# TM 332 - Estática II

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UFPR

TM-332, 2012

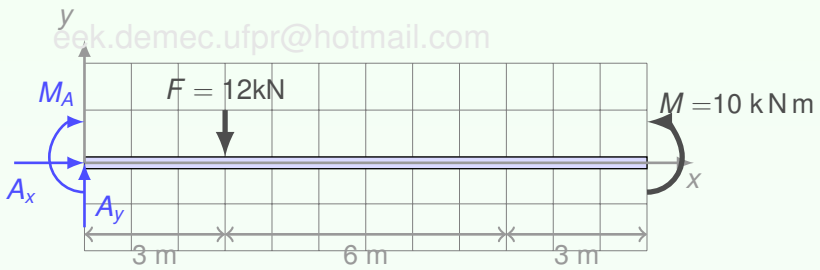
# TÓPICOS

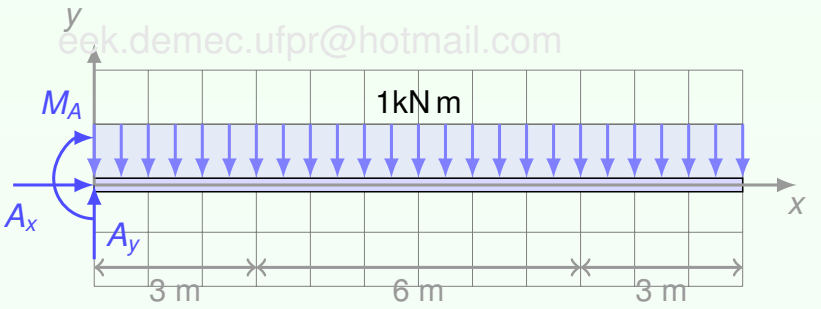
## Correção do diagrama M e V

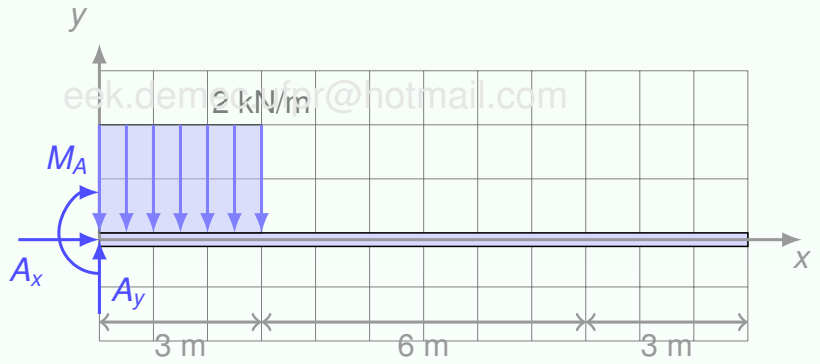
## RESOLVENDO CADA GRUPO DE CARREGAMENTO

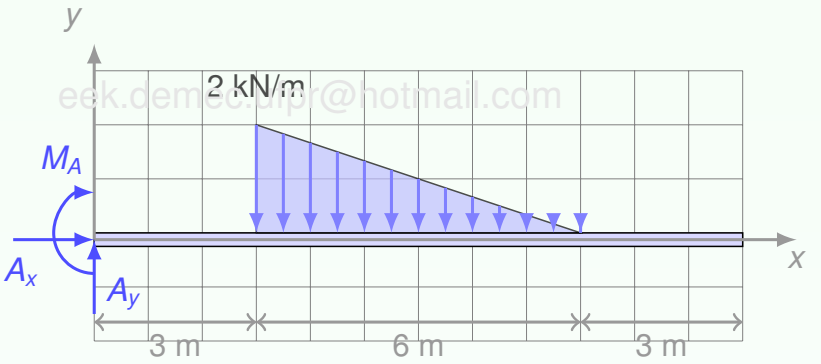
# Correção do diagrama M e V

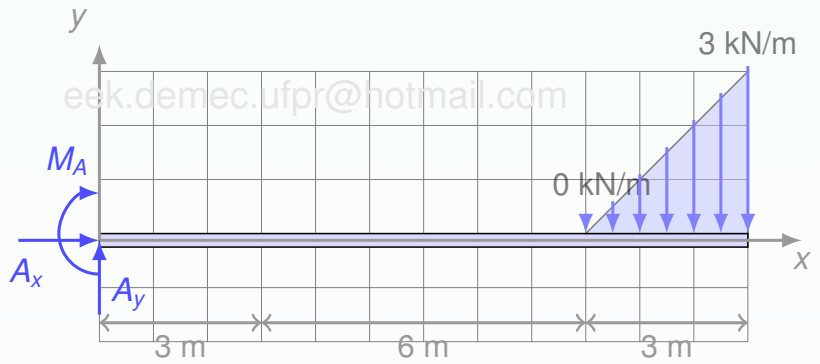
O problema apresentado é:


y
$M_A$
$F = 12\text{kN}$
$M = 10\ \text{kN m}$
x
$A_x$
$A_y$
3 m
6 m
3 m


y
$M_A$
1kN m
x
$A_x$
$A_y$
3 m
6 m
3 m


y
2 kN/m
$M_A$
x
$A_x$
$A_y$
3 m
6 m
3 m

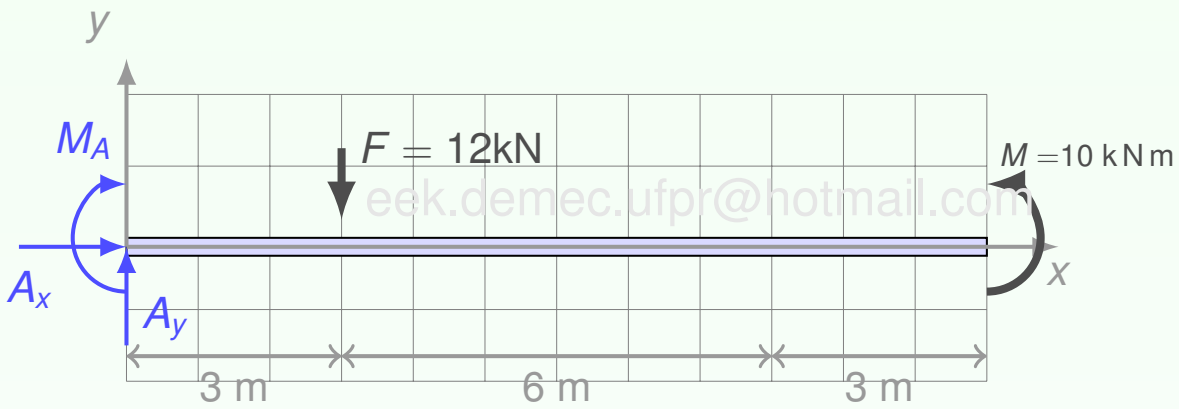

y
2 kN/m
$M_A$
x
$A_x$
$A_y$
3 m
6 m
3 m


y
3 kN/m
$M_A$
0 kN/m
x
$A_x$
$A_y$
3 m
6 m
3 m

# TÓPICOS

Correção do diagrama M e V

## RESOLVENDO CADA GRUPO DE CARREGAMENTO

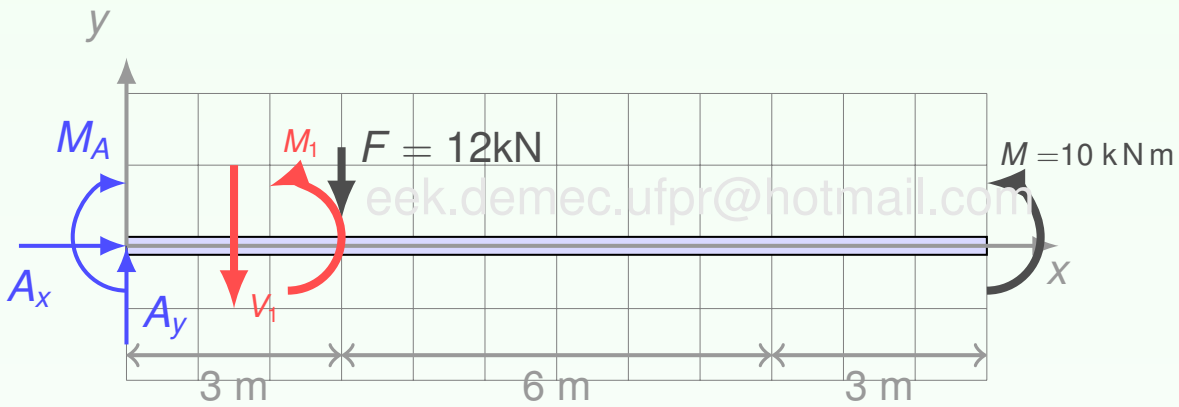





Cálculo das reações de apoio:

$$\sum M_A = 0 \rightsquigarrow -M_A - 3 \cdot 12 + 10 = 0 \rightsquigarrow \boxed{M_A = -26\text{kN m}}$$

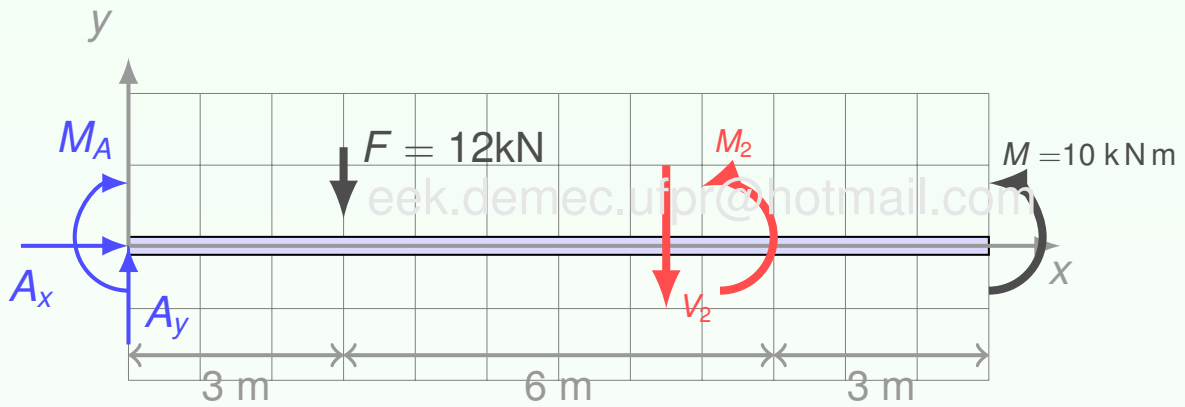
$$\sum F_y = 0 \rightsquigarrow \boxed{A_y = 120\text{kN}}$$




Cálculo das forças internas para $x \in [0, 3]$:

$$\sum M_C = 0 \rightsquigarrow M_1 - M_A - x\,A_y = 0 \rightsquigarrow \boxed{M_1 = (12x - 26)\text{kN m}}$$

