Classification and Definitions

One of the most remarkable features of the fracture or rupture of polymers is the great variety of ways in which different materials respond to stress. The elongation at break varies between less than one percent and several thousand percent: breaking stresses vary between less than 10^8 dynes/cm^2 and 2×10^{10} dynes/cm^2; cracks may travel catastrophically at near sonic velocities, or so slowly that little change can be observed in a day; the depth of residual deformation may be measured in centimeters or in microns. This variety of response forms a means of determining the mechanism of response to stress by different polymers, and of exploring the factors affecting this mechanism through tests consisting of the application of stresses leading to fracture in relatively short times.

To bring some order into this variety, it is necessary to find some means of classifying the different types of fracture. One possible basis for classification is the appearance of the specimen, during deformation and after fracture, but this approach can be rather subjective and can lead to difficulties. It is less ambiguous to use the shape of a load–deformation curve as the primary basis for classification, supplementing it where necessary by observations of the appearance of deformed and broken specimens.

Many different test methods have been used for studying fracture but the majority give useful information only in limited regions of behavior; as an obvious example, the breaking stress of a rubbery polymer is not usually measured in flexure. The only common test method which can be used to determine fracture properties, whether the material is soft or hard, oriented or isotropic, brittle or highly extensible, is the simple tensile test. This sugggests that the best basis for the classification of short-term fracture behavior is the shape of the load–extension curve, supplemented by the specimen appearance, in a simple tensile test. Using this criterion, it is possible to distinguish five principal types of behavior: uniform extension, cold drawing, necking rupture, brittle fracture, and necking rupture of the second kind. These will be discussed in detail in this section. As with most systems of classification, this one is not entirely unambiguous; there are borderline cases and variations which are difficult to classify, but this method appears to give a reasonably clear definition of the great majority of observed fractures.

A tensile test having been decided upon, it is necessary to choose the shape of the specimen. Different standards organizations have chosen different profiles; even a single organization may not specify the same profile for testing all materials. This has led to the confusing situation wherein a large number of specimen profiles are used in laboratories throughout the world and comparison of results obtained in different laboratories has become difficult. In the absence of one internationally accepted profile, the fundamental definitions and many of the results quoted in this article are based on tests made in the author's laboratory by the following method. A specimen is machined, cut, stamped, or molded to the shape shown in Figure 1. Its width and thickness are measured at the smallest section and it is inserted in clamps in a tensile

testing machine. The specimen is then extended at a constant speed, which is standardized at ½ in./min. (about 0.2 mm/sec) unless the effect of the changes in extension speed is being studied. (Unless otherwise noted, the results given in this article were obtained at this speed.) The load on the specimen during extension is measured by a load cell based on electric resistance strain gages; the signal is fed to a pen recorder giving a permanent record of the load–time curve. It is this curve which is used to define the type of failure.

It has been found in practice that the specimen profile shown in Figure 1 combines negligible slipping at the clamps, a small size convenient for use with a limited quantity of experimental material, and a radius of curvature large enough for the stress concentration to be small (the elastic stress-concentration factor is 1.03% (1)). The specimen profile specified in ASTM D 638 is less easy to machine and more often suffers from slippage and fracture near the clamps. The microtensile specimen specified in ASTM D 1708 has a fillet radius of only 3 mm and is therefore excessively liable to fracture at the fillets.

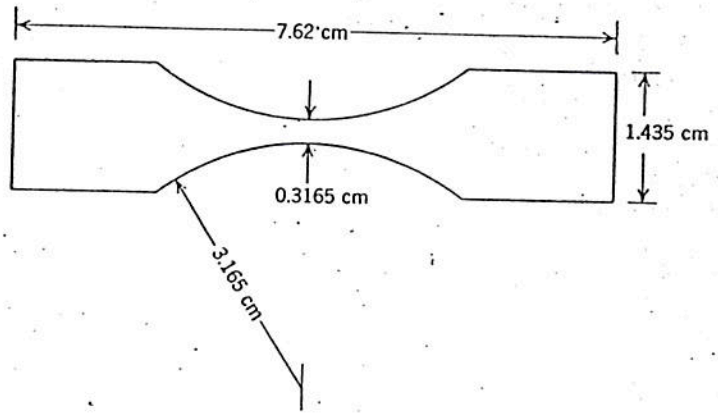

Fig. 1. The tensile test specimen profile used for the definitions in this article.

The disadvantage of the specimen shown in Figure 1 is that, because it is short and has no parallel gage length, measurements of strain are not accurate. However, three reasons may be offered for the belief that precise measurements of strain are not desirable in this type of experiment: (*1*) For studying stress–strain relations at low strains, more accurate measurements can be made in creep or stress-relaxation experiments with accurate extensometers attached to the test specimens. It is undesirable to attach an extensometer to a specimen being strained to fracture because it can cause premature failure. (*2*) At higher strains the deformation is frequently not uniform: Under such circumstances, strain measurements based on clamp separation or on extensometer readings are meaningless averages. (*3*) The point of fracture is so seriously affected by trivial experimental details, such as the specimen shape, the quality of the machining in a machined specimen, or the molding conditions in a molded specimen, that the strain at break is often not a reliable measure of the fracture behavior.

