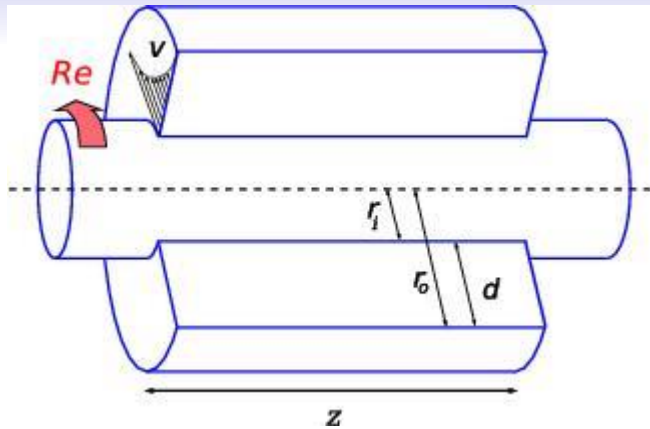


Plane Couette turbulence in a periodic cell with large aspect ratio

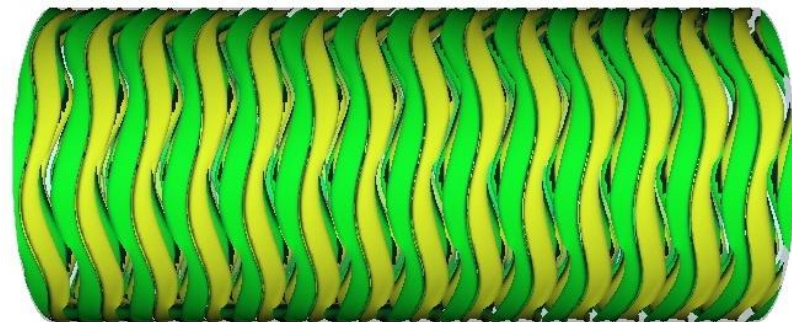
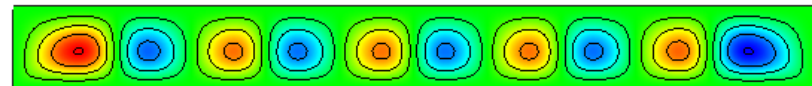
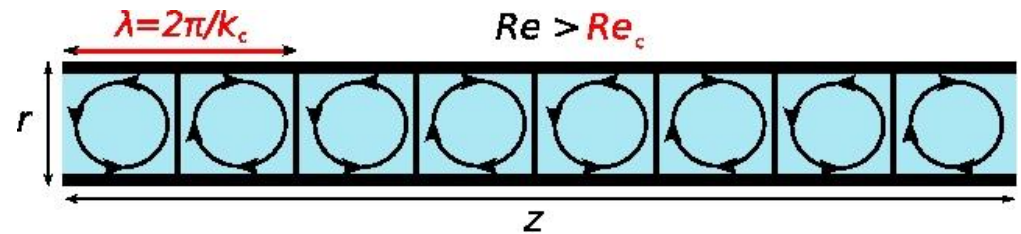


