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APPLICATION AND CORRECTION OF THE EXPONENTIAL WINDOW FOR FREQUENCY RESPONSE FUNCTIONS

W. FLADUNG AND R. ROST

Structural Dynamics Research Laboratory, University of Cincinnati, Cincinnati, OH 45221-0072, *U.S.A.*

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Applying windows to experimental data is a common practice in modal testing to minimise the effects of leakage, and the exponential window is used for the transient signals measured with impact testing and burst random excitation. Used properly, the exponential can minimise leakage errors on lightly damped signals and can also improve the signal-to-noise ratio of heavily damped signals. The time constant of the exponential window is specified typically by the user, and this paper discusses guidelines for specifying the window for both types of response signal. The effect of the exponential window is to increase the apparent damping of the measured system, and the correction for this effect on the estimated modal parameters is developed by utilising the shift property of the Laplace transform. In addition, the transfer function of a half-period sine pulse, which is a representative model of an impact force signal, is studied to show the need for, and consequences of, applying the exponential window to both the force signal and response signals. Finally, several numerical simulation test cases of an sdof system are presented to demonstrate the issues discussed in preceding sections.

1. INTRODUCTION

Windowing is a common signal processing technique available on all modern data acquisition systems. The exponential window is a time domain weighting function that has been developed for use with transient-type signals, such as those of impact testing and burst random excitation [1, 2]. The principal purpose of the exponential window is to reduce the effects of leakage, which are caused by violating the assumptions of the discrete Fourier transform, but it can also improve the signal-to-noise ratio of certain types of measured signals. A review of the exponential window and its applications is given in the next section.

Proper use of the exponential window requires that it be applied to all measured time signals. This means that it must be applied to input signals (in addition to the force window for the impact case) as well as to output signals. Although a frequency response function (FRF) is a division of the output spectrum by the input spectrum, applying the exponential window to both the input and output does not, however, cancel the effect of the window from the FRF. The effect of the exponential window on the estimated FRF is to increase the apparent damping of the measured system. This effect of the estimated modal parameters is predictable and the correction for the exponential window is presented in the paper [3].

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2. REVIEW OF THE EXPONENTIAL WINDOW

The exponential window is simply an exponential function as defined by either of the two forms in equation (1), where the parameter τ is the time constant of the exponential function. The time variable for the exponential function starts at zero, regardless if a pretrigger delay is used in the data acquisition.

$$w(t) = e^{-t/\tau} \quad \text{or} \quad w(t) = e^{-\beta t}, \qquad \text{where } \beta = \frac{1}{\tau} \tag{1}$$

When impact testing lightly damped structures, the purpose of the exponential window is to reduce the effects of leakage by forcing the data to meet the requirements of a completely observed transient more closely. By definition, a completely observed transient must start and end within the measured time record. For lightly damped systems, the response of the structure will typically continue beyond the time period of data collection as shown in Fig. 1(a). Since the response does not decay to near zero at the end of the time record, the exponential window is applied to reduce the signal at the end of the time record to approximately 1%. Figure 1(b) shows the same signal after the window has been applied and also shows the exponential window, the windowed signal more closely represents an observable transient.

When impact testing heavily damped systems, the response will often decay to zero much before the end of the time record, as seen in Fig. 2, and the measured signal is a completely observed transient. An exponential window is used in this case to improve the signal-to-noise ratio of the measured output by attenuating the noise on the signal after the response has decayed due to system damping. For heavily damped systems, the exponential window should follow the damping of the system, also shown in Fig. 2.

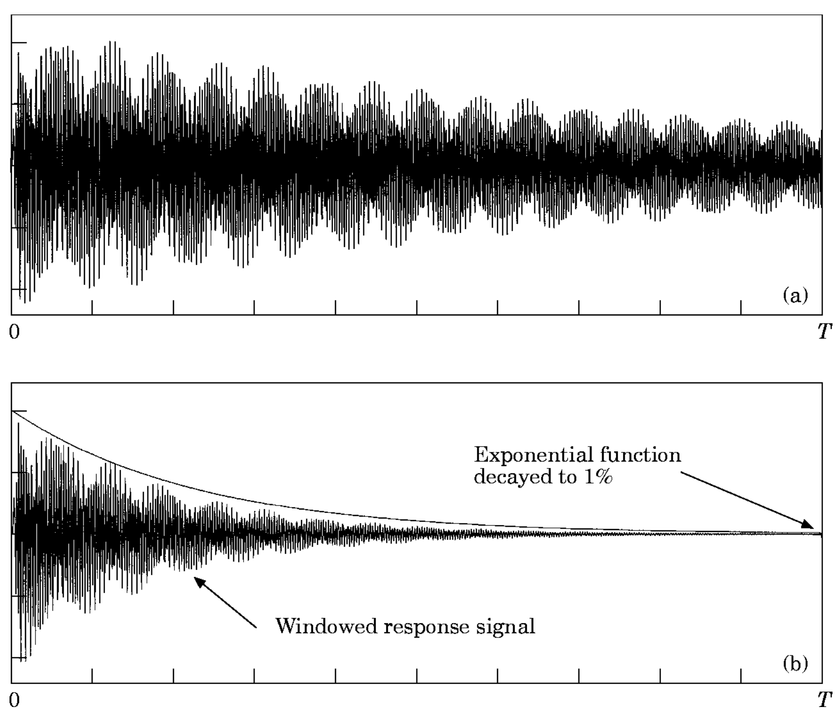

Figure 1. (a) Unwindowed and (b) windowed response signal of a lightly damped system for impact excitation and the exponential window, which forces the measured response towards zero at the end of the time record.

