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SHORT COMMUNICATIONS

ROUND-OFF ERRORS IN RICHARDSON'S EXTRAPOLATION METHOD*

S.YA. VILENKIN AND A.N. KALASHYAN

The effect of round-off error in Richardson's method in calculations made on a computer with a small wordlength is suppressed by the introduction of an additional mesh point.

In /1/ it is assumed that the round-off error is negligibly small and it is ignored when choosing the weighting factors. This assumption is unacceptable when the increase in accumulation of the round-off errors significantly affects the accuracy of the solution. The small wordlength of a computer is usually the source of this increase.

The worsening of the accuracy of the solution at a sequence of meshes when the integration step is increased (see Figs.1 and 2 where *1* is the error of the solution by the Crank-Nicholson scheme, and *2, 3, 4* are the errors of the extrapolated solutions $U^H(2)$, $U_{31}{}^H(2)$, $U_1{}^H(2)$ respectively) is explained by the fact that the total round-off error $\varepsilon_h(x)$ is inversely proportional to the integration step /2, 3/. Thus to determined the weighting factors the round-off errors must also be taken into account.

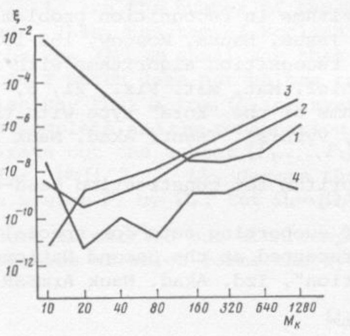

Fig.1

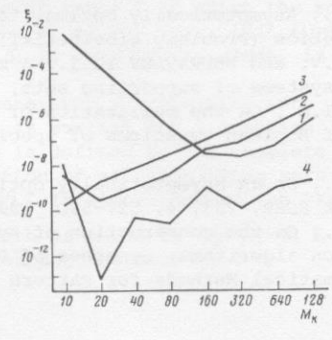

Fig.2

We shall fix an arbitrary point x in $\overline{\Omega}_h$. On the basis of Theorem 2.1 in /1/ at this node the expression

$$u^h(x)=u(x)+\sum_{j=1}^{l}h^{ij}v_j(x)+\eta^h(x), \tag{1}$$

is obtained where

$$l=\begin{cases} m, & \text{if } i=1,\\ [m/2], & \text{if } i=2\end{cases}$$

($i=2$ corresponds to the case when the regular part of (1) contains only even degrees of h), $v_j(x)$ does not depend on h and the remaining term satisfies the estimate

$$|\eta^h(x)|\leqslant\|\eta^h(x)\|_{\Omega_h}\leqslant ch^{m+\beta},$$

where c is some constant and β is directly related to the approximation method of differential operators with difference relations (see /1, p.23).

For $h_1>\ldots>h_{l+2}>0$ let

$$\overline{\Omega}_H=\bigcap_{k=1}^{l+2}\overline{\Omega}_{h_k}\neq\varnothing.$$

Let u^{h_k} denote the solution for each h_k. Taking into account the round-off error for every value of the parameter h_k we have the unique solution

$$\tilde{u}^{h_k}=u^{h_k}+\varepsilon_{h_k},$$

where $\varepsilon_{h_k}\approx\omega_k/h_k{}^p$, $p>0$ and ω_k are random independent equally distributed quantities with expectation ω and finite variance. Finding ω_k for each k is quite a difficult problem. We will therefore

**Zh.vychisl,Mat.mat.Fiz.*,27,1,128-130,1987

consider $\varepsilon_{h_k}=\omega/h_k^p$. Such a representation is justified as indicated by the numerical results.

The sequence of steps h_k in the extrapolation method is given as a function of the basic step h_1, i.e.

$$h_k=h_1/\varphi_k, \quad k=1, 2, \ldots, l+2,$$

where $1=\varphi_1<\ldots<\varphi_{l+2}$. For example $\varphi_k=k$ or $\varphi_k=2^{k-1}$.