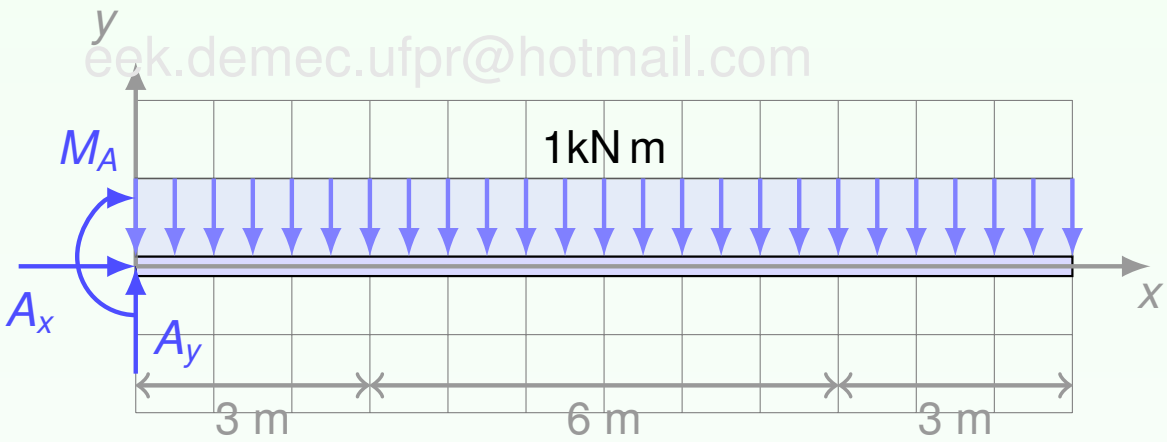
$$\sum F_y = 0 \rightsquigarrow \boxed{V_1 = 12\text{kN}}$$




Cálculo das forças internas para $x \in [3, 12]$:

$$\sum M_C = 0 \rightsquigarrow M_2 - M_A - x\,A_y + (x-3)12 = 0 \rightsquigarrow \boxed{M_2 = 10\text{kN m}}$$

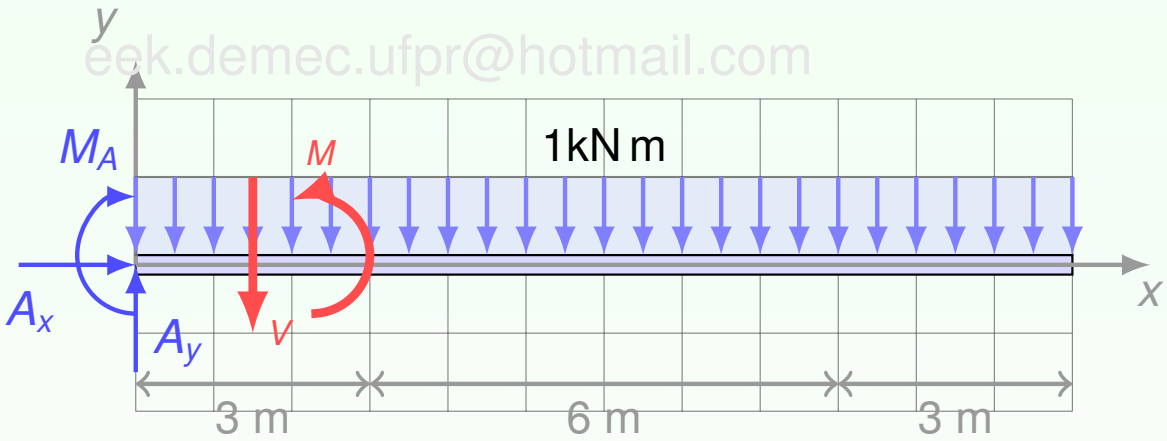
$$\sum F_y = 0 \rightsquigarrow \boxed{V_2 = 0\text{kN}}$$




Cálculo das reações de apoio:

$$\sum M_A = 0 \rightsquigarrow -M_A - 6 \cdot 1 \cdot 12 = 0 \rightsquigarrow \boxed{M_A = -72\text{kN m}}$$

$$\sum F_y = 0 \rightsquigarrow \boxed{A_y = 12\text{kN}}$$

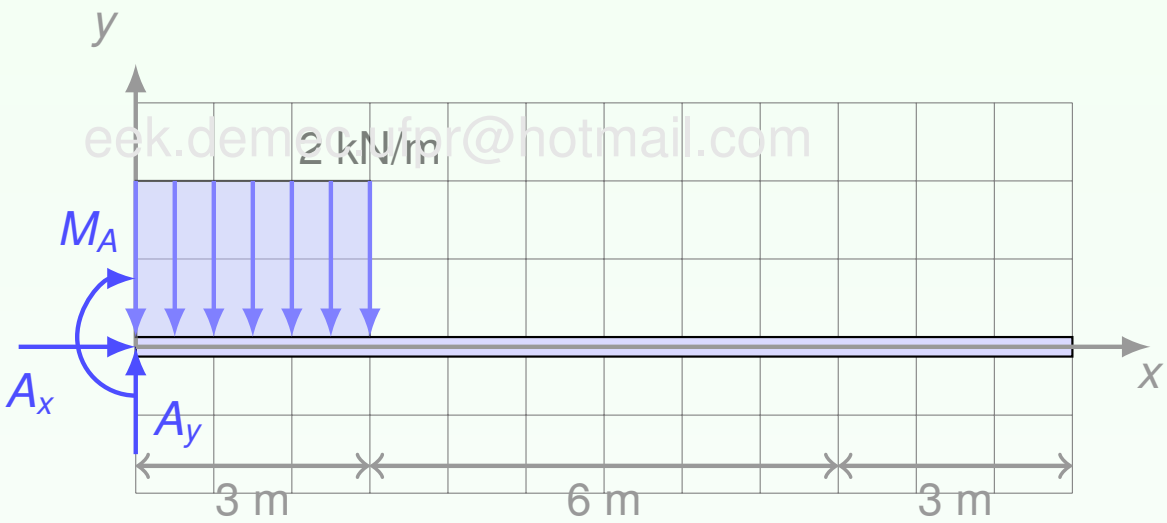



Cálculo das forças internas para $x \in [0, 12]$:

$$\sum M_C = 0 \rightsquigarrow M - M_A - x\,A_y - 1x\frac{x}{2} = 0$$

$$\boxed{M_1 = \left(72 + 12x + \frac{x^2}{2}\right)\text{kN m}}$$

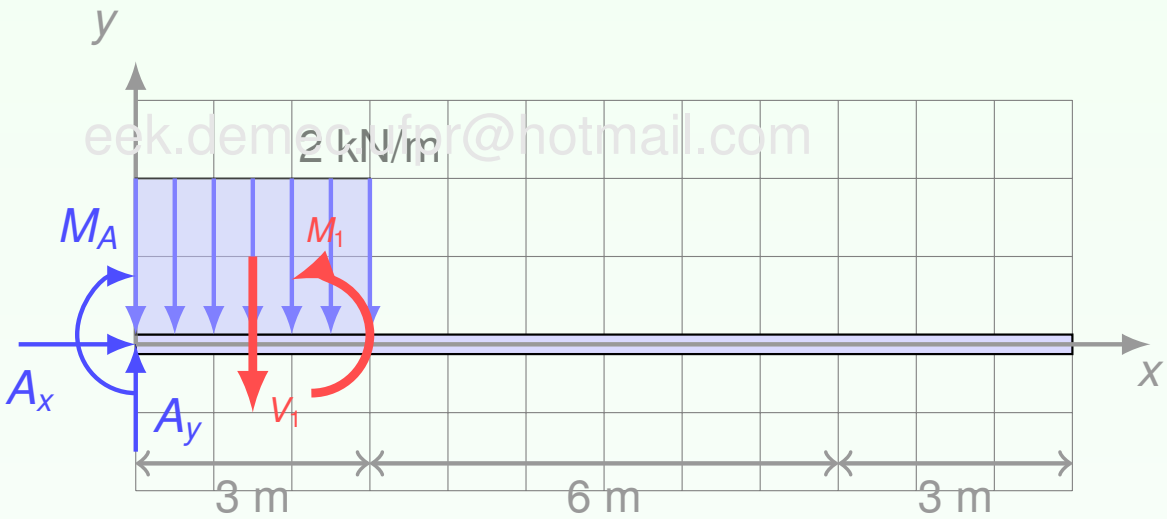
$$\sum F_y = 0 \rightsquigarrow V - A_y + 1x \rightsquigarrow \boxed{V = (12 - x)\text{kN}}$$




Cálculo das reações de apoio:

$$\sum M_A = 0 \rightsquigarrow -M_A - 3 \cdot (1.5) = 0 \rightsquigarrow \boxed{M_A = -4.5\text{kN m}}$$

$$\sum F_y = 0 \rightsquigarrow A_y = \frac{6 \cdot 2}{2} \rightsquigarrow \boxed{A_y = 3\text{kN}}$$

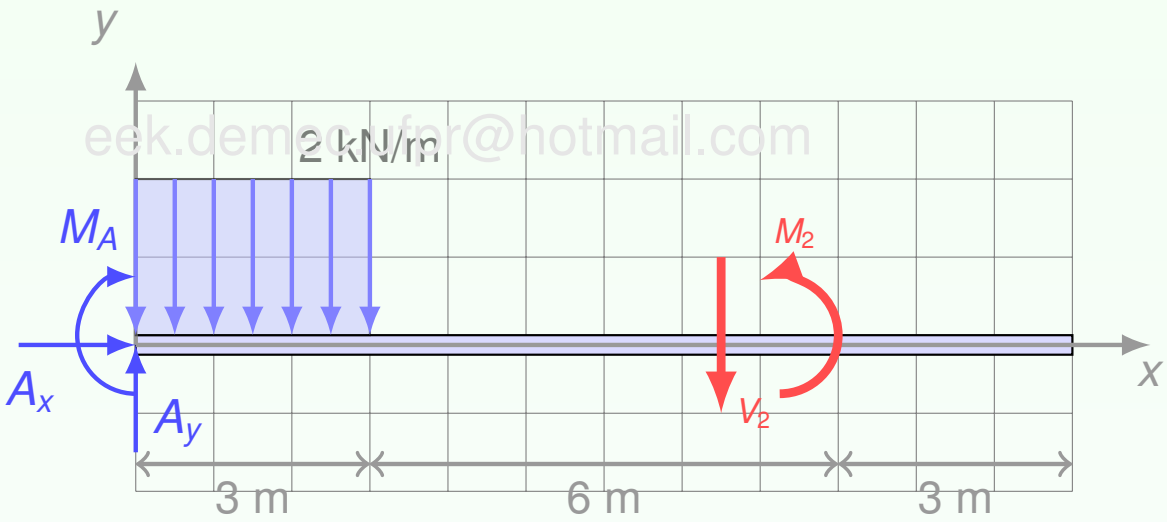



Cálculo das forças internas para $x \in [0, 3]$:

$$\sum M_C = 0 \rightsquigarrow M = 0$$

$$\boxed{M = \left(-4.5 + 3x + x^2\right) \text{kN m}}$$

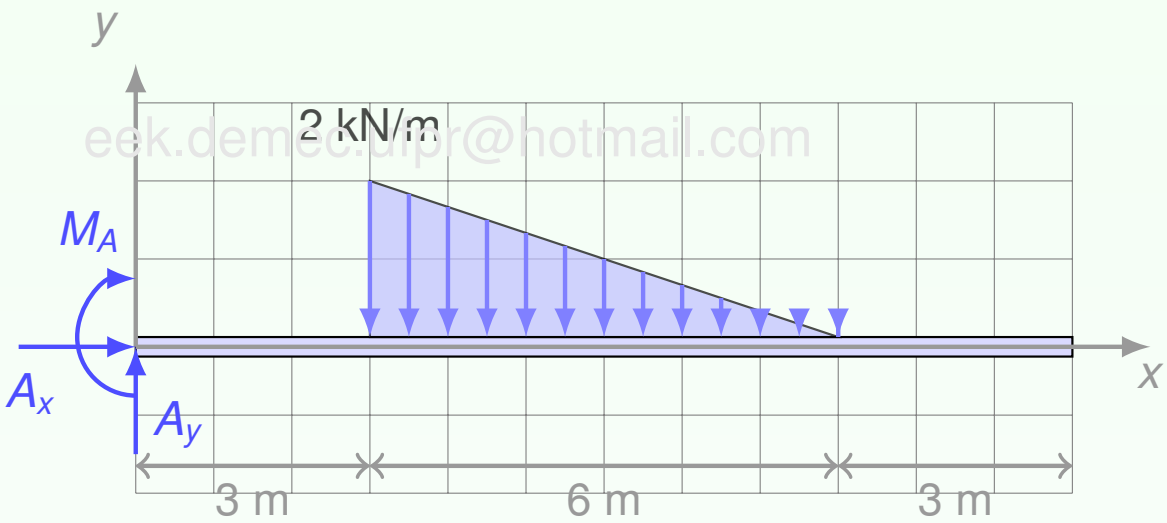
$$\sum F_y = 0 \rightsquigarrow V - A_y + 2x \rightsquigarrow \boxed{V = (3 - 2x)\text{kN}}$$




Cálculo das forças internas para $x \in [3, 12]$:

$$\sum M_C = 0 \rightsquigarrow \boxed{M_2 = 0}$$

$$\sum F_y = 0 \rightsquigarrow \boxed{V_2 = 0}$$

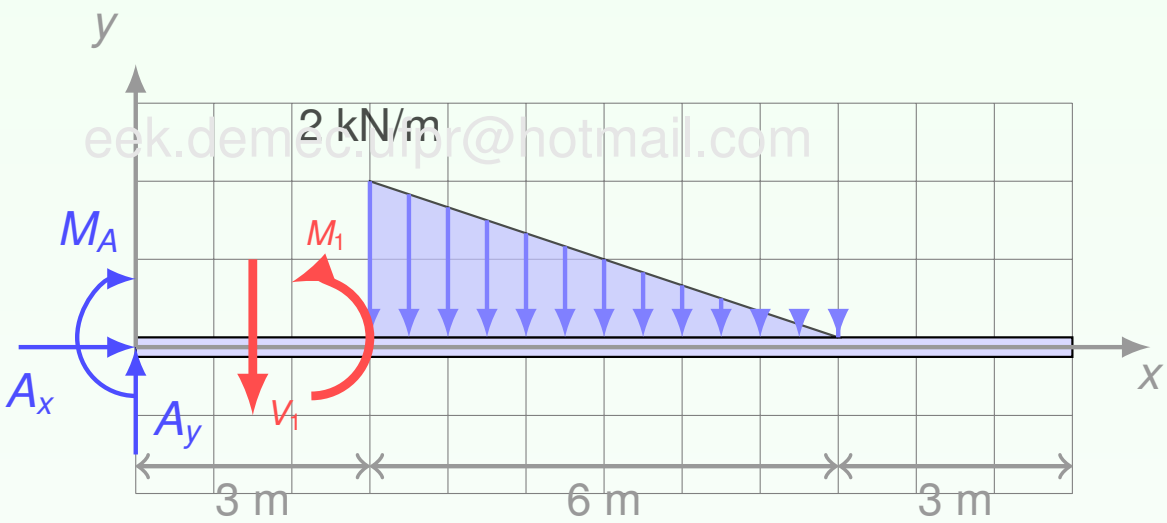





Cálculo das reações de apoio:

$$\sum M_A = 0 \rightsquigarrow -M_A - 6 \cdot (2+3) = 0 \rightsquigarrow \boxed{M_A = -30\text{kN m}}$$

$$\sum F_y = 0 \rightsquigarrow A_y = \frac{6 \cdot 2}{2} \rightsquigarrow \boxed{A_y = 6\text{kN}}$$