http://www.cns.gatech.edu/~gibson/PCF-movies/bigbox_rand.mp4

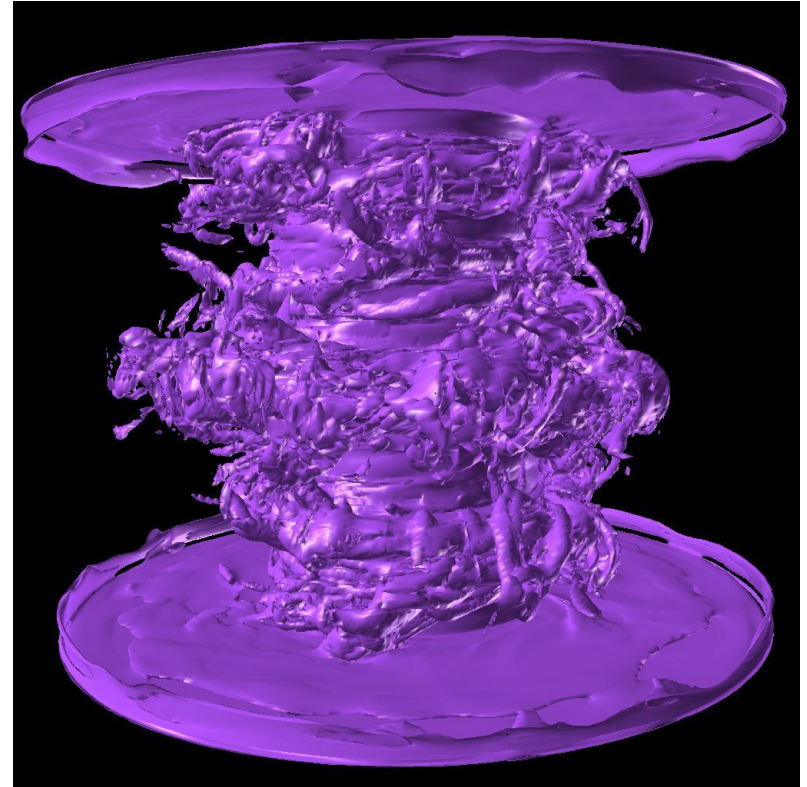
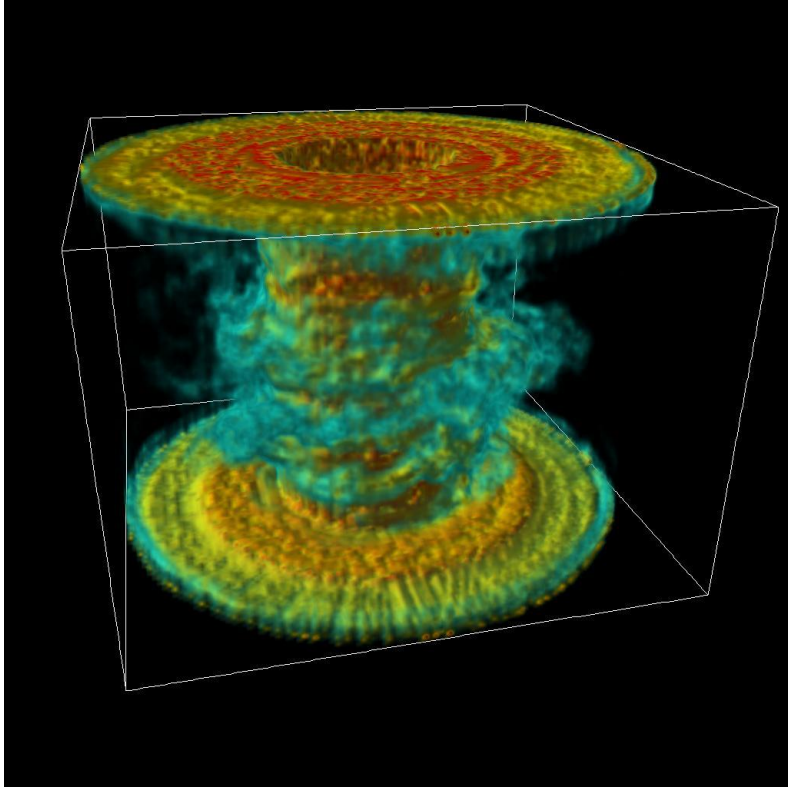
Escoamento de Taylor-Couette



www-fa.upc.es/websfa/fluids/marc/img

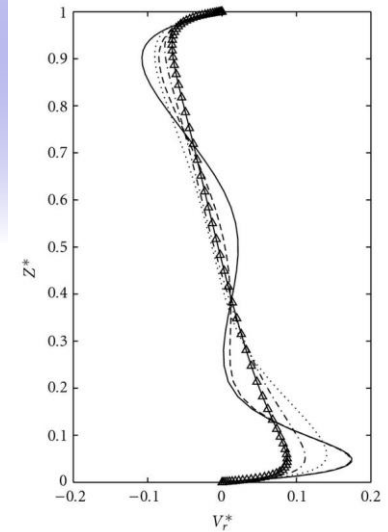
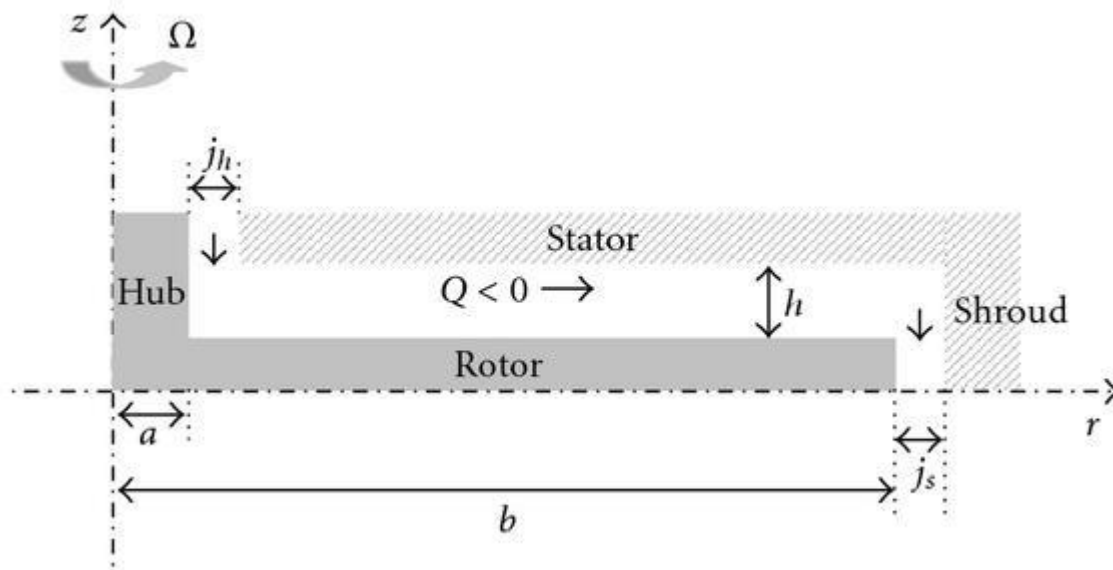


