Fig. 12. The development of a neck of the second kind.

as necking rupture, since the load rises and then falls again before fracture, but this would confuse two different phenomena. In order to distinguish between them, the behavior illustrated in Figure 11 will be called "necking rupture of the second kind." The two types of necking rupture occur for the same polymer in separate temperature regions; the distinction between them will be clarified when the effect of temperature is discussed under Factors Affecting Fracture.

Test Methods Other than the Tensile Test

It was pointed out in the section on Classification and Definitions that the only common test which can be used to study fracture over the whole range of polymer behavior is the simple tensile test and for this reason it was used as a basis for definition and classification. However, a tensile test is not always the best method for making measurements of specific physical properties; various different tests are better adapted for studying fracture and measuring the relevant quantities in particular regions of behavior. Some of these methods will be described and discussed in this section.

Flexure. A tensile testing machine can be adapted for testing specimens in flexure. For example, a specimen may rest on two supports and be bent by loading a third, central, support. This arrangement is known as three-point loading; flexural tests can also be performed in four-point loading or cantilever loading. The curve of load against deformation can be obtained in the same way as the load–extension curve in a tensile test. The stress and strain vary from point to point in the specimen but, with certain assumptions (11), the maximum stress and strain can be calculated from standard formulas. For example, consider a specimen with a uniform rectangular cross section bent in three-point loading. Suppose L is the load on the specimen, l is the span (distance between outer supports), b is the specimen width, d is the specimen thickness, and δ is the deformation or deflection at the center. Then the maximum tensile stress, σ, which occurs on the outer surface of the specimen opposite the central loading point, is given by equation 11 and the corresponding maximum strain,

$$\sigma = 3Ll/2bd^2 \qquad (11)$$

ϵ, at the same point is given by equation 12. The strain energy per unit volume at

$$\epsilon = 6d\delta/l^2 \qquad (12)$$

the point of maximum stress is given by equation 13. This expression is the area

$$E = \sigma\epsilon/2 = 9L\delta/2lbd \text{ or } (\tfrac{1}{2}L\delta)/(\tfrac{1}{9}lbd) \qquad (13)$$

under the load–deflection curve divided by one ninth of the stressed volume. The factor of 9 is necessary because of the variation in strain energy along and through the specimen.

If the load is proportional to the deflection up to the point of fracture, then the load and deflection at break can be inserted in the appropriate equations to give the following quantities: *(1)* σ_B = the brittle strength. It is also called the flexural strength, the cross-breaking strength, and the modulus of rupture in bending. *(2)* ϵ_B = the brittle breaking strain. *(3)* E_B = the energy to break per unit volume. If the load is not proportional to the deflection, then, although the load and deflection at break and the area under the load–deflection curve can still be inserted in the equations to give measures of σ_B, ϵ_B, and E_B, there is some error involved in this procedure because the formulas are calculated on the assumption of linearity. It is possible to make a more precise analysis, without assuming linearity (Ref. 6, Chap. 22) but from the literature it seems that this more complicated approach has not often been felt necessary. If the nonlinearity becomes so marked that the load drops before the specimen breaks, then the errors in the formulas become so large that the test method loses its usefulness.

A second assumption in the calculations is that the deflection is not too large, relative to the specimen thickness; for this reason, the span should not be too large relative to the thickness. The span should also not be too small, to avoid excessive shear deformation (12). Experience has shown that it is reasonable to use a ratio of span to thickness in the range 10 to 14.

As well as adapting tensile testing machines, it is also possible to stress specimens in flexure by means of falling weights or swinging pendulums, in order to determine their flexural impact strengths (eg, ASTM D 256 Method B, and BS (British Standard) 2782 Method 306 D). From the results of such a test, the energy to break per unit volume can be calculated by dividing the energy to break by one ninth of the stressed volume, as before (eq. 13). If the specimen is subjected to three-point loading, this would be called a Charpy impact test; cantilever loading would make it an Izod impact test. The same errors can arise in flexural impact tests as in slow speed tests; the formula is accurate only for brittle fractures and the ratio of span to thickness should be about 12.