Because of these difficulties, the load–time curve obtained on the pen recorder cannot be immediately transformed into a stress–strain curve. It is, however, possible to deduce an approximate curve relating the load developed to the extension of the specimen.

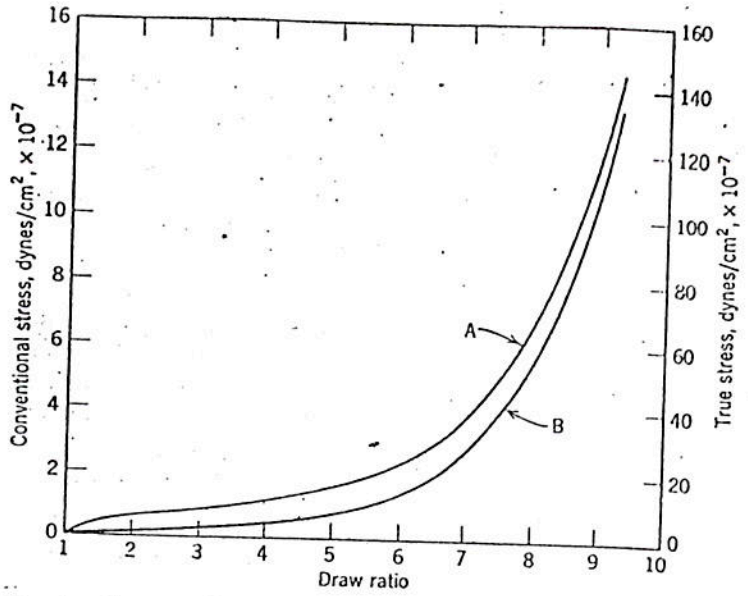

Fig. 2. A, a load–extension curve for a specimen of natural rubber at room temperature, exemplifying uniform extension. B, the true stress-strain curve deduced from A.

If it is assumed that the specimen is made from a linearly elastic material (ie, it obeys Hooke's law), it can be calculated that it deforms initially as if it were a parallel-sided strip about 25 mm long with a uniform cross section equal to that of the narrowest section. Thus one can say that the initial straining rate is about 50%/min at the standardized speed of 0.5 in./min. However, when the deformation is large, and particularly when it becomes nonuniform, the straining rate varies both with time and also along the specimen. It follows that it is preferable to quote the applied velocity of extension rather than a nominal straining rate.

The primary basis for classification in this article of the different types of fracture will therefore be the shape of the load–extension curve recorded in the tensile test described above.

Uniform Extension. Curve A in Figure 2 shows an example of the first class of behavior, which may be called "uniform extension." It is defined by the fact that the load does not fall until the point of fracture is reached.

The true stress may be arrived at by consideration of a volume element of the specimen which is sufficiently small for the stress and strain in it to be considered uniform uniaxial tension, but sufficiently large for the material to be considered homogeneous. This element has an initial length l_0 in the direction of the applied tension and a uniform cross-sectional area A_0 perpendicular to the applied tension. Under load L the length increases to l and the cross-sectional area decreases to A. Assuming that the density remains constant under stress

$$l_0 A_0 = lA \tag{1}$$

Then the following quantities may be defined:

$$\text{nominal stress} = \sigma = L/A_0 \tag{2}$$

$$\text{true stress} = \sigma_T = L/A \tag{3}$$

$$\text{engineering strain} = \epsilon = (l - l_0)/l_0 = (l/l_0) - 1 = (A_0/A) - 1 \quad (4)$$

$$\text{extension ratio or draw ratio} = R = l/l_0 = A_0/A = \epsilon + 1 \quad (5)$$

It follows that

$$\sigma_T = R\sigma \quad (6)$$

The deformation of the specimen in this test is characterized by the dependence of σ on ϵ, which is the conventional stress–strain curve, or by the dependence of σ_T on R (the true stress–strain curve). The true stress–strain curve may be computed from the conventional stress–strain curve using the relations given above. Curve B in Figure 2 shows the true stress–strain curve for the specimen whose load–extension curve is curve A.

In order to characterize the fracture of this specimen, it is necessary to know the stress–strain curve, either true or conventional, and also some quantity which specifies the point on the stress–strain curve at which fracture occurs. This quantity may be one of the following:

$$\text{the breaking stress (conventional or true)} = \sigma_B \text{ or } \sigma_{BT}$$

$$\text{the breaking strain} = \epsilon_B \text{ or } R_B$$

$$\text{the energy to break per unit volume} = \int_0^{\epsilon_B} \sigma d\epsilon$$

If the stress–strain curve and one of these quantities are known, then the others can be calculated.

The shape of the stress–strain curve of an amorphous polymer above its glass-transition temperature is well understood in general terms. Treloar (Ref. 2, Chap. 6) concludes that the non-Gaussian statistical theory "is capable of reproducing in a rather striking manner the form of the experimental curves for vulcanized rubber."