Exemplo - Escoamento de Taylor Couette

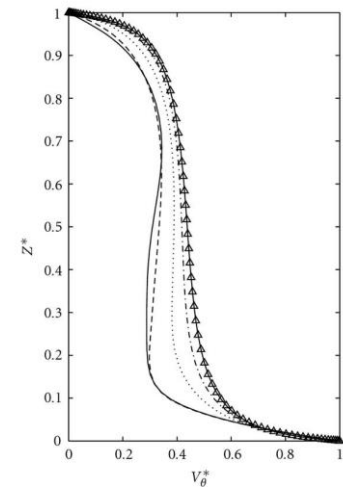


- http://flash.uchicago.edu/~cattaneo/Pages/image_gallery.htm

Aplicações

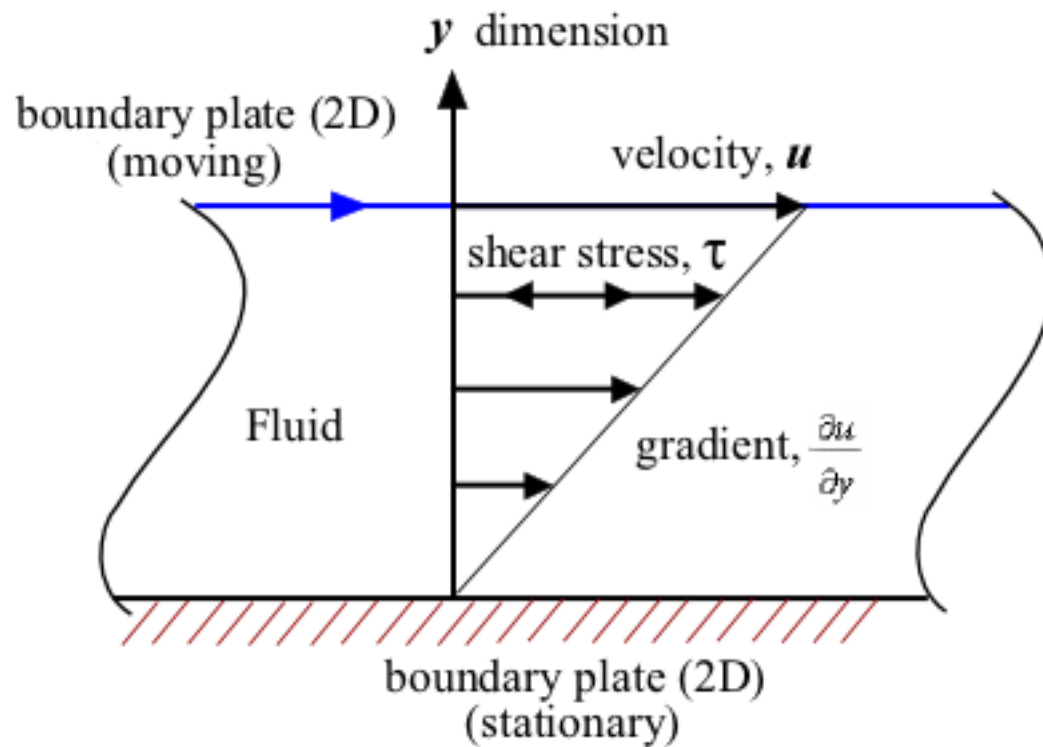


— $r^* = 0.31$ - - - $r^* = 0.7$
 - - - $r^* = 0.44$ -▲- $r^* = 0.83$
 $r^* = 0.57$