The principal advantage of flexural tests on brittle specimens is that the fracture usually can be made to originate on a molded surface, so that the results are more reliable and less scattered than those of tensile tests on brittle specimens in which the fracture often originates at a machined or cut surface. The main disadvantages of flexural tests are that the formulas become inaccurate when the load–deformation relation becomes nonlinear and that the test is inapplicable when the specimen does not break.

Biaxial Flexure. Another type of loading, more commonly used in impact testing than at slow speeds, may be called biaxial flexure (or "dart-drop" testing) and is exemplified by BS 2782 Methods 306 B and C. In the British Standard forms of this test, the specimen is supported on a hollow steel cylinder with an internal diameter of 2 in. (50.8 mm) and impacted by a striker with a hemispherical striking surface 0.5 in. (12.7 mm) in diameter. From this test it is possible to determine the median energy to break. However, this result cannot be readily transformed into the energy to break per unit volume because the theory is complicated, particularly for materials with nonlinear stress–strain curves, and requires measurement of the depth to which the striker indents the specimen before fracture.

The main advantages of this type of test method are that it appears to give a realistic simulation of a type of hazard encountered in practice and, also, that the specimens do not require accurate machining since the fracture originates on a molded surface. The main disadvantage is that there is no satisfactory theory for taking into consideration changes in thickness or for extracting the energy to break per unit volume from the result. Also, the British Standard specifies a method of calculating the median which assumes that the energy to break is normally distributed. It can happen that the distribution is so far from normal as to be markedly bimodal and then the result can be very misleading. This particular disadvantage is not inherent in the type of stress but only in the way in which it is usually applied.

Stress Concentrations. The behavior of specimens and the results of tensile tests, flexural tests, and impact tests can be varied, sometimes greatly, by drilling holes or machining notches in the test specimens. Notched specimens are frequently used in flexural impact tests such as the Izod (eg, ASTM D 256 Method A or BS 2782 Method 306 A) or the Charpy (eg. ASTM D 256 Method B or BS 2782 Method 306 E). The dimensions of the notches used vary between countries; some idea of the difference in severity of the different tests can be obtained by inserting the specified dimensions of the notch into the formula given by Roark (11) for the elastic stress concentration factor, k, for a notched bar in bending (eq. 14, where c is the depth of the notch and r is the radius of the tip of the notch). Table 1 gives the figures for the

$$k = 1 + 2\sqrt{c/r} \tag{14}$$

Table 1. Effect of Notch Dimensions on Stress Concentration

Method	Notch dimensions, mm		Elastic stress concentration factor
	Depth	Radius	
ASTM D 256 A	2.5	0.25	7.3
BS 2782 306 A	2.5	1.0	4.2

Izod test specifications cited above. Clearly, it cannot be expected that the two tests will give identical results.

In the usual form of these impact tests, the energy lost by the pendulum is taken as the energy to break the specimen and the result may be expressed in various ways. The following two ways are the most common:

1. The energy to break the specimen is divided by the width of the specimen (b in Figure 13). The energy to break per unit width is quoted in ft-lb/in. of notch, ergs/cm of notch, or some equivalent. This method is adopted in the standard tests quoted above.

2. The energy to break the specimen is divided by the area beneath the notch ($b(d - c)$ in Fig. 13). The energy to break per unit area may be quoted in dynes/cm or ergs/cm², in ft-lb/in.² or lb/in., or in some equivalent. This method was urged by Koon (13) in 1942 and seems to be gaining acceptance.