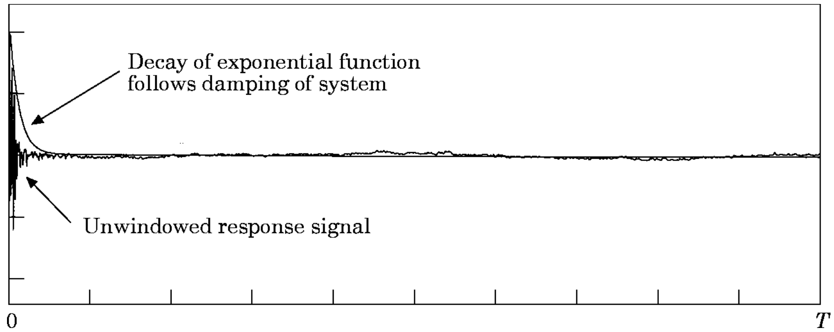

Figure 2. Unwindowed response signal of a heavily damped system for impact excitation and the exponential window, which follows the decay of the system response.

The exponential window is also used with burst random excitation to minimise leakage errors when testing lightly damped systems. The burst length is adjusted to achieve the requirement of a completely observed transient, which should ideally decay to zero by the end of the time record. For heavily damped systems, this requirement can typically be met with burst lengths of as much as 80% of the time record. For lightly damped systems however, the response may not decay by the end of the time record for burst lengths of less than 50%. While shorter burst lengths might eliminate this problem, another potential problem is putting insufficient energy into the system and possibly increasing the signal-to-noise ratio of the measurement. The signal-to-noise ratio is especially a concern for the force signal which uses the internal, feedback damping of the shaker to reduce the force to zero and therefore has little signal for 50% of the measurement. Figure 3(a) shows the unwindowed response of a lightly dampled system for a 50% burst excitation which does not decay by the end of the time record. Figure 3(b) shows the windowed response and the exponential window that was applied. Note that the exponential decay of the window starts at the beginning of the time record, and not at the end of the burst length.

3. SELECTION OF WINDOW PARAMETERS

In some data acquisition software, the exponential window is defined by specifying the time constant (τ) in s, while in other software, the exponential window is defined by specifying the reciprocal parameter (β) in rad/s. A common suggestion is that the time constant should be one quarter of the time record length, which creates an exponential function that decays to approximately 2% at the end of the time record. However, this is not appropriate for heavily damped systems, as explained above. Although calculating either quantity (τ or β) for the desired exponential window properties is not difficult, a much more intuitive definition for the exponential window is proposed in equation (2) [3].

Let

$$e^{-t_{co}[\%]T/\tau} = w_{co}[\%], \quad \text{or} \quad \tau = \frac{-t_{co}[\%]T}{\ln(w_{co}[\%])} \tag{2}$$

In the equations above, the exponential time constant is calculated from the cut-off time (t_{co}), the cut-off value (w_{co}), and the time record length (T). The cut-off time is defined as a percentage of the time record length, and the cut-off value is defined as percentage of unity. The exponential window has a value of unity at the beginning of the time record and will decay to the value specified by the cut-off value at the percentage of the time record specified by the cut-off time. In effect, the cut-off time and cut-off value specify

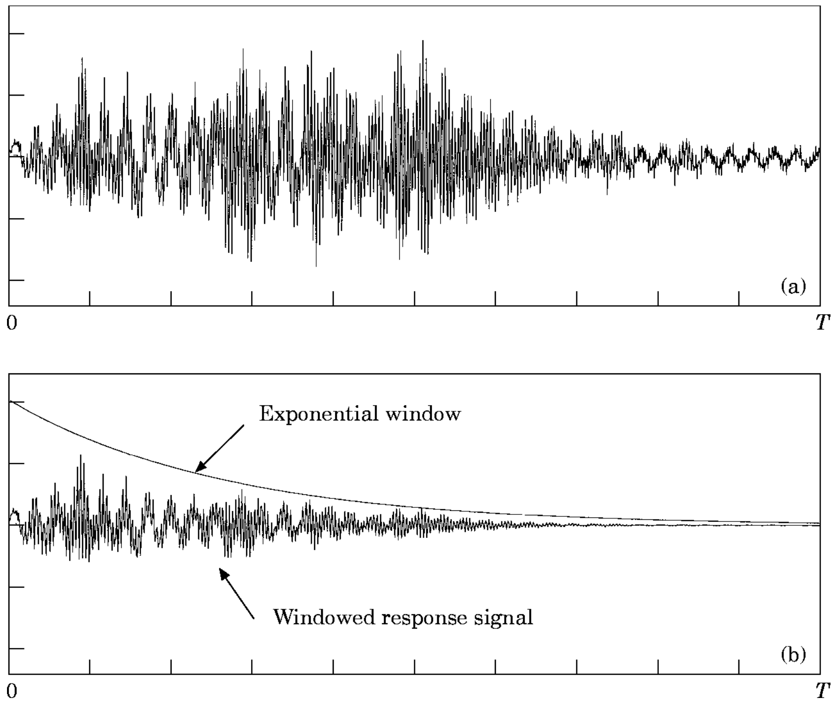

Figure 3. (a) Unwindowed and (b) windowed response signal of a lightly damped system for burst random excitation and the exponential window, which forces the measured response towards zero at the end of time record.

another point on the exponential curve. The time record length is defined by standard signal processing definitions and can be determined by the frequency span and the number of spectral lines. If the time record length is modified, the exponential time constant is also updated accordingly, such that the relative shape of the window remains constant.