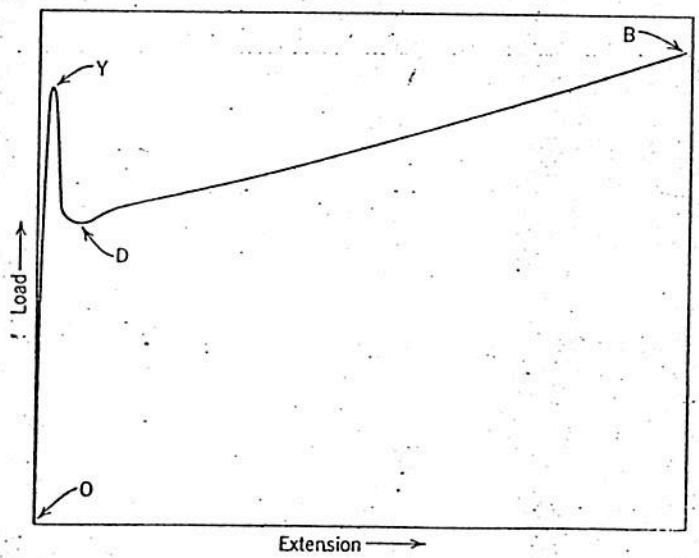

Fig. 3. Load–extension curve for a specimen of isotactic polypropylene at 20°C, exemplifying cold drawing. Key: Y, yield stress; D, drawing stress; B, point of fracture; O, origin.

cold drawing
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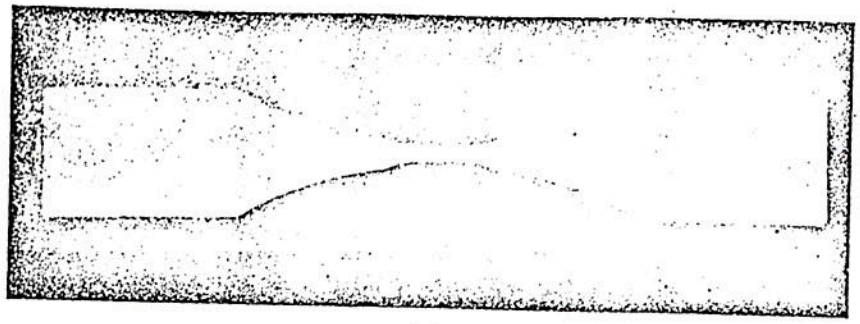
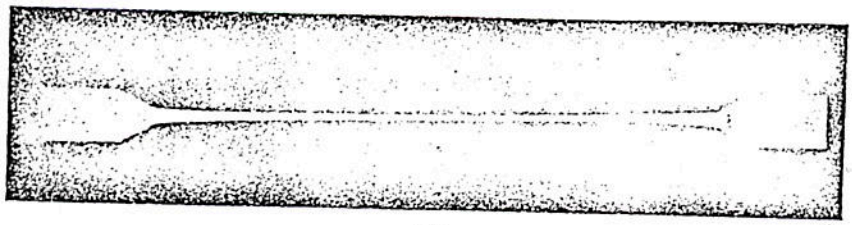


(a)



(b)

Fig. 4. (a) The beginning of cold drawing. (b) Cold drawing well developed.

The rapidly increasing stress at high extension ratios is a consequence of "the high extension of the molecules which occurs as the network approaches its maximum extension." Based on an approximate theory, the maximum extension ratio, at which the force becomes infinite, is equal to the square root of the number of links in the chain. We therefore have the picture that, as the specimen is extended in simple tension, the molecules become oriented toward the direction of the applied force. This molecular orientation makes the specimen harder and more difficult to extend. This process has a superficial similarity to strain hardening in metals, but with the significant difference that polymers, unlike metals, become harder only in the direction of the applied tension. It is therefore preferable not to call it strain hardening or work hardening since these terms can cause confusion in comparisons of the mechanical properties of polymers and metals. In the remainder of this article the process of hardening at high extensions will be called "orientation hardening" because this expression has implications of both the anisotropy of the process and its mechanism.

Cold Drawing. Figure 3 shows a typical load–extension curve for a sample of polypropylene at 20°C. This is an example of the second class of behavior, which is called cold drawing. It is defined by the characteristic shape of the load–extension curve; the load first rises, then falls, and then rises again. Figure 4 illustrates the behavior of the specimen during the cold-drawing process. During the first rise in load (OY in Fig. 3), the specimen extends uniformly. The stress at Y is called the yield stress and is denoted by σ_Y. During the fall in load (YD in Fig. 3), the specimen no longer deforms uniformly. The deformation is localized near the narrowest section and a neck forms; an example of this can be seen in Figure 4a. The nominal stress at the point D in Figure 3 is called the drawing stress and is denoted by σ_D. At the point D the load reaches a minimum, remains temporarily constant, and then starts increasing. During the second and final rise in load (DB in Fig. 3), the shoulders of the neck travel along the specimen so that an increasing volume of the specimen becomes highly de-