For quality-control testing, when all specimen dimensions must be rigidly controlled, it is not too important which way the results are expressed. For other purposes, however, such as engineering design, comparison of the results of different test methods, and understanding of fracture behavior, it is desirable to have a good method of allowing for changes in specimen dimensions. In this situation, it is unfortunate

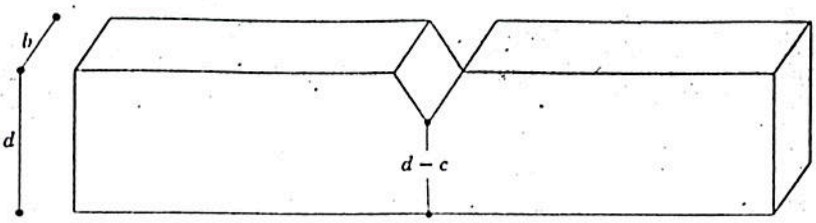

Fig. 13. Diagram of a notched specimen for flexural impact tests.

that neither of the possibilities listed above provides a universally satisfactory method of quoting the results of impact tests on notched specimens in such a way that different tests give the same answer. There seems to be no alternative but to face the fact that the result of an impact test is an arbitrary number which applies only to that particular test on a specimen of that particular shape. See also IMPACT RESISTANCE.

The principal advantage of making measurements on notched specimens is that it gives information which is not readily deducible from other types of test. In particular, notched specimens may be brittle, even when unnotched specimens from the same sample are not. By varying the radius and depth of the notch, it is possible to test a sample over a range of notch severities; this procedure gives a better chance of understanding and predicting behavior in service from laboratory experiments than does relying on results obtained with only one size of notch. The main difficulty stems from the lack of a reliable theory for considering changes in the dimensions of the specimen and the notch. An additional practical point is that careful and accurate machining is essential.

Natural Cracks. In the impact test methods discussed in the preceding section, the notch radius was not less than ¼ mm, which is about the lower limit for routinely machining notches in a controlled and accurate manner without excessive precautions. There may be much lower radii at the tips of natural cracks and interesting information can be obtained by studying the behavior of precracked specimens. The analysis of the behavior of this type of specimen originates with the work of Griffith (14) on brittle fracture. A useful summary of the application of this approach to polymers has been given by Berry (Ref. 15, Chap. IIA and B). Griffith pointed out that when a natural crack increases in length, the energy needed to create the new surfaces comes from the strain energy of the specimen; he assumed that the condition for rapid propagation of the crack was that, for a given increase in crack length, the energy to create the new surfaces should be no greater than the reduction in strain energy. For precracked specimens this line of thought leads to equation 15, where σ_c is the tensile

$$\sigma_c = \alpha\sqrt{Y\gamma/c} \tag{15}$$

strength of the precracked specimen based on the total cross-sectional area (ie, not allowing for the reduction in area due to the cracks); Y is Young's modulus; γ is the fracture surface energy or crack-propagation energy or energy needed to create unit area of new surface, allowing for the fact that two new surfaces are created; c is the crack length; and α is a constant of proportionality whose magnitude depends on the shape of the specimen. This approach introduces a new quantity, γ, which characterizes the toughness of precracked specimens. It may be given in ergs/cm², dynes/cm, lb/in., or some equivalent form. It should be emphasized that the use of the quantity γ implies that a graph of $1/\sigma_c$ against $\sqrt{c}$ for a range of values of c is a straight line through the origin. Since such a graph is not always straight nor, if straight, does it always pass through the origin, care is needed in the use of values of fracture surface energy.