Let $\gamma_k^{(pi)}$ be the solutions of the system of equations

$$\sum_{k=1}^{l+2} \gamma_k^{(pi)} \frac{\omega}{h_k^p}=0, \quad \sum_{k=1}^{l+2} \gamma_k^{(pi)}=1, \quad \sum_{k=1}^{l+2} \gamma_k^{(pi)} h_k^{ij}=0, \quad j=1,2,\ldots,l. \tag{2}$$

For example $(k=1, 2, \ldots, l+2)$,

$$\gamma_k^{(1,1)}=(-1)^{k+1-l}\varphi_k^l\Big(\sum_{j=1}^{l+2}\varphi_j-\varphi_k\Big)\Big[\prod_{j-1}^{k-1}(\varphi_k-\varphi_j)\prod_{j=k+1}^{l+2}(\varphi_j-\varphi_k)\Big]^{-1},$$

$$\gamma_k^{(2,1)}=(-1)^{k+1-l}\varphi_k^l\Big(\sum_{j=1}^{l+2}\varphi_j^2+\sum_{s=1}^{l+1}\varphi_s\sum_{j=s+1}^{l+2}\varphi_j-\varphi_k\sum_{j=1}^{l+2}\varphi_j\Big)\times$$

$$\Big[\sum_{j=1}^{l+2}\varphi_j\prod_{j=1}^{k-1}(\varphi_k-\varphi_j)\prod_{j=k+1}^{l+2}(\varphi_j-\varphi_k)\Big]^{-1},$$

$$\gamma_k^{(1,2)}=(-1)^{k+1-l}\varphi_k A(jk)\Big\{\sum_{s=1}^{k-1}(-1)^{s-l+1}\Big(1-\frac{\varphi_s^2}{\varphi_k^2}\Big)A(j,s)+$$

$$\sum_{s=1}^{l+2}(-1)^{s-l}\Big(\frac{\varphi_s^2}{\varphi_k^2}-1\Big)A(j,s)\Big\}\Big[\sum_{s=1}^{l+2}(-1)^{s-l}A(j,s)\Big]^{-1},$$

where

$$A(j,s)=\varphi_s^{2l+1}\Big[\prod_{j=1}^{s-1}(\varphi_s^2-\varphi_j^2)\prod_{j=s+1}^{l+2}(\varphi_j^2-\varphi_s^2)\Big]^{-1},$$

$$\gamma_k^{(2,2)}=(-1)^{k+1-l}\varphi_k^{2l}\Big(\sum_{s=1}^{l+2}\varphi_s^2-\varphi_k^2\Big)\Big[\prod_{s=1}^{h-1}(\varphi_k^2-\varphi_s^2)\prod_{s=k+1}^{l+2}(\varphi_s^2-\varphi_k^2)\Big]^{-1},$$

and for $p=1, 2$, $i=1, 2$ we form the linear combination

$$U^H(x)=\sum_{k=1}^{l+2}\gamma_k^{(pi)}\tilde{u}^{h_k}(x).$$

The following assertion holds.

Suppose that for the mesh domains Ω_{h_k} with parameters $h_1>\ldots>h_{l+2}>0$ the conditions of Theorem 2.1 in /1/ are satisfied with a uniform norm and $\varepsilon_{h_k}=\omega/h_k^p$, $p=1, 2$. Then for the extrapolated solution $U^H(x)$ the estimate

$$\max|U^H(x)-u(x)|\leqslant d\Big(\sum_{k=1}^{l+2}h_k\Big)^{m+\beta}, \tag{3}$$

is valid, where u is the solution of the original problem and d is a constant that does not depend upon h_k.

Proof. Taking into account expansion (1) and the fact that $\gamma_k^{(pi)}$ can be found from system (2), we obtain

$$|U^H(x)-u(x)|\leqslant c\max_{1\leqslant k\leqslant l+2}|\gamma_k^{(pi)}|\sum_{k=1}^{l+2}h_k^{m+\beta}.$$

Since the function φ_k is monotonically increasing we can always find an $a>0$ such that $\varphi_{k+1}/\varphi_k\geqslant 1+a$, $k=1, 2, \ldots, l+1$.