For example, if the cut-off time is defined to be 50% of the time record length and the cut-off value is 5% of unity, the value of the window at the midpoint of the time record will be 0.05, for any time record length. By defining the exponential window in this manner, the properties of the window can very easily be specified. Figure 4 illustrates the definition of the cut-off and cut-off value of the exponential window.

For this example, if $T = 2$ s, the time constant τ is:

$$\tau = \frac{-0.5 * 2\text{ s}}{\ln(0.5)} = 0.33\text{ s}$$

For the exponential windows in Figs 1(b) and 3(b), the lightly damped systems, the cut-off time is 100% and the cut-off value is 1%. While for the exponential window in Fig. 2, the heavily damped system, the cut-off time is 5% and the cut-off value is 2%.

4. EXPONENTIAL WINDOW CORRECTION

To explain the effect of the exponential window on the estimated modal parameters, consider the following series of equations. In this derivation, the Laplace variable (s) is used, but the Fourier transform is equivalent to the Laplace transform evaluated at the imaginary ($j\omega$) axis. The effect of the exponential window is governed by the shift property

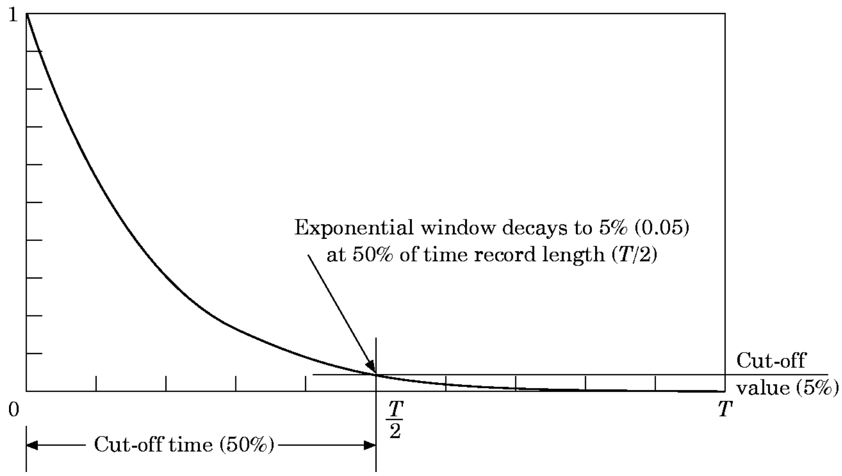

Figure 4. Definition of exponential window cut-off time and cut-off value. T, time record length.

of Laplace and Fourier transform theory [4]. Multiplying a time signal $y(t)$ by an exponential function shifts the independent variable of the associated Laplace transform $Y(s)$.

$$\text{If} \quad y(t) \Leftrightarrow Y(s) \quad \text{then} \quad e^{at}y(t) \Leftrightarrow Y(s-a) \tag{3}$$

The windowed ($e^{-t/\tau}$) input (f_q) and output (x_p) time signals are tranformed to the frequency domain, where q denotes the input degrees of freedom (dof) and p denotes the output dof.

$$e^{-t/\tau}f_q(t) \Leftrightarrow F_q\left(s+\frac{1}{\tau}\right) \quad \text{and} \quad e^{(-t/\tau)} \Leftrightarrow X_p\left(s+\frac{1}{\tau}\right) \tag{4}$$

Since the transfer function is computed from the shifted input and output functions, it is a function of the same independent variable. The transfer function is inverse Laplace transformed to the impulse response function (IRF), where $h_{pq}(t)$ is the IRF of the true (i.e. unwindowed) system.

$$H_{pq}\left(s+\frac{1}{\tau}\right) = \frac{X_p\left(s+\frac{1}{\tau}\right)}{F_q\left(s+\frac{1}{\tau}\right)} \quad \text{and} \quad H_{pq}\left(s+\frac{1}{\tau}\right) \Leftrightarrow e^{-t/\tau}\, h_{pq}(t) \tag{5}$$

The IRF can be written as a summation of damped exponential terms, where λ_r is the complex eigenvalue of mode r, A_{pqr} is the residue of dof pair (p, q) for mode r, and the caret notation (^) denotes a parameter associated with the measured (i.e. windowed) system.

$$e^{-t/\tau}h_{pq}(t) = \hat{h}_{pq}(t) \tag{6}$$

$$e^{-t/\tau}\sum_r e^{\lambda_r t}A_{pqr} = \sum_r e^{\hat{\lambda}_r t}\hat{A}_{pqr} \tag{7}$$

Equating the like terms in equation (7) and inserting the real and imaginary parts of the eigenvalues yields the correction for the exponential window, where ω_r and σ_r are the damped natural frequency and damping factor of mode r, respectively.

$$e^{-t/\tau}\, e^{\lambda_r t} = e^{\hat{\lambda}_r t} \quad \text{and} \quad A_{pqr} = \hat{A}_{pqr} \tag{8}$$

$$-\frac{1}{\tau} + \lambda_r = \hat{\lambda}_r \tag{9}$$

$$-\frac{1}{\tau} + \sigma_r + j\omega_r = \hat{\sigma}_r + j\hat{\omega}_r \tag{10}$$

$$\sigma_r = \hat{\sigma}_r + \frac{1}{\tau} \quad \text{and} \quad \omega_r = \hat{\omega}_r \tag{11}$$

Equations (8) and (11) indicate that the damped natural frequencies and residues of the measured system are identical to those of the true system and the difference of the damping factors between the true and measured systems is a function of the exponential window time constant. The effects of the exponential window described in the above equations are illustrated on the complex plane in Fig. 5. Since the damping factors of a natural system are expected to be negative, the damping factors of the measured system will have larger negative values than that of the true system, which causes the apparent increased damping in the measurements. Note that since the measured FRFs are computed from data modified by the exponential window, residue and modal scaling calculations and FRF synthesis should use the uncorrected poles. Another possible effect of the exponential window which should be noted is that it may complicate separation of closely spaced modes due to the increased damping.