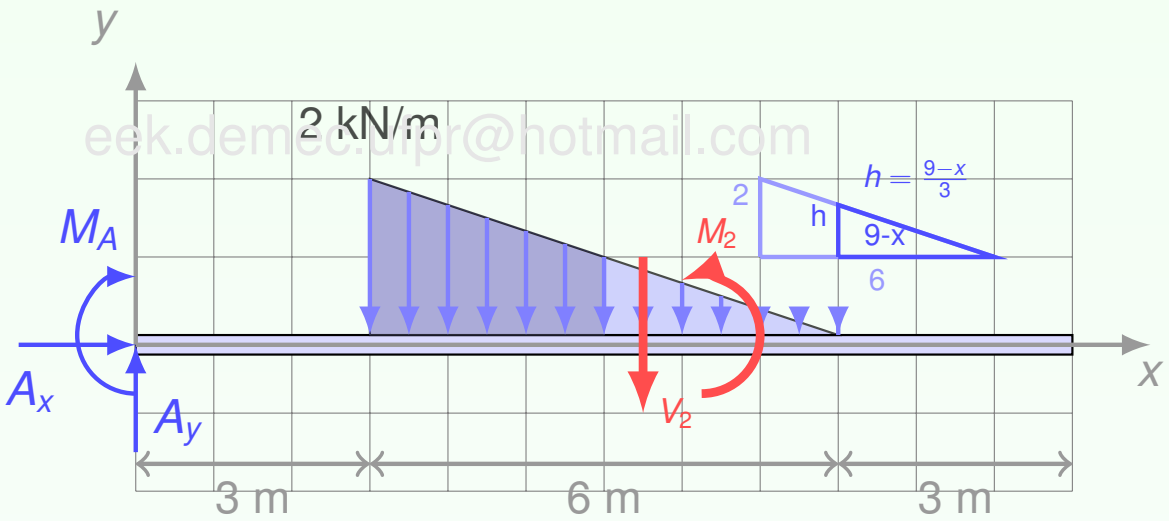



Cálculo das forças internas para $x \in [0, 3]$:

$$\sum M_C = 0 \rightsquigarrow M_1 - M_A - x\, A_y = 0$$

$$\boxed{M_1 = (-30 + 6x)\,\text{kN m}}$$

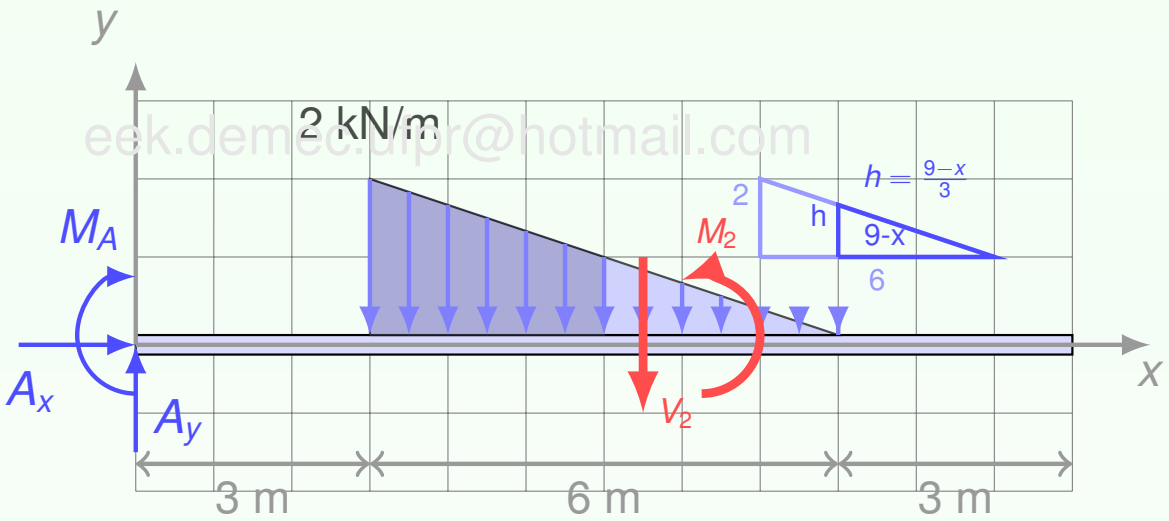
$$\sum F_y = 0 \rightsquigarrow V_1 - A_y \rightsquigarrow \boxed{V = 6\text{kN}}$$




Cálculo das forças internas para $x \in [3, 9]$:

$$\sum F_y = 0 \rightsquigarrow V_2 - A_y + \overbrace{\overbrace{(x-3)}^{\text{alt.}} \overbrace{\frac{\left(2 + \frac{9-x}{3}\right)}{2}}^{\text{base média}}}^{\text{Área}}$$

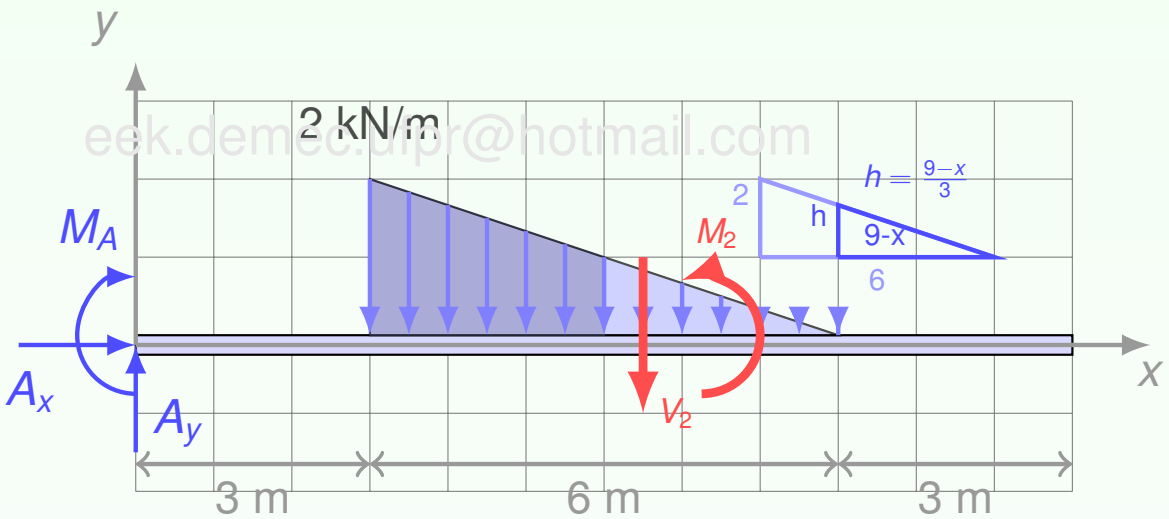
$$\boxed{V_2 = 6 - \frac{x-3}{2}\frac{15-x}{3}\text{kN}}$$