It is not difficult to show that

$$|\gamma_k^{(p1)}|\leqslant p(l+2)\Big(\frac{1+a}{a}\Big)^{l+1}\varphi_{l+2}, \quad |\gamma_k^{(p2)}|\leqslant(l+1)\Big(\frac{1+\bar a}{\bar a}\Big)^{l+1}\varphi_{l+2}^2,$$

where $\bar a=a^2+2a$. Thus

$$\max|U^H(x)-u(x)|\leqslant d\Big(\sum_{k=1}^{l+2}h_k\Big)^{m+\beta}.$$

Numerical results. Calculations were carried out for an example taken from /1/:

$$u'+xu=(x^2+x+1)\exp(x), \quad x\in(0, 2), \quad u(0)=0,$$

the solution of which is the function $u(x)=\exp(x)$.

The calculations were carried out on a 32-bit computer (of which 24 bits are assigned to a number's mantissa) according to the Crank-Nicholson scheme

$$u^h(x_{j+1})=\frac{2-ha(x_{j+1/2})}{2+ha(x_{j+1/2})}u^h(x_j)+\frac{2h}{2+ha(x_{j+1/2})}f(x_{j+1/2}),\qquad u(x_0)=0, \tag{4}$$

where $a(x)=x$, $f(x)=(x^2+x+1)\exp(x)$.

For problem (4) the assumptions that allow the extrapolation to be carried out in two or more steps h_k are satisfied, and for it the regular part of expansion (1) contains only even degrees of h and $\varepsilon_h=\omega/h$. This determines the choice of weighting factors $\gamma_k^{(1,2)}$.

Initially we shall solve some of the difference problems (4) with steps $h_k=1/M_k$. In finding u^{h_k} at the point $x=2$ we construct an extrapolated solution in two $(U^H(2))$ and three $(U_{31}{}^H(2)$ for $\varphi_k=k$ and $U_{32}{}^H(2)$ for $\varphi_k=2^{k-1})$ approximations with weighting factors given in /1/.

With the coefficients $\gamma_k^{(1,2)}$ we shall find the extrapolated solution $U_1{}^H(2)$ $(\varphi_k=k)$ and $U_2{}^H(2)$ $(\varphi_k=2^{k-1})$ in three approximations.

We construct a graph of the dependence

$$\xi_k(M_k)=|u^{h_k}(2)-u(2)|$$

of the points of the mesh $\omega_{h_k}=\{x_j=jh_k,\ j=0,\ 1,\dots,M_k\}$ on the number M_k at the point $x=2$. This graph is plotted in logarithmic coordinates in Figs.1 and 2. Analogous graphs are constructed for the extrapolated solutions. From Figs.1 and 2 it is clear that the proposed method for determining the weighting factors actually allows the accuracy of the solutions to be increased for calculations on computers with a small wordlength.

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DIFFERENCE SCHEMES FOR NON-AUTONOMOUS STIFF SYSTEMS OF ORDINARY DIFFERENTIAL EQUATIONS*

P.D. SHIRKOV

The properties of Rosenbrock schemes with complex coefficients and implicit Runge-Kutta schemes for the numerical solution of Cauchy problems for stiff systems of ordinary differential equations are investigated. The model problem chosen is a linear non-autonomous system. A one-stage second-order approximation Rosenbrock scheme is constructed, which is monotonic and damped to second order when applied to non-autonomous linear systems. This scheme enables one to compute with large step-size outside the boundary layer and can be used for the numerical solution of a broad range of stiff problems which are nearly linear (including, e.g., computations of transients in electrical circuits).

Introduction.

A considerable number of difference schemes have been proposed for the numerical integration of the Cauchy problem for stiff systems of ordinary differential equations (o.d.e.)

$$d\mathbf{u}/dt=\mathbf{f}(t,\mathbf{u}),\qquad \mathbf{u}(0)=\mathbf{c}, \tag{1}$$

where $\mathbf{u}=(u_1,\dots,u_m)^T$, $\mathbf{f}=(f_1,\dots,f_m)^T$, $\mathbf{c}=(c_1,\dots,c_m)^T$ (see e.g. /1-3/ and the references cited therein). This profusion often complicates the already difficult task of selecting a numerical algorithm to solve specific applied problems.

In order to ensure good qualitative and quantitative behaviour of difference solutions in nearly linear stiff problems. It is best to use schemes that meet the following conditions

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