A damping correction for the exponential window that has been previously presented in the literature, which will be shown below to be actually an approximate method, deals with damping ratio of a mode and is given by [2, 5, 6]

$$\zeta'_r = \hat{\zeta}_r - \frac{\beta}{\hat{\Omega}_r} \tag{12}$$

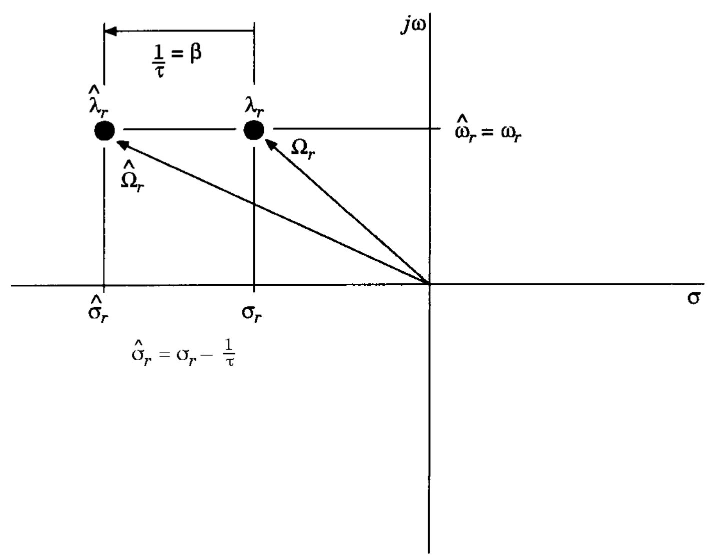

Figure 5. The exponential window effects and correction on the complex plane.

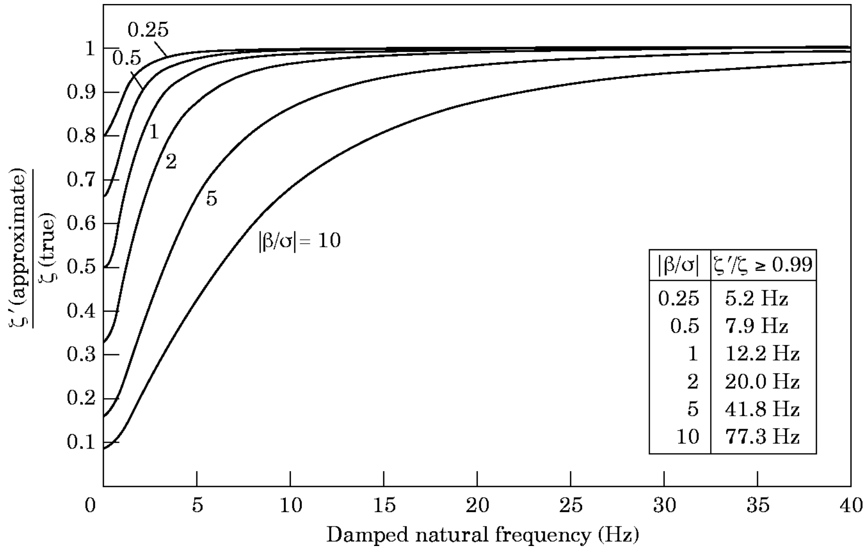

Figure 6. Comparison of the approximate correction of the damping ratio to the true damping ratio as a function of damped natural frequency and the ratio of damping factor to exponential decay constant.

where the prime notation (′) denotes a parameter associated with the above correction method. The damping ratio (ζ_r) and the undamped natural frequency (Ω_r) of mode r are defined by

$$\zeta_r = \frac{-\sigma_r}{\Omega_r} \quad \text{and} \quad \Omega_r = \sqrt{\sigma_r^2 + \bar{\omega}_r^2} \tag{13}$$

To show the relationshiip between the damping correction given in equation (11) to that in equation (12), the definition of the damping ratio is substituted in equation (11) and τ is replaced by β.

$$-\zeta_r\Omega_r = -\hat{\zeta}_r\hat{\Omega}_r + \beta \quad \text{or} \quad \zeta_r = \hat{\zeta}_r\frac{\hat{\Omega}}{\Omega_r} - \frac{\beta_r}{\Omega_r} \tag{14}$$

In order to reduce the damping ratio correction in equation (14) to that of equation (12), the approximation that the estimated undamped natural frequency is equal to the true undamped natural frequency must be made. The result of this approximation is shown below.

$$\zeta_r\left(\frac{\Omega_r}{\hat{\Omega}_r}\right) = \left[\hat{\zeta}_r\frac{\hat{\Omega}_r}{\Omega_r} - \frac{\beta}{\Omega_r}\right]\left(\frac{\Omega_r}{\hat{\Omega}_r}\right) = \hat{\zeta}_r - \frac{\beta}{\hat{\Omega}_r} = \zeta_r' \quad \text{or} \quad \zeta_r' = \zeta_r\left(\frac{\Omega_r}{\hat{\Omega}_r}\right) \tag{15}$$

Next, since $|\sigma_r| < |\hat{\sigma}_r|$, $\sigma_r^2 < \hat{\sigma}_r^2$ and $\Omega_r < \hat{\Omega}_r$ or $(\Omega_r/\hat{\Omega}_r) < 1$, thus $\zeta' < \zeta$.