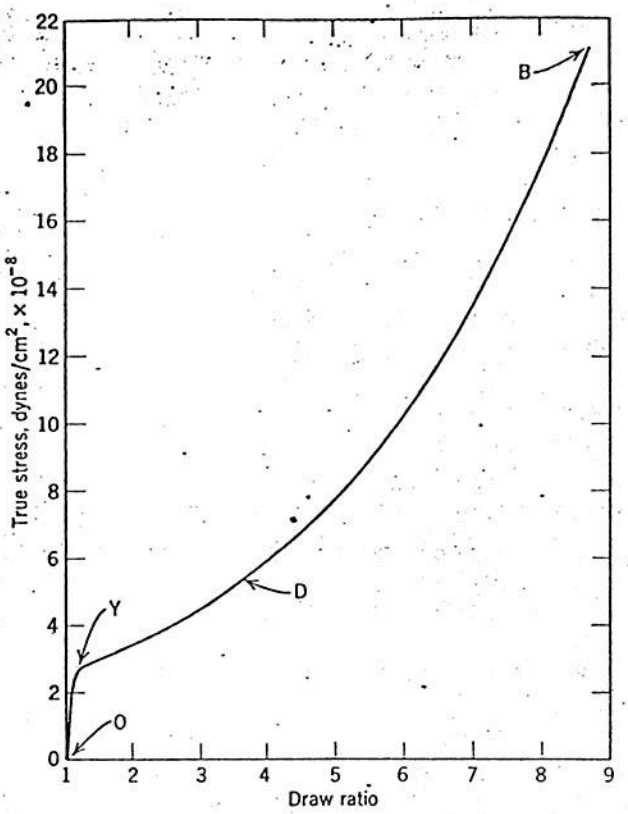


Fig. 5. The true stress-strain curve corresponding to the load–extension curve of Figure 3.

formed (Fig. 4b). The specimen fractures at the point B. The nominal stress at the point of fracture is denoted by σ_B. Cold drawing is not confined to uniaxial tension, but can be observed in, for example, polyethylene which has been fractured in biaxial flexural impact (discussed below under Biaxial Flexure). In this case, a large amount of deformation occurs near the point of impact and it can be inferred that the process under this different type of applied stress is similar in nature to the cold-drawing process in simple uniaxial tension.

When a specimen deforms by cold drawing, the load–extension curve does not immediately represent the general behavior of the material but only the behavior of a specimen of a particular shape. For example, if a 10 cm long test specimen has been extended a further 10 cm, this does not mean that the strain is 100% but only that the average strain is 100%; some of the specimen may be strained, for example, 400% and other parts less than 10%. For this reason, it is necessary to concentrate attention on a uniformly strained element such as that defined in the section on Uniform Extension.

A device is therefore needed which permits measurement of the strain of one small part of the specimen under load. Ekvall and Low (3) adopted a technique in which selenium was vacuum evaporated onto the test specimen to produce a pattern of dots; the separation of the dots was measured during extension so that R (the draw ratio or extension ratio of a selected element of the specimen) could be determined at any load

L. From the relations given in equations 1–6 it is possible to calculate the true stress, σ_T, from L and R and so construct the curve relating σ_T with R, ie, the true stress–strain curve. It will be noted that these relations involve the assumption that the specimen density remains constant during deformation. Some evidence suggests that this assumption is reasonable for low-density polyethylene but there may well be other cases where the assumption involves some error (4). Another simple and convenient device, which still involves the assumption of constant density, has been used in the author's laboratory to provide the true stress–strain curves which are given in this article. The specimen is photographed during extension and the dimensions of the smallest cross section are measured on the negatives at a series of loads. This gives a direct measure of true stress ($\sigma_T = L/A$) and, assuming constant volume, the draw ratio ($R = A_0/A$). Figure 5 shows the true stress–strain curve corresponding to the load–extension curve of Figure 3 and it can be seen that it gives a different impression of the behavior of the material. One valuable feature of concentrating attention on the true stress–strain curve rather than on the load–extension curve is that the former is more nearly a property of the material and less dependent on the arbitrary choice of specimen profile. Other advantages of the true stress–strain curve will become obvious later in the article when it is used to assist in the understanding of fracture processes.