The calculation of the stresses at which preexisting cracks extend rapidly and catastrophically in specimens of different shapes and sizes has been developed considerably by Irwin and others (16–19); this approach has been called "fracture mechanics," or, more precisely, linear elastic fracture mechanics. To follow this approach, it is necessary to define four new parameters: K, K_c, G, and G_c. (*1*) Stress field intensity, K, is not a property of the material but of the applied load and the specimen dimensions. It has units such as kg-cm$^{-3/2}$, lb-in.$^{-3/2}$, or some equivalent. (*2*) The value of K at which the crack extends rapidly and catastrophically is K_c. It has the same units as K but the distinction is that K_c depends on the material and the testing conditions. Thus one can say that fracture occurs when $K = K_c$. (*3*) Strain energy release rate or crack-extension force, G, and K are related by equations 16 and 17, where Y = Young's modulus and ν = Poisson's ratio. G has the units of energy

$$K^2 = YG \quad \text{for plane stress (thin sheets)} \tag{16}$$

$$K^2 = YG(1 - \nu^2) \quad \text{for plane strain (thick sheets)} \tag{17}$$

per unit area (ergs/cm², lb/in. or some equivalent). (*4*) Fracture toughness, G_c, and K_c are related by equations 18 and 19. G_c has the same units as G and is equivalent to

$$K_c^2 = YG_c \quad \text{for plane stress} \tag{18}$$

$$K_c^2 = YG_c(1 - \nu^2) \quad \text{for plane strain} \tag{19}$$

twice the fracture surface energy or crack-propagation energy, γ, defined and used in the Griffith theory, ie, $G_c = 2\gamma$.

The use of these parameters may be made clearer by an example. Suppose that a tensile stress σ is applied to a specimen of width W containing two edge cracks of depth c. Then it can be calculated (17) that the stress field intensity is given by equation 20. If such a specimen is found to break at a gross section stress σ_c, then

$$K = \sigma[W \tan \pi(c/W) + 0.1W \sin 2\pi(c/W)]^{1/2} \tag{20}$$

equation 21 can be written. The fracture toughness, G_c, can then be calculated from

$$K_c = \sigma_c[W \tan \pi(c/W) + 0.1W \sin 2\pi(c/W)]^{1/2} \tag{21}$$

K_c and the elastic constants. Thus fracture mechanics provides, on the one hand, a means of determining a quantity (G_c or K_c) which characterizes the fracture toughness of cracked specimens and, on the other hand, equations relating this quantity to the applied fracture stress for cracked specimens of other shapes and sizes.

This approach has two main advantages. It permits engineering design calculations in circumstances where other methods are inapplicable or inaccurate. Also, it may give interesting information about the growth of natural cracks. However, the method must be used with caution for the following reasons:

1. It is a fundamental assumption of the theory that the specimen contains a natural crack. In laboratory experiments, such a crack is deliberately introduced into the test specimen and its length can be measured; it is possible to ensure that this assumption of the theory is satisfied. In a practical situation, however, it may not be possible to

observe a natural crack and, therefore, calculations of fracture mechanics cannot be precise.

2. It is possible to determine the stress which would, in practice, lead to catastrophic fracture by assuming that the article contains a long natural crack; knowing c and K_c, the stress for fracture can be calculated. For example, suppose that it is required to know the pressure at which a pressure vessel would crack explosively. One could assume that the vessel contained a crack with length no longer than the wall thickness, on the grounds that a crack of such a length would cause the vessel to leak and would therefore be detected. For given K_c, it would then be possible to calculate the pressure at which the vessel would leak before breaking explosively. Such an approach provides a design stress which should be very safe and, therefore, valuable when the consequences of fracture would be particularly dangerous or expensive. On the other hand, if the consequences of fracture would not be too serious, this design stress might well be too low in many practical situations; then the fracture mechanics calculation would lead to uneconomic designs.

3. Comparison of materials on the basis of the value of G_c or K_c is not always realistic.

4. The calculations assume that the gross section stresses and strains are linearly related, ie, it is assumed that Young's modulus and Poisson's ratio are independent of strain. This assumption is often not sufficiently accurate for polymers.

5. K_c cannot always be readily measured. The specimen may reach the macroscopic yield stress before catastrophic fracture and then no value of K_c can be obtained.

6. At present, very little information is available concerning the dependence of K_c or G_c on temperature, rate of crack propagation, specimen thickness, environment, fabrication conditions, and material variables. Though this is a difficulty which could be reduced if more work were done on this problem, it does restrict the current application of the method.