The approximate correction method underestimates the damping ratio by a factor that is dependent on the undamped natural frequency. For a given damping factor and exponential time constant, the relative error is greatest for low damped natural frequencies and decreases as the damped natural frequency increases. This is because as the damped natural frequency increases, the damping factor becomes less significant in the magnitude of the undamped natural frequency, and does so in a quadratic fashion. This characteristic is shown in Figs 6 and 7, which indicates that the approximate, corrected damping ratio approaches the true value at high damped natural frequencies.

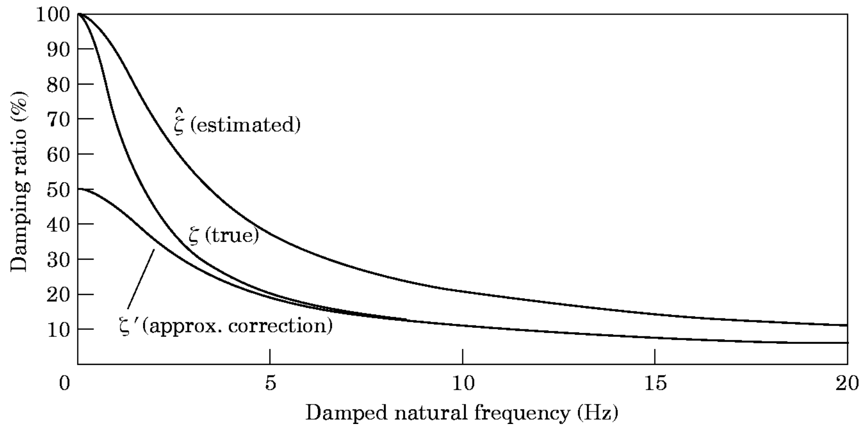

Figure 7. Comparison of the estimated damping ratio and the approximate correction for the damping ratio with the true damping factor as a function of damped natural frequency for $|\beta/\sigma| = 1$.

5. EFFECTS OF THE EXPONENTIAL WINDOW ON AN IMPACT FORCE SPECTRUM

The shift property of the Laplace transform governs the effect of the exponential window and essentially changes the independent variable of the associated s domain functions. Equation (5) clearly shows that the exponential window must be applied to both the response and force time signals so that the independent variable of the transfer function is unambiguously defined. For instance, if the exponential window is applied to the response, its transform is a function of $s + 1/\tau$, but if the window is not applied to the force, its transform is a function of just s. The practical consequences of applying the exponential window to an impact excitation are investigated below.

An impact force time signal can be modeled as a half-period sine pulse, which is defined by

$$f(t) = \begin{cases} \sin(\pi t/a) & 0 \leqslant t \leqslant a \\ 0 & t > a \end{cases} \tag{16}$$

where the parameter a defines the width of the pulse. The Laplace transform of this function is given by [4]

$$F(s) = \frac{\pi a(1 + e^{-as})}{a^2 s^2 + \pi^2} \tag{17}$$

The Laplace transform for a typical force pulse is shown in Fig. 8, where the pulse width was specified such that the main lobe has a 10-dB roll-off over a frequency range of 0–400 Hz ($a = 2.6$ ms, $T = 1$ s). The exponential window shifts the imaginary axis to the right, or shifts the function to the left. Thus, the portion of the function in the first quadrant is of interest for a damping shift of the exponential window. As can be seen from the s domain function, a windowed input spectrum, $F(\beta + j\omega)$, can differ significantly from the unwindowed input spectrum, $F(j\omega)$. However, for typical values of damping shift (the reciprocal of the time constant), Fig. 9, the Laplace transform is basically constant along the positive real axis, as shown in Fig. 10, and little error is introduced by not applying the exponential window to the force signal.

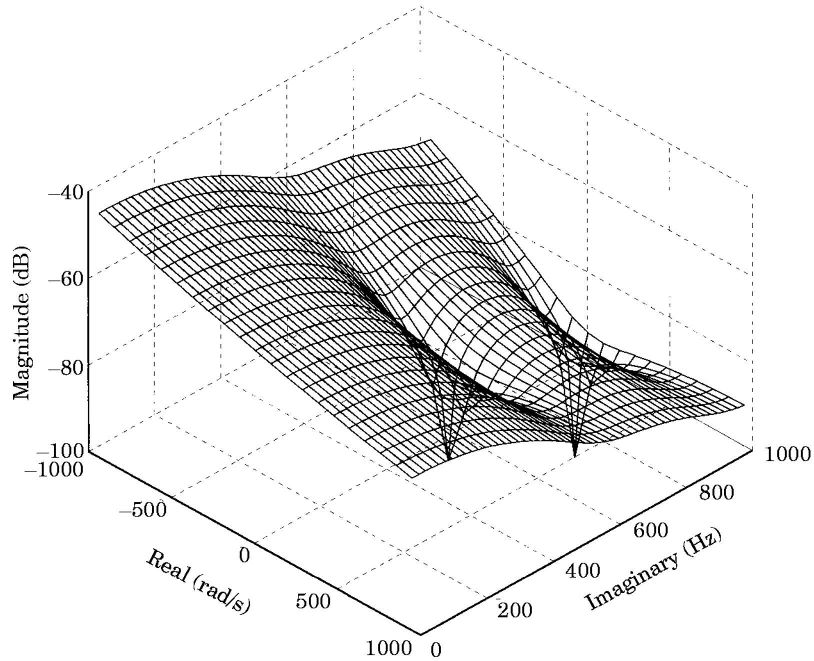

Figure 8. The transfer function of a half-period sine pulse, which is representative of a typical impact force time signal.