In cold drawing, because the extensions are not uniform, the true stress–strain curve cannot be constructed from the load–extension curve without measuring the local strains. However, given the true stress–strain curve, it is possible to understand the general shape of the load–extension curve. In the region OY the extension is uniform and the distinction between the load–extension curve and the true stress–strain curve is minor and usually unimportant. Study of the details of the behavior of thermoplastics shows that between O and Y the slope of the isochronous stress–strain curve ($d\sigma/dR$) decreases as the strain increases (5). This nonlinearity has not been explained in quantitative detail but it is possible to make some points about it: (*1*) Nonlinearity and yielding cannot be caused by an increase in free volume because they also occur in shear, where there is no volume change, and in compression, where the volume decreases under stress. (*2*) Nonlinearity, yielding, and necking are not caused by heating of the material, because they can occur very slowly in creep tests when the temperature is constant. (*3*) Nonlinearity and yielding are not connected with viscous flow in the usual sense, because the deformations are recoverable. (*4*) Nonlinearity, yielding, and necking are attributable to a shear component of the applied stress (see the section on Type of Stress). (*5*) It is usually observed that the moduli of polymers decrease with increases in the temperature or the time under load. It seems plausible to suggest that, whatever the molecular mechanism may be whereby the moduli decrease, a similar mechanism is responsible for a decrease in the slope of the stress–strain curve with increasing shear stress.

It can be shown how curvature of the stress–strain curve leads to instability and necking (6). By definition

$$\sigma = \sigma_T/R$$

so that

$$\frac{d\sigma}{dR} = \frac{1}{R}\frac{d\sigma_T}{dR} - \frac{\sigma_T}{R^2} = \frac{1}{R^2}\left(R\frac{d\sigma_T}{dR} - \sigma_T\right) \qquad (7)$$

When

$$\frac{d\sigma_T}{dR} = \frac{\sigma_T}{R}$$

then

$$\frac{d\sigma}{dR} = 0$$

ie, the nominal stress, or the load developed by the specimen, is constant as the extension increases. This means that the condition for yielding is that there should be a point on the true stress–strain curve where

$$\frac{d\sigma_T}{dR} = \frac{\sigma_T}{R}$$

This condition for yielding can also be expressed by a simple construction, known as Considère's construction; the yield point is the point on the true stress–strain curve to which a tangent to the curve can be drawn from the point $\sigma_T = 0, R = 0$. In other words, if the shape of the true stress–strain curve of a material is such that a point exists on it to which a tangent can be drawn from $\sigma_T = 0, R = 0$, this point appears as a yield point in the normal tensile test. If the slope of the true stress–strain curve continues to decrease after the yield point, the specimen becomes unstable and a neck forms at the smallest cross section. It will be seen later that instabilities of this type are of fundamental importance in the understanding of fracture in polymers and it is necessary to appreciate that instability can be a straightforward consequence of the behavior expressed by the shape of the true stress–strain curve. As the strain increases, the area of the narrowest section decreases; if the increase in true stress is not sufficient to counteract the decrease in area, this section becomes more deformable than the remainder, it becomes thinner preferentially, and a neck is formed. For cases such as the natural rubber specimen of Figure 2 there is no point on the true stress–strain curve where $d\sigma_T/dR = \sigma_T/R$ so that the specimen does not become unstable but extends uniformly without yielding or necking.

Returning to the specimen used to exemplify cold drawing in Figure 3, $d\sigma_T/dR$ is always less than σ_T/R between the points Y and D and the instability persists: the neck continues to thin and the load continues to fall. At the point D, another phenomenon occurs which can be treated by an inverse form of Considère's construction (7). After the yield point the slope of the true stress–strain curve increases until there is a second point, D, to which a tangent can be drawn from the point $\sigma_T = 0, R = 0$ (Fig. 6). After this point, $d\sigma_T/dR$ becomes greater than σ_T/R; this means that the material in the thinnest part of the neck has become so much harder and less deformable than the material in the adjacent parts of the specimen that this compensates for the reduction in cross-sectional area. Consequently, the neck restabilizes and stops thinning; the adjacent parts deform instead and the shoulders travel along the specimen until, at point B, the specimen fractures. The nominal stress at the point D is called the drawing stress, σ_D, and the draw ratio at the point D is called the natural draw ratio, R_N. R_N is important in practice because it is the smallest uniform draw ratio which can be given to a fiber or a film when it is oriented by cold drawing rather than by uniform extension.