— $r^* = 0.31$ - - - $r^* = 0.7$
 - - - $r^* = 0.44$ -▲- $r^* = 0.83$
 $r^* = 0.57$

Escoamento de Couette



Conjunto de Equações

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \rho g_x$$

$$\rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \rho g_y$$

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \mu \left\{ \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + 2 \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right] \right\} + \dot{q}$$

$$u \frac{\partial C_A}{\partial x} + v \frac{\partial C_A}{\partial y} = D_{AB} \left(\frac{\partial^2 C_A}{\partial x^2} + \frac{\partial^2 C_A}{\partial y^2} \right) + \dot{N}_A$$

Com as aproximações p/ escoamento de Couette

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (v=0) \quad (\text{ECD=Escoamento completamente desenvolvido})$$

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \rho g_x$$

(CM, ECD) (v=0) sem ∇p (CM) sem F_{Bx}

$$\rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = - \frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \rho g_y$$

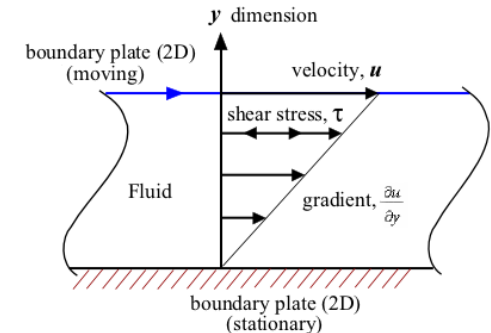
(v=0) (v=0) sem F_{By}

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \mu \left\{ \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + 2 \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right] \right\} + \dot{q}$$

(ECD) (v=0) (ECD) (v=0) (CM) (v=0) sem geração de calor

$$u \frac{\partial C_A}{\partial x} + v \frac{\partial C_A}{\partial y} = D_{AB} \left(\frac{\partial^2 C_A}{\partial x^2} + \frac{\partial^2 C_A}{\partial y^2} \right) + \dot{N}_A$$

(ECD) (v=0) (ECD) sem geração de M



Finalmente para o escoamento de Couette

$$\frac{\partial u}{\partial x} = 0$$

$$\mu \left(\frac{\partial^2 u}{\partial y^2} \right) = 0$$

$$\frac{\partial p}{\partial y} = 0$$

$$k \left(+ \frac{\partial^2 T}{\partial y^2} \right) + \mu \left\{ \left(\frac{\partial u}{\partial y} \right)^2 \right\} = 0$$

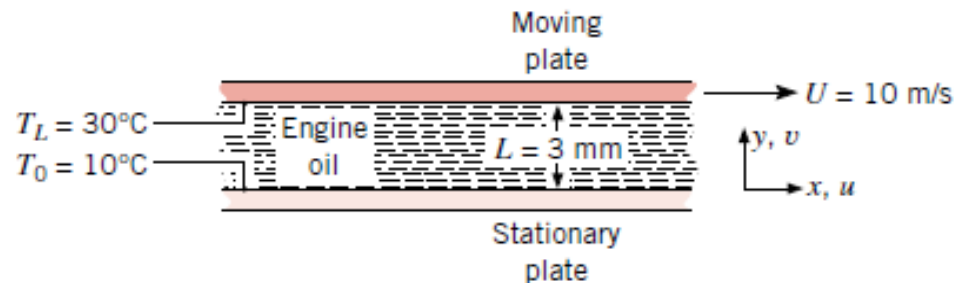
$$D_{AB} \left(+ \frac{\partial^2 C_A}{\partial y^2} \right) = 0$$

Known: Couette flow with heat transfer.

Find:

1. Form of the continuity equation.
2. Velocity distribution.
3. Temperature distribution.
4. Surface heat fluxes and maximum temperature for prescribed conditions.

Schematic:



Assumptions:

1. Steady-state conditions.
2. Two-dimensional flow (no variations in z).
3. Incompressible fluid with constant properties.
4. No body forces.
5. No internal energy generation.

Properties: Table A.8, engine oil (20°C): $\rho = 888.2 \text{ kg/m}^3$, $k = 0.145 \text{ W/m} \cdot \text{K}$, $\nu = 900 \times 10^{-6} \text{ m}^2/\text{s}$, $\mu = \nu\rho = 0.799 \text{ N} \cdot \text{s/m}^2$.

Analysis:

1. For an incompressible fluid (constant ρ) and parallel flow ($v = 0$), Equation D.1 reduces to

$$\frac{\partial u}{\partial x} = 0$$



The important implication of this result is that, although depending on y , the x velocity component u is independent of x . It may then be said that the velocity field is *fully developed*.

2. For two-dimensional, steady-state conditions with $v = 0$, $(\partial u / \partial x) = 0$, and $X = 0$, Equation D.2 reduces to

$$0 = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial y^2} \right)$$

However, in Couette flow, motion of the fluid is not sustained by the pressure gradient, $\partial p/\partial x$, but by an external force that provides for motion of the top plate relative to the bottom plate. Hence $(\partial p/\partial x) = 0$. Accordingly, the x -momentum equation reduces to

$$\frac{\partial^2 u}{\partial y^2} = 0$$

The desired velocity distribution may be obtained by solving this equation. Integrating twice, we obtain

$$u(y) = C_1 y + C_2$$

where C_1 and C_2 are the constants of integration. Applying the boundary conditions

$$u(0) = 0 \quad u(L) = U$$

it follows that $C_2 = 0$ and $C_1 = U/L$. The velocity distribution is then

$$u(y) = \frac{y}{L} U \quad \triangleleft$$

3. The energy equation (D.4) may be simplified for the prescribed conditions. In particular, with $v = 0$, $(\partial u/\partial x) = 0$, and $\dot{q} = 0$, it follows that

$$\rho c_p u \frac{\partial T}{\partial x} = k \frac{\partial^2 T}{\partial x^2} + k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2$$

However, because the top and bottom plates are at uniform temperatures, the temperature field must also be fully developed, in which case $(\partial T/\partial x) = 0$. The appropriate form of the energy equation is then

$$0 = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2$$

The desired temperature distribution may be obtained by solving this equation. Rearranging and substituting for the velocity distribution,

$$k \frac{d^2 T}{dy^2} = -\mu \left(\frac{du}{dy} \right)^2 = -\mu \left(\frac{U}{L} \right)^2$$

Integrating twice, we obtain

$$T(y) = -\frac{\mu}{2k} \left(\frac{U}{L} \right)^2 y^2 + C_3 y + C_4$$

The constants of integration may be obtained from the boundary conditions

$$T(0) = T_0 \quad T(L) = T_L$$

in which case

$$C_4 = T_0 \quad \text{and} \quad C_3 = \frac{T_L - T_0}{L} + \frac{\mu U^2}{2k L}$$

and

$$T(y) = T_0 + \frac{\mu}{2k} U^2 \left[\frac{y}{L} - \left(\frac{y}{L} \right)^2 \right] + (T_L - T_0) \frac{y}{L}$$



4. Knowing the temperature distribution, the surface heat fluxes may be obtained by applying Fourier's law. Hence

$$q_y'' = -k \frac{dT}{dy} = -k \left[\frac{\mu}{2k} U^2 \left(\frac{1}{L} - \frac{2y}{L^2} \right) + \frac{T_L - T_0}{L} \right]$$

At the bottom and top surfaces, respectively, it follows that

$$q_0'' = -\frac{\mu U^2}{2L} - \frac{k}{L}(T_L - T_0) \quad \text{and} \quad q_L'' = +\frac{\mu U^2}{2L} - \frac{k}{L}(T_L - T_0)$$

Hence, for the prescribed numerical values,

$$q_0'' = -\frac{0.799 \text{ N} \cdot \text{s/m}^2 \times 100 \text{ m}^2/\text{s}^2}{2 \times 3 \times 10^{-3} \text{ m}} - \frac{0.145 \text{ W/m} \cdot \text{K}}{3 \times 10^{-3} \text{ m}} (30 - 10)^\circ\text{C}$$

$$q_0'' = -13,315 \text{ W/m}^2 - 967 \text{ W/m}^2 = -14.3 \text{ kW/m}^2 \quad \triangleleft$$

$$q_L'' = +13,315 \text{ W/m}^2 - 967 \text{ W/m}^2 = 12.3 \text{ kW/m}^2 \quad \triangleleft$$

The location of the maximum temperature in the oil may be found from the requirement that

$$\frac{dT}{dy} = \frac{\mu}{2k} U^2 \left(\frac{1}{L} - \frac{2y}{L^2} \right) + \frac{T_L - T_0}{L} = 0$$

Solving for y , it follows that

$$y_{\max} = \left[\frac{k}{\mu U^2} (T_L - T_0) + \frac{1}{2} \right] L$$