The parameter a in equation (16), which defines the width of the pulse, controls the position of the first zero in the transform function and the frequency span of the main lobe of the input spectrum. Increasing a produces a wider pulse and decreases the frequency of the first zero, decreasing a produces a narrower pulse and increases the frequency of the first zero, but the overall shape of the function remains similar to that in Fig. 8. That is, for different values of a, the transform function is constant along the positive real-axis in the first quadrant for typical values of damping shift used in impact testing.

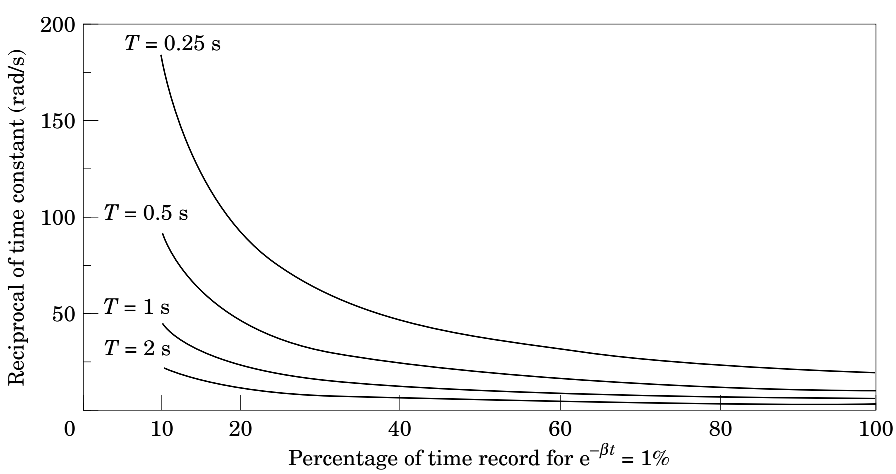

Figure 9. Typical values of the reciprocal of the exponential window time constant (β) used for impact testing. T, time record length.

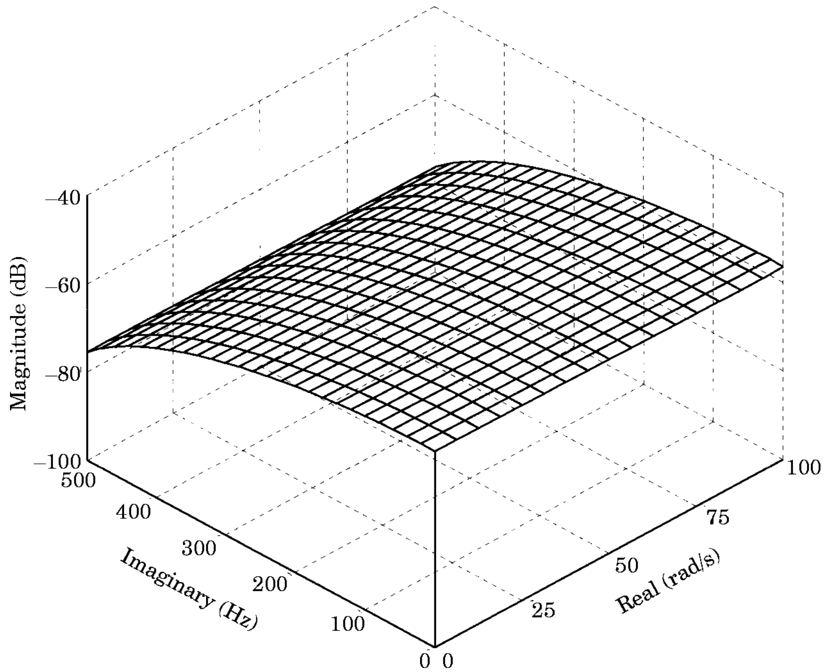

Figure 10. The first quadrant of the transfer function of a half-period sine pulse with no pretrigger delay, for the usable frequency range and typical values of damping shift.

Another issue of concern is the use of a pretrigger delay with impact testing: the force pulse does not occur at the start of the time record. A pretrigger delay corresponds to shifting the time variable which is also governed by the shift property of Laplace transform. Shifting a time signal $y(t)$ by an amount t_0 multiplies the associated Laplace transform $Y(s)$ by an exponential function in the s domain.

$$\text{If} \quad y(t) \Leftrightarrow Y(s) \quad \text{then} \quad y(t - t_0) \Leftrightarrow \mathrm{e}^{-t_0 s} Y(s) \tag{18}$$

This function is shown in Fig. 11 for a 10% pretrigger delay and indicates that a windowed input spectrum can differ significantly from the unwindowed input spectrum for values of damping shift that may be typically encountered with impact testing. Figure 12 compares the FRFs generated from a numerical simulation of an sdof system, in one case applying the exponential window to both the response and force signals, and in the other case applying the exponential window only to the response signal. The FRF for windowing both the response and the force matches an analytically synthesised FRF exactly, while the FRF for windowing only the response has an incorrect magnitude. This is because the unwindowed input spectrum has a larger magnitude than the shifted spectrum, thus decreasing the magnitude of the FRF.