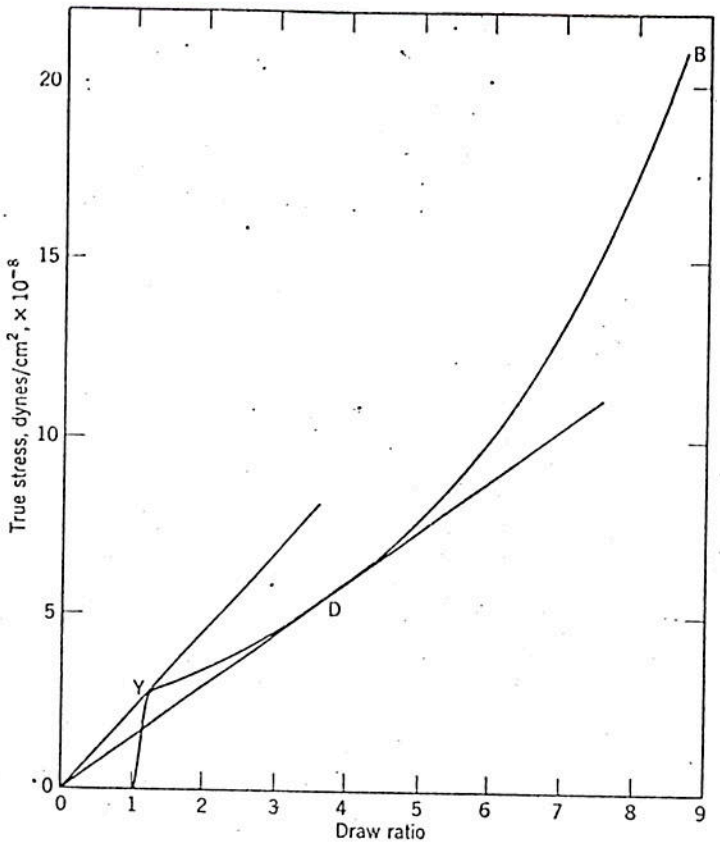


Fig. 6. An example of the extension of Considère's construction for a typical polymer.

To summarize, the condition for necking is that the slope of the true stress–strain curve decreases until it becomes less than σ_T/R and the condition for cold drawing is that this slope should subsequently increase until it becomes again greater than σ_T/R. Thus necking and cold drawing are both consequences of the shape of the true stress–strain curve. The general shape of the curve can be explained in principle though not in quantitative detail. The reduction in slope before yield has already been attributed to the effect of the shear component of the stress in increasing deformability by the same mechanism whereby the modulus is decreased by increasing temperature or time under load. The increase in slope after yield occurs as the molecules become oriented in the direction of the applied tensile force and has a marked similarity to orientation hardening in elastomers (compare Figs. 2 and 5). It seems probable, therefore, that orientation hardening during cold drawing is explicable in the same way as during uniform extension, that is, it is a consequence of the high extension of the molecules. For a cold-drawing plastic that is not chemically crosslinked, it is not possible to define a quantity analogous to the number of links in the chain as used in the statistical theory of rubber elasticity (2), but it is likely that the material deforms as a network with physical crosslinks such as entanglements and intermolecular bonds.

In order to characterize the fracture of a test specimen which has failed by cold drawing, it is necessary to know the true stress–strain curve and also some quantity

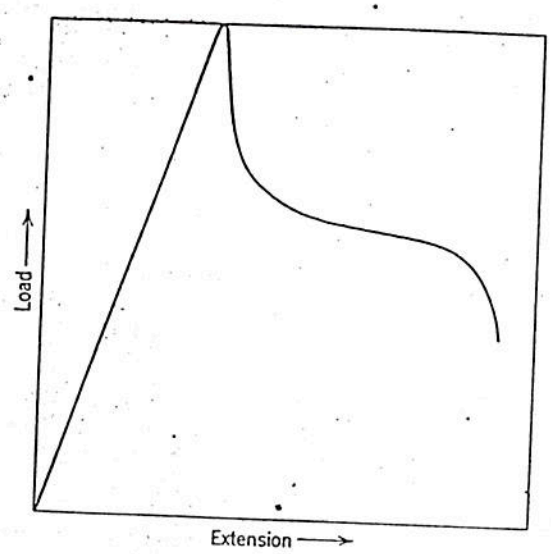


Fig. 7. Load-extension curve for a sample of rigid poly(vinyl chloride) at 20°C and 0.76 cm/sec exemplifying necking rupture.

which specifies the point on the curve at which fracture occurs. This quantity may be one of the following: (a) true breaking stress = σ_{BT} or (b) draw ratio to break = R_B. Either quantity may be calculated from the true stress–strain curve if the other is known. Alternatively, one can measure the area beneath the load–extension curve and divide it by the volume, in order to obtain the energy, or work, to break per unit volume, although this quantity is likely to be trivially dependent on the specimen profile. See also DEFORMATION; VISCOELASTICITY.

Necking Rupture. Figure 7 shows an example of the third class of behavior which may be called "necking rupture" because the specimen necks and then breaks without restabilization of the neck. It is defined by the shape of the load–extension curve; the load first rises and then falls until the specimen breaks. Necking rupture can occur under types of stress other than uniaxial tension; for example, it has been observed in fracture on impact under biaxial flexure. Frequently, though not invariably, specimens which fail by necking rupture whiten in the neck. This is usually attributed to the occurrence of very small voids, presumably as a consequence of the triaxial tensile component of the applied tension (see the section on Mechanism of Fracture); the scattering of light at the microvoids is the cause of the white appearance. It is not ordinarily easy to demonstrate the existence of these microvoids in a photomicrograph because they are below the limit of resolution but Figure 8 shows an example of particularly coarse-textured voiding; the photograph was taken by transmitted ordinary light (S) and the voids appear black because the light is scattered.

As in cold drawing, the nominal yield stress, σ_Y, can be determined from the load–extension curve; yield, instability, and necking are consequences of the downward curvature of the true stress–strain curve. The significant difference from cold drawing is that in necking rupture there is no point D at which $d\sigma_T/dR$ becomes equal to σ_T/R for a second time; the neck never restabilizes because the specimen breaks before the orientation hardening is sufficient. The fracture of a test specimen which has failed by necking rupture is characterized in the same way as the fracture of a specimen which

Fig. 8. Microvoids in a neck.