7. There is some evidence that the values obtained depend on the way in which the test specimen has been precracked (20).

Cleavage. A test method was devised by Benbow and Roesler (21) which gives a direct measure of $Y\gamma$ or K_c, and, by assuming Y, gives γ or G_c. The test specimen is a flat strip which is split lengthwise by gradually propagating a crack down the middle. The crack is made stable in direction by holding the specimen in a state of lengthwise compression. Variations of this type of experiment have also been described by Svensson (22), Berry (Ref. 15, Chap. IIB), and Broutman and McGarry (23).

The cleavage technique is useful for giving information about fracture as a physical process because it can be used to make natural cracks grow in a stable, controlled way (see also "controlled fracture" in the article on MACHINING). As a means of providing data for practical application, it has similar disadvantages to the fracture mechanics approach summarized in the previous section. The experiments are rather complex for routine use, the predictions may be unrealistic, the calculations assume a linear relation between stress and strain, and the technique is not always applicable to materials which are not entirely brittle.

Tear. The problem of analyzing the results of tear tests on rubbers has been considerably clarified by the work of Thomas, Greensmith, and others (24–27). Many different tear tests have been used at different times and in different places but they do not all rate a given set of materials in the same order. However, it was found that if the results of tear tests were analyzed by an energy-balance theory derived from

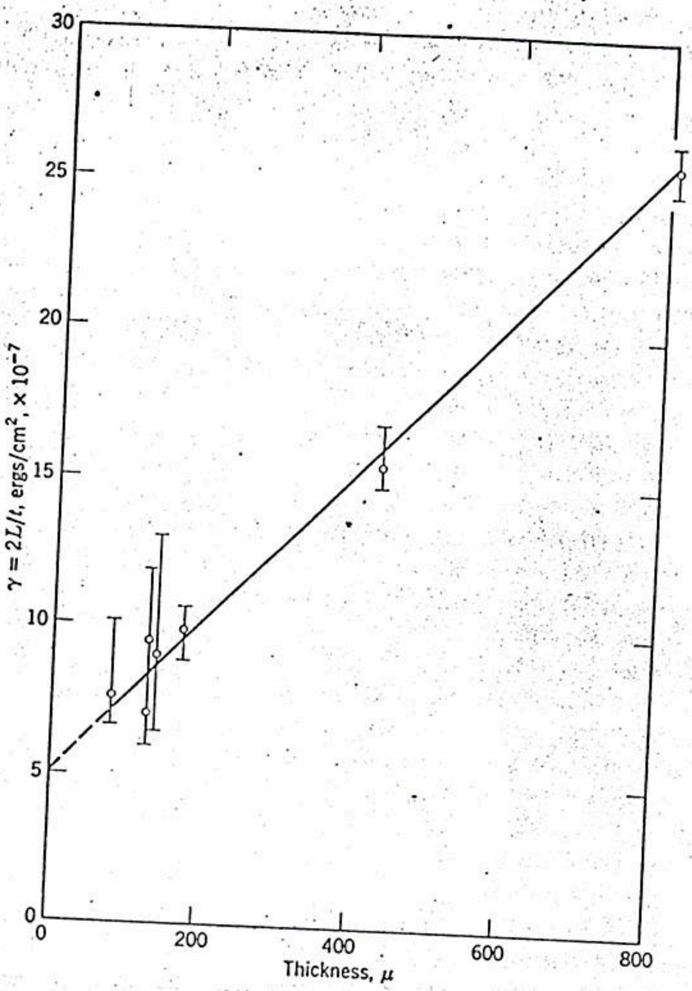

Fig. 14. Tearing energy as a function of thickness for poly(ethylene terephthalate) film.

that of Griffith mentioned on p. 308 (14), then the fracture surface energy, γ, had the same value in experiments on three different test specimens. For example, if a load L is needed to tear in "simple extension" a specimen of thickness t, then $\gamma \simeq 2L/t$, provided the extension of the "legs" of the "trouser-shaped" test piece is small.