The errors introduced by not windowing the force signal will not affect the estimated frequency or damping but will affect the scaling of the residue due to the incorrect FRF magnitude. The function in Fig. 11 has a slope along the real axis of -0.88 dB/rad/s, and for $\beta = 4.6$ rad/s, which is the value for the exponential window used in this example, the magnitude of the windowed spectrum is -4.05 dB less than the magnitude of the unwindowed spectrum. A difference in magnitude of -4.05 dB between the measured FRF and the true FRF translates into a residue estimated from the measured FRF that is 37% less than the true residue of the system.

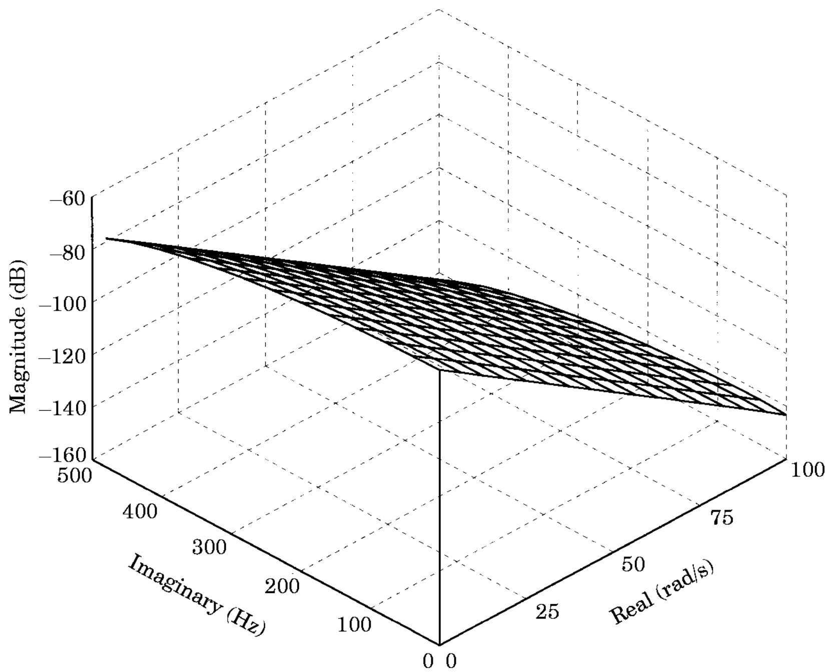

Figure 11. The first quadrant of the transfer function of a half-period sine pulse with 10% pretrigger delay, for the usable frequency range and typical values of damping shift. This transfer function has a much greater slope along the real-axis than does the transfer function in Fig. 10.

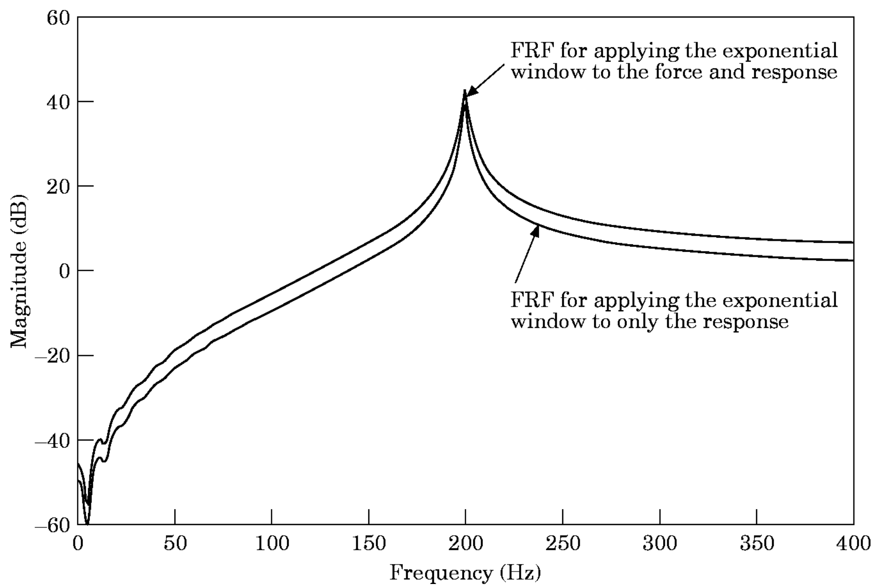

Figure 12. Simulated FRFs of a sdof system using the time function corresponding to the transfer function in Fig. 11 as the input to the system and an exponential window which decays to 1% at the end of the time record.

6. EXAMPLE TEST CASES

Several analytical test cases were run to demonstrate the correction for the exponential window and the effects of not applying the window to the force signal. Acceleration response signals were generated from a numerical simulation of an sdof system, as in the previous section, using the time signals corresponding to the transfer functions in Figs 10 and 11 as inputs to the system. The system's damping and the sampling parameters were specified such that the response did not decay to near zero at the end of the time record, and the exponential window that was applied decayed to 1% at the end of the time record. FRFs were calculated with the exponential window applied to the force and response signals and with the exponential window applied to only the response signal, for both an input force with no pretrigger delay and an input force with a 10% pretrigger delay. The system poles were estimated from the FRFs using a single reference, sdof, frequency domain, rational fraction polynomial algorithm, and the residues were estimated using a single reference, frequency domain algorithm [7].

The results of the test cases and the true modal parameters of the analytical system are listed in Table 1 (only the positive frequency pole of the complex conjugate pair is listed). For each test case, the estimated damping factor, the real part of the eigenvalue, is much greater in absolute value than the true value due to the damping added by the exponential window, but the damped natural frequency, the imaginary part of the eigenvalue, is not changed by the exponential window. The estimated eigenvalue corrected by equation (11), which corrects the damping factor with the exponential window time constant, are in agreement with the true eigenvalue of the system. The damping ratio calculated from the estimated damping factor and undamped natural frequency are also much greater for each test case that the true damping ratio of the system. Using the corrected eigenvalue to calculate the damping ratio naturally yields the true value because the corrected eigenvalue is an accurate estimate of the true eigenvalue. For this example, the approximate correction for the damping ratio of equation (12) also yields accurate values for the damping ratio. This is because for $|\beta/\sigma| = 1.8$, the approximate correction for the damping ratio approaches to within 1% of the true value at less than 20 Hz (refer to Fig. 6), and thus the error in the approximation is very small.

Note that the estimated eigenvalue could be corrected for the effects of the exponential window if it is applied to both the force and response or to only the response, for cases 1 and 2 with no pretrigger delay and for cases 3 and 4 with a 10% pretrigger delay. That is, not applying the exponential window to force does not significantly affect the estimated eigenvalue. For test case 2, with the exponential window applied only to the response and no pretrigger delay, the estimate of the residue is not significantly affected because the unwindowed input spectrum and the input spectrum shifted by 4.6 rad/s are nearly identical (refer to Fig. 10). However for test case 4, with the exponential window applied only to the response and a 10% pretrigger delay, the estimated residue is much less than the true value. This result is evident, as discussed in the previous section, from the FRFs in Fig. 12, which were processed in test cases 3 and 4, and the transfer function in Fig. 11. Thus, not applying the exponential window to force does not significantly affect the estimated residue if there is no pretrigger delay, but does cause an underestimation of the residue if there is a pretrigger delay.

7. CONCLUSION

The exponential window is applied to the transient signals of impact testing and burst random excitation to either minimise leakage errors or to improve the signal-to-noise ratio

TABLE 1

Comparison of estimated and corrected modal parameters of test cases to true values of analytical system

	Analytical system	Test case 1	Test case 2	Test case 3	Test case 4
Eigenvalue (rad/s), *estimated*	$-2.5133 + j1256.6345$	$-7.1185 + j1256.6352$	$-7.1186 + j1256.6347$	$-7.1187 + j1256.6290$	$-7.1188 + j1256.6284$
Eigenvalue (rad/s), *corrected*		$-2.5133 + j1256.6352$	$-2.5135 + j1256.6347$	$-2.5135 + j1256.6290$	$-2.5136 + j1256.6284$
Undamped natural frequency (Hz), *estimated*	200	200.003	200.003	200.002	200.002
Undamped natural frequency (Hz), *corrected*		200.000	200.000	199.999	199.999
Damping ratio (%), *estimated*	0.2	0.5665	0.5665	0.5665	0.5665
Damping ratio (%), *corrected*		0.2000	0.2000	0.2000	0.2000
Damping ratio (%), *approx. correction*		0.2000	0.2000	0.2000	0.2000
Residue, *estimated*	$-j6.2832 \times 10^{-4}$	$-j6.2083 \times 10^{-4}$	$-j6.1499 \times 10^{-4}$	$-j6.2551 \times 10^{-4}$	$-j3.9074 \times 10^{-4}$

Test case 1: no pretrigger, exponential window on force and response.
Test case 2: no pretrigger, exponential window on response only.
Test case 3: 10% pretrigger, exponential window on force and response.
Test case 4: 10% pretrigger, exponential window on response only.
Time record length = 1 s
Exponential window cut-off value = 1%
Exponential window cut-off time = 100%
Exponential window time constant: $\tau = 0.2171$ s or $\beta = 4.6052$ rad/sec, $|\beta/\sigma| = 1.8$.

of the measurements. To minimise leakage errors on lightly damped signals, the exponential window typically decays to 1% at the end of the time record and constrains the measured response to be a completely observed transient. To improve the signal-to-noise ratio of a heavily damped signal, the exponential should follow the damping of the system and attenuate the noise on the trailing segment of the signal. The method for defining the exponential window presented in equation (2) allows an appropriate window to be readily specified for either type of the above signals.

The exponential window artificially increases the damping of the measured FRFs, which affects only the estimated damping factors, but not the damped natural frequencies or the residues, if applied to both the response and the input. The effect of the window is governed by the shift property of the Laplace transform and the method developed in this paper corrects the estimated damping factor with reciprocal of the time constant of the applied exponential function. The other correction method discussed, which operates on the damping ratio, was shown to be an approximation that approaches the above method as the damped natural frequency of the mode increases.

The effects of the exponential window on an impact force spectrum are revealed by studying the Laplace transform of a representative impact force time signal. Although the function is basically constant along the positive real axis in the s domain for the values of damping shift used in impact testing, the combined effects of the exponential window and a pretrigger delay require that the window must be applied to both the response and the input signals when measuring frequency response functions. As was shown with the test cases, the error on the estimated residue introduced by not applying the exponential window to the force is typically small if there is no pretrigger delay, but can be very significant if there is a pretrigger delay.

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