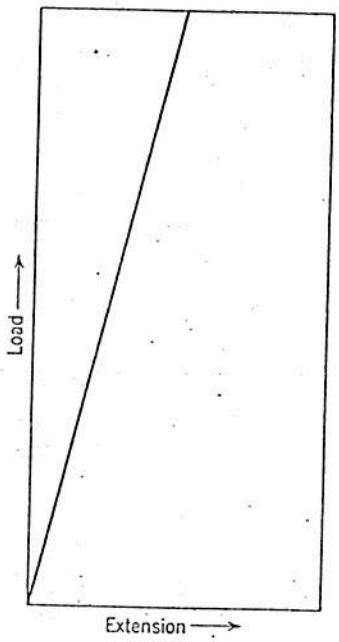


Fig. 9. Load–extension curve exemplifying brittle fracture.

has failed in cold drawing (see the end of the section on Cold Drawing). The energy to break per unit volume is particularly likely to be trivially dependent on the specimen profile.

Brittle Fracture. Figure 9 shows a schematic load–extension curve of a polymer failing by the fourth class of behavior to be discussed here, brittle fracture; this type of failure is characterized by a curve in which the load rises until the specimen breaks. It is perhaps necessary to consider how to distinguish between brittle fracture and uniform extension since in both cases the load rises monotonically to fracture. One pos-

sible approach is to say that there is no difference; Cottrell and Kelly (9) seem to take this view, since they refer to rubber as a brittle material, presumably because of the absence of residual or plastic deformation. However, the practical differences in behavior between a rubber and a brittle plastic are so extreme that it is necessary to treat their fracture behavior as belonging to separate classes; there is never any serious difficulty in distinguishing them. As an arbitrary dividing line, one could say that, for a fracture to be brittle, the load must rise monotonically to break and the strain at break must be low, probably less than 20%.

Because the strains in brittle fracture are usually less than 5%, the distinction between the conventional stress–strain curve and the true stress–strain curve is not ordinarily important. The point of fracture may be defined by any one of the following quantities: (a) the breaking stress = the brittle strength = σ_B; (b) the breaking strain = the elongation to break = ϵ_B; or (c) the energy to break per unit volume = E_B = the area under the stress–strain curve = area under the load–extension curve divided by the specimen volume.

If the material obeys Hooke's law all the way to the point of fracture, then it is sufficient to know the relevant modulus, Y, and just one of the three above quantities defining the point of fracture. For example, if the brittle strength, σ_B, is known, then ϵ_B and E_B can be calculated (eqs. 8 and 9). If the material does not obey Hooke's

$$\epsilon_B = \sigma_B/Y \tag{8}$$

$$E_B = \sigma_B^2/2Y \tag{9}$$

law, then it is necessary to know the complete stress–strain curve. Then, if the brittle strength is known, ϵ_B follows immediately and

$$E_B = \int_0^{\epsilon_B} \sigma d\epsilon \tag{10}$$

The yield stress can be measured in a tensile test only when the specimen fails in necking rupture, or cold draws, but not when it fails in brittle fracture. However, it is possible to assess what the yield stress would have been, if the specimen had not been brittle, by extrapolation from experimental conditions where yielding is observed, such as in compression tests (p. 328), in tensile tests at higher temperatures (p. 317), or on oriented specimens (p. 342). This permits a useful definition of brittle fracture as fracture which occurs before the average stress reaches the yield stress; for this definition, the condition for brittle fracture is $\sigma_B < \sigma_Y$. Both the breaking stress (brittle strength) and the yield stress are sometimes called the tensile strength. This usage is loose and liable to lead to confusion because σ_B and σ_Y are measures of different material properties and are affected in different ways by changes in test conditions and material structure. It is important to distinguish between them and, when a tensile strength is given, to state whether it is a yield stress or a breaking stress (brittle strength).

The fracture surface of a tensile specimen that has broken in a brittle manner has a characteristic appearance, illustrated in Figure 10. Leeuwerik (10) has discussed the appearance of fracture surfaces and, with acknowledgments to Smekal, distinguishes four different regions: (a) the primary fracture source, P; (b) the mirror, so called because of its smoothness; (c) the transition region; and (d) the rough region.

As the primary crack advances, secondary cracks may originate; interactions between these cracks create hyperbolic or parabolic fracture traces. As the crack progresses further, its velocity increases and the surface becomes rougher. It is sometimes useful to be able to locate the primary fracture source and this can usually be done by finding the mirror region, which is prominent in Figure 10.