Cálculo das forças internas para $x \in [3, 9]$:

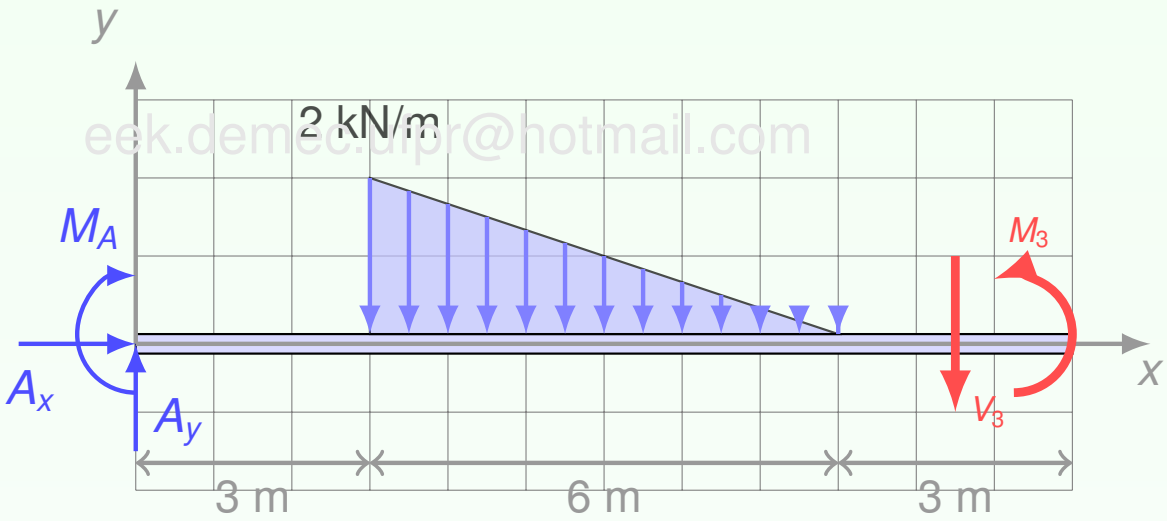
$$\sum M_C = 0$$

$$M_2 - M_A - x\,A_y - \overbrace{\left(\frac{2\,x}{3} + \frac{24}{x-15} + 3\right)}^{\text{braço}} \overbrace{\overbrace{(x-3)}^{alt.}\,\overbrace{\frac{\left(2 + \frac{9-x}{3}\right)}{2}}^{\text{base média}}}^{\text{Área}} = 0$$




Cálculo das forças internas para $x \in [3, 9]$:

$$\boxed{M_2 = -\frac{x^3}{9} + \frac{3\,x^2}{2} + 6\,x - \frac{81}{2}\,\text{kN m}}$$

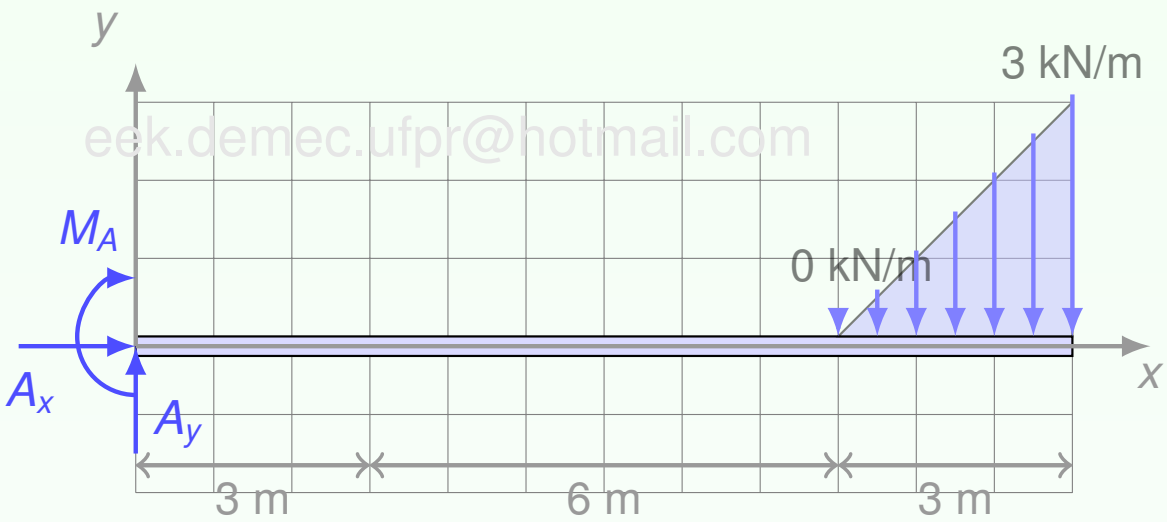



Cálculo das forças internas para $x \in [9, 12]$:

$$\sum M_C = 0 \rightsquigarrow M - M_A - x\, A_y - \overbrace{F_{eq}}^{\frac{2\,6}{2}}(x-7) = 0$$