It is particularly useful to use this approach for calculating the results of tear tests on elastomers because it gives the same results on specimens of different types. Also, it provides good predictions of the growth of cuts, tears, and cracks.

The tear strength of thermoplastic films can also be measured by such a simple tear test. In some ways, such tests are better than tensile tests; the specimens are easier to prepare because the results are not seriously dependent on the preparation technique. Also, when the specimen is torn stably and continuously at constant speed in a tensile testing machine, the tearing load can be measured throughout the test; in this way, one test gives a large number of results and it is possible to measure an average tearing load and also a range of values. Unfortunately, this attractive approach has disadvantages for testing thermoplastics: (*1*) It is applicable only to thin films. Thick specimens generally do not tear and moderately thick specimens produce spurious bending loads. The actual limit of specimen thickness varies with the material and the test conditions. (*2*) The method cannot be used on brittle materials as they fracture uncontrollably across the specimen at right angles to the initial cut. (*3*) The result depends on the thickness of the specimen. Figure 14, for example, shows γ ($= 2L/t$) as a function of thickness for samples of amorphous, isotropic poly(ethylene terephthalate) film (28). Clearly, it cannot be assumed for thermoplastic films that γ is a material property, independent of thickness. Apparently no work has been published concerning the effect of sample thickness on γ for any rubbery polymer.

Compression. If a specimen is compressed uniaxially at constant speed, the load developed can be recorded as a function of the deformation (29). This can be converted into a true stress–strain curve. A compression test has the following two advantages over a tensile test for determination of deformation: (*1*) When a material is brittle in a tensile test, the tensile yield stress cannot be measured directly. It may be that such a material is not brittle in a compression test; it is then possible to measure the yield stress in compression. (*2*) When a specimen fails by necking rupture in a tensile test, the true stress–strain curve can be obtained only in a limited range of strains. Since such a material is generally more ductile in compression, the compressive true stress–strain curve can be obtained up to higher strains.

The main difficulty with compression tests lies in ensuring that the stress is uniform. Long thin specimens buckle and short fat specimens barrel because of friction between the loading plates and the specimen ends. A preferable form of compression test was devised by Watts and Ford (30) for metal sand has been applied to thermoplastics by Williams and Ford (31). This test uses a technique in which the center of a specimen is compressed between two dies. When the dies are moved together with constant speed, the specimen is compressed in one direction and expands outward at right angles. Because of frictional restraint there is no strain in the third orthogonal direction except for relatively small end errors. Because the strains are thus confined to a single plane, this is called a plane-strain compression test. The curve of load against deformation can be recorded and, because the area of contact is constant throughout the test, the curve can be converted directly into a true stress–strain curve; the load recorded is proportional to the true stress, unlike the situation in uniaxial tensile and compressive tests where the loads must be corrected to allow for the changing cross-sectional areas. Of course, this test is not designed to give a direct measure of fracture but its particular value is that it provides a true stress–strain curve directly, even when this is difficult in tension because of necking and fracture. It has the following two disadvantages: (*1*) The results are not extended reliably beyond nominal strains of about 70%, which is the equivalent of a tensile draw ratio of about 3. Behavior at much higher draw ratios than this can be investigated in tension in the favorable cases when the specimens do not break. (*2*) It may be necessary to make tests with a series of different dies of varying length-to-width ratios to overcome experimental errors caused by friction and specimen geometry (30, 31).

Mechanism

It is usual to start a discussion on the mechanism of fracture by pointing out that the measured tensile strengths of solids are much lower than the theoretical strengths as calculated from the atomic binding forces. For organic polymers, the maximum theoretical strength is that of a carbon–carbon bond, which has been given by de