Necking Rupture of the Second Kind. Figure 11 shows a load–extension curve for a sample of poly(methyl methacrylate) at 180°C; this specimen develops a neck which becomes very thin before fracture (Fig. 12). This failure could be classed

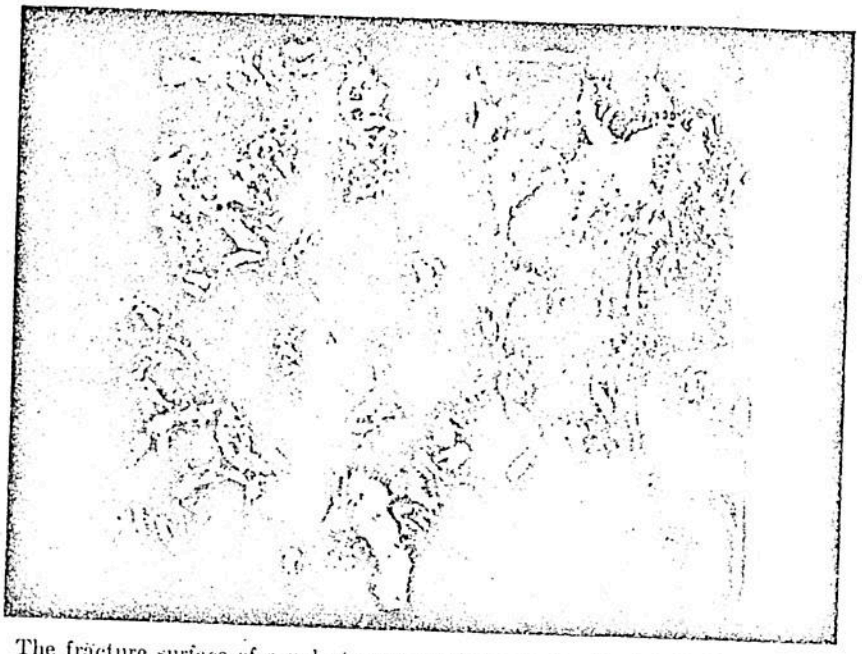


Fig. 10. The fracture surface of a polystyrene specimen broken by brittle fracture, showing the mirror region and the rough region.

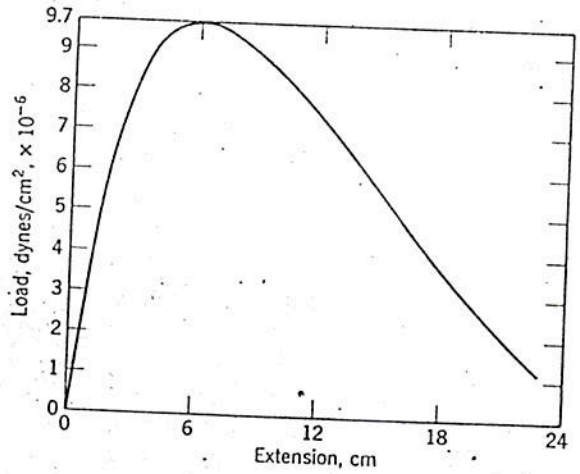


Fig. 11. Load–extension curve for a specimen of poly(methyl methacrylate) at 180°C, exemplifying necking rupture of the second kind.

Fig. 12. The development of a neck of the second kind.

as necking rupture, since the load rises and then falls again before fracture, but this would confuse two different phenomena. In order to distinguish between them, the behavior illustrated in Figure 11 will be called "necking rupture of the second kind." The two types of necking rupture occur for the same polymer in separate temperature regions; the distinction between them will be clarified when the effect of temperature is discussed under Factors Affecting Fracture.

Test Methods Other than the Tensile Test

It was pointed out in the section on Classification and Definitions that the only common test which can be used to study fracture over the whole range of polymer behavior is the simple tensile test and for this reason it was used as a basis for definition and classification. However, a tensile test is not always the best method for making measurements of specific physical properties; various different tests are better adapted for studying fracture and measuring the relevant quantities in particular regions of behavior. Some of these methods will be described and discussed in this section.

Flexure. A tensile testing machine can be adapted for testing specimens in flexure. For example, a specimen may rest on two supports and be bent by loading a third, central, support. This arrangement is known as three-point loading; flexural tests can also be performed in four-point loading or cantilever loading. The curve of load against deformation can be obtained in the same way as the load–extension curve in a tensile test. The stress and strain vary from point to point in the specimen but, with certain assumptions (11), the maximum stress and strain can be calculated from standard formulas. For example, consider a specimen with a uniform rectangular cross section bent in three-point loading. Suppose L is the load on the specimen, l is the span (distance between outer supports), b is the specimen width, d is the specimen thickness, and δ is the deformation or deflection at the center. Then the maximum tensile stress, σ, which occurs on the outer surface of the specimen opposite the central loading point, is given by equation 11 and the corresponding maximum strain,

$$\sigma = 3Ll/2bd^2 \tag{11}$$

ϵ, at the same point is given by equation 12. The strain energy per unit volume at

$$\epsilon = 6d\delta/l^2 \tag{12}$$

the point of maximum stress is given by equation 13. This expression is the area

$$E = \sigma\epsilon/2 = 9L\delta/2lbd \text{ or } (\tfrac{1}{2}L\delta)/(\tfrac{1}{9}lbd) \tag{13}$$