$$\boxed{M_3 = 0}$$

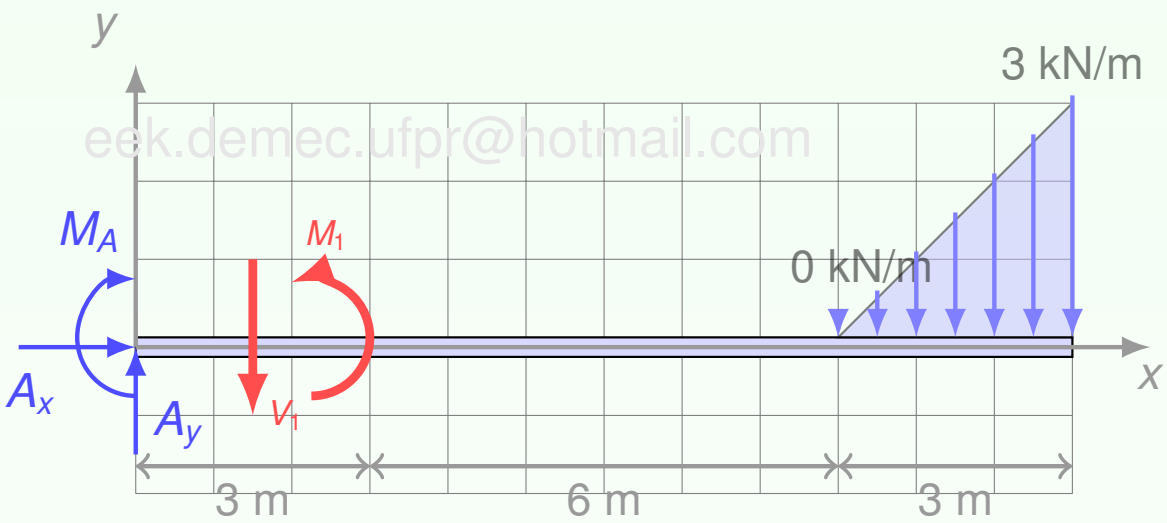
$$\sum F_y = 0 \rightsquigarrow \boxed{V_3 = 0}$$




Cálculo das reações de apoio:

$$\sum M_A = 0 \rightsquigarrow -M_A - 4.5 \cdot (2+9) = 0 \rightsquigarrow \boxed{M_A = -49.5\text{kN m}}$$

$$\sum F_y = 0 \rightsquigarrow A_y = \frac{3 \cdot 3}{2} \rightsquigarrow \boxed{A_y = 4.5\text{kN}}$$

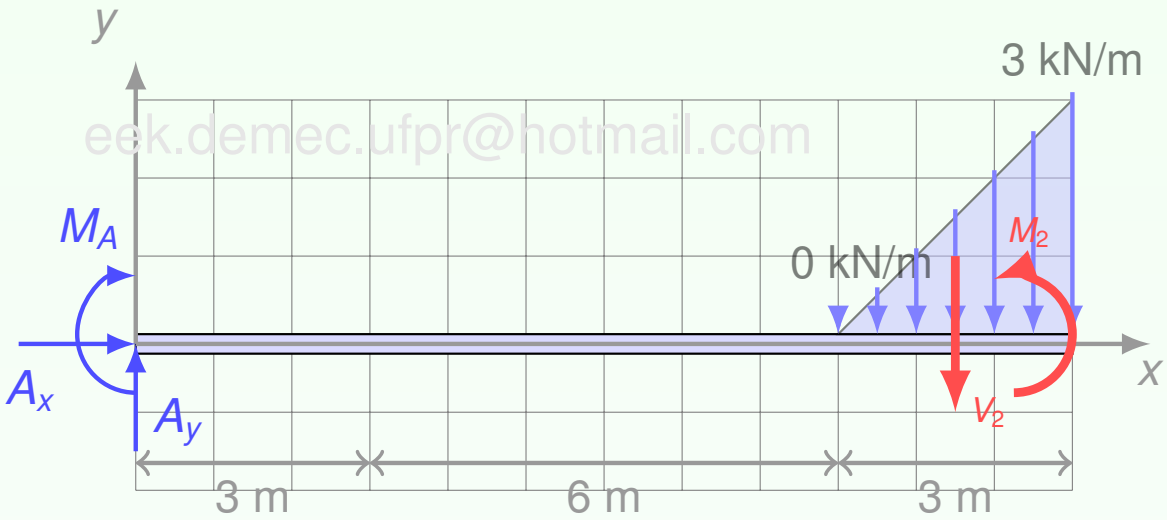



Cálculo das forças internas para $x \in [0, 9]$:

$$\sum M_C = 0 \rightsquigarrow M - M_A - x\, A_y = 0$$

$$\boxed{M_1 = (-49.5 + 4.5x)\,\text{kN m}}$$

$$\sum F_y = 0 \rightsquigarrow V_1 - A_y \rightsquigarrow \boxed{V_1 = 4.5\text{kN}}$$




Cálculo das forças internas para $x \in [9, 12]$:

$$\sum M_C = 0 \rightsquigarrow M - M_A - x\,A_y + \frac{(x-9)^2}{2}\frac{(x-9)}{3} = 0$$

$$\boxed{M_2 = \left(-49.5 + 4.5x - \frac{(x-9)^3}{6}\right)\text{kN m}}$$

$$\sum F_y = 0 \rightsquigarrow V - A_y + \frac{(x-9)^2}{2} \rightsquigarrow \boxed{V_2 = (4.5 - \frac{(x-9)^2}{2})\text{kN